DIRECT-CURRENT MACHINERY

A Text-book on the Theory and Performance of Generators and Motors

BY

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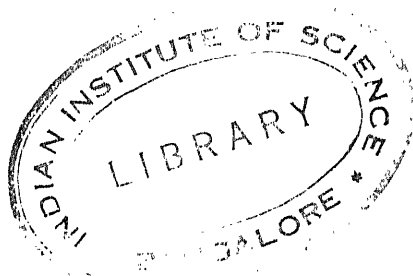
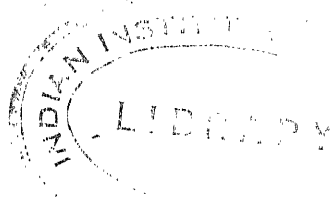
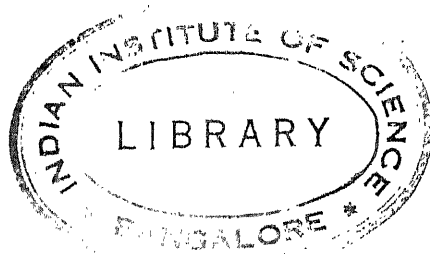

PREFACE

THE author knows of no text-book on direct-current machinery which gives a thorough treatment of the theory and performance of such machines, without at the same time going into the details of design to such an extent as to be confusing to the mind of the ordinary undergraduate student. It is with the desire to provide such a text that this book is offered to the teachers and students of Electrical Engineering.

In the author's opinion, a course in the design of electric machinery is out of place in the undergraduate curriculum. A student of electrical engineering should, however, acquire a thorough knowledge of the theory of such machines and of the basic principles upon which their design depends. Only to this extent, therefore, is the matter of design gone into in this book. Those interested in the details of design are referred to such books as Arnold's *Die Gleichstrommaschine* or Langsdorf's *Principles of Direct-current Machines*.

Although but few graduates in electrical engineering ever have occasion to design a generator or a motor, the majority of them, at some time in their careers, will be called upon to operate or test such machines, or to pass upon their suitability for a particular service. The greater part of this book is therefore devoted to the performance, application, and testing of the various types of direct-current generators and motors.

The graphical method, given in Chapters VI and VII, for determining the performance characteristics of generators and motors, was developed by the author several years ago, with the assistance of Mr. H. R. West. This method, although in its simple form only approximate, brings out in a readily understood manner, the dependence of the performance of the machine upon the nature of its electric and magnetic circuits.

In the chapter on Commutation the attempt has been made to treat this subject in an understandable, and yet rigorous, manner. The nature of the effects produced by the various factors involved,

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DIRECT-CURRENT MACHINERY

CHAPTER I

DEFINITIONS AND FUNDAMENTAL PRINCIPLES

1. Introduction.—The treatment of direct-current machinery given in this book assumes a knowledge of the elementary principles of electricity and magnetism. For convenience of reference a summary of these elementary principles, as developed in detail in the author's *Electricity and Magnetism for Engineers*, Vol. I *, is given in this chapter.

2. Power.—The rate, with respect to time, at which work is done, or at which energy is transformed or transferred, is called power. The power input to a machine is the rate at which energy is supplied to it from some other source, and the power output of a machine is the rate at which this machine supplies energy to some other machine or apparatus.

In the English system of units the fundamental unit of power is the horsepower, which is the rate at which work must be done to raise 1 pound 550 feet in 1 second. In the metric system the fundamental unit of power is the watt, which is equal to 10^7 ergs per second. The power output of electric generators and the power input to electric motors are usually expressed in kilowatts, a kilowatt being 1000 watts.

$$1 \text{ horsepower} = 0.746 \text{ kilowatt (approximately } \tfrac{3}{4})$$

$$1 \text{ kilowatt} = 1.341 \text{ horsepower (approximately } \tfrac{4}{3})$$

The energy corresponding to 1 kilowatt for 1 hour is called a kilowatt-hour.

* Published by the McGraw-Hill Book Co., New York City.

Power is also equal to force times linear speed, or to torque times angular speed. Let F=force in pounds, S=linear speed in feet per minute, T=torque in pound-feet, N=number of revolutions per minute, P=horsepower. Then

$$P=\frac{FS}{33,000} \quad (1)$$

$$P=\frac{2\pi NT}{33,000}=\frac{NT}{5252} \quad (2)$$

3. Efficiency and Losses.—In any machine or apparatus for transforming energy from one form into another a certain amount of energy is always converted into forms which cannot be readily utilized. In general this useless energy appears as heat energy. The difference between the power input and the useful power output is called the power loss.

The ratio of the useful power output P_o to the power input P_i, for any given load on the machine, is called the efficiency of the machine at this load. This ratio is usually expressed as a percentage, viz.,

$$\text{Percent Efficiency}=100\frac{P_o}{P_i} \quad (3)$$

4. Electric Current.—An electric current is a flow of electricity. By the "intensity" of an electric current is meant the rate at which positive electricity flows through any cross-section of the path of flow, plus the rate at which negative electricity flows through this same cross-section in the opposite direction. The intensity of an electric current is commonly referred to simply as the current.

By the direction of an electric current is meant the direction in which the positive electricity in the stream moves.

The practical unit of electric current is the **ampere.** The ampere is defined as that current which, when caused to flow through a solution of silver nitrate in water, in accordance with certain standard specifications, deposits silver on the cathode of a voltameter at the rate of 0.00111800 gram per second.

An electric current which does not vary appreciably with time is called a **direct current.**

The **quantity of electricity** which flows through any cross-section of the path of an unvarying electric current in an interval of time t is equal to the product of the intensity I of this current by the time t, viz.,

$$Q = It \tag{4}$$

When I is expressed in amperes and t in seconds, the quantity Q is in **coulombs.**

5. Electric Resistance.—The flow of electricity through any substance is always accompanied by a production of heat energy in this substance. This phenomenon may be looked upon as due to a frictional resistance offered by the substance to the flow of electricity through it. Substances which offer a relatively small resistance to the flow of electricity through them are called **conductors,** and those which offer a very high resistance to the flow of electricity through them are called **insulators.**

When electricity flows through a substance, entering it at a fixed terminal A and leaving it at a fixed terminal B, the rate at which heat energy is developed in this substance is found to vary as the square of the strength of the current through it. The quotient of the rate P_h at which heat energy is developed in the substance divided by the square of the strength I of the current through it, namely, the quotient

$$r = \frac{P_h}{I^2} \tag{5}$$

is called the **electric resistance** of the given substance between the two terminals A and B.

If the temperature of the substance is kept constant this quotient is constant, independent of the strength I of the current.

The relation expressed by equation (5), which also may be written

$$P_h = rI^2 \tag{5a}$$

is known as "Joule's Law."

The practical unit of electric resistance, corresponding to P_h in watts and I in amperes, is called the **ohm,** and is the resistance of a column of mercury 106.300 centimeters in length, of uniform cross-section throughout, having a mass of 14.4521 grams, the

mercury being at 0° C. This column of mercury is approximately 1 square millimeter in cross-section.

The resistance of a wire, or straight bar of uniform cross-section, to the flow of a direct current through it parallel to its axis, varies directly as the length and inversely as the cross-section of the wire or bar, viz.,

$$r=\rho\frac{l}{S} \tag{6}$$

where l is the length, S the cross-section, and ρ a quantity which depends upon the material of the bar and its temperature, and upon the units employed. This quantity ρ is called the **resistivity** of the material. For the values of the resistivity of various substances see any electrical engineer's handbook.*

The reciprocal of the resistivity of a substance is called the **conductivity** of this substance, and is usually represented by the symbol γ, viz.,

$$\gamma=\frac{1}{\rho} \tag{7}$$

The resistance of metallic conductors (with the exception of certain special alloys) increases with increase of temperature. The relation between resistance and temperature for such substances may be expressed, to a close degree of approximation, by the formula

$$r=r_o(1+\alpha t) \tag{8}$$

where r_o is the resistance of the conductor at 0° C., r its resistance at any other temperature t° C., and α a constant, called the **temperature coefficient** of the conductor. The value of the temperature coefficient of a conductor depends upon the material of the conductor.

The value of α for copper of 100 percent conductivity (i.e., for copper having a resistivity of 1.7241 microhm-centimeters at 20° C.) is 0.00427. Ordinary copper wire (which has a percent

* In this country there are three comprehensive electrical engineer's handbooks in general use, viz., *Pender's Handbook for Electrical Engineers* (John Wiley and Sons), the *Standard Handbook for Electrical Engineers* (McGraw-Hill Book Co.) and *Foster's Electrical Engineers' Pocket Book* (D. Van Nostrand Co.)

conductivity ranging from about 96 to 100 percent) has a temperature coefficient of approximately the same value.

Taking for α the value 0.00427, and applying equation (8), the ratio of the resistance r_1 of a coil of copper wire at $t_1°$ C. to its resistance r at $t°$ C. may be written

$$\frac{r_1}{r}=\frac{234.5+t_1}{234.5+t} \tag{8a}$$

From this formula the temperature t_1 of a coil may be readily determined by measuring its resistance r_1 at this temperature, provided its resistance r at some other temperature t is known.

6. Difference of Electric Potential and Electromotive Force.— The flow of electricity is always accompanied by the transformation or transfer of energy. For example, when a wire is connected to the two terminals of a dry cell, an electric current is established in the wire and in the cell, and there is a loss of chemical energy by the cell and a production of heat energy in the wire.

From the principle of the conservation of energy, the chemical energy lost by the cell may be looked upon as converted, within the cell, into some new form, which may be called **electric energy.** Similarly, the heat energy developed in the wire may be looked upon as due to the transfer of this energy (or a part of it) from the cell to the wire, where it is converted into heat energy.

In general, any portion of an electric circuit in which there exists a force which produces, or tends to produce, a flow of electricity may be looked upon as a **source of electric energy,** and any portion of an electric circuit in which there exists a force which opposes the flow of electricity through it may be looked upon as a **receiver of electric energy.**

Since every substance offers an opposition to the flow of electricity through it, due to its electric resistance, the conductors which form the path of a current through a source of electric energy are themselves receivers of electric energy to the extent of the heat energy developed in them by the current. The electric power *output* P_o from any portion of an electric circuit is therefore less than the electric power P *developed in* this portion

of the circuit by an amount rI^2, where r is the resistance of this portion of the circuit and I the current in it, viz.,

$$P_o = P - rI^2 \tag{9}$$

When the electric energy transferred to a receiver is all converted into heat energy in the conductors which form this portion of the circuit, due to their electric resistance, the power input to the receiver is simply rI^2, where r is the total resistance of the conductors which form the path of the current through the receiver, and I is the current.

In general, however, the electric energy input to a receiver may, in part at least, be converted into other forms than heat energy, as illustrated by the chemical energy produced in a storage battery which is being charged, or by the mechanical work done by the armature of an electric motor. The rate P at which these other forms of energy are developed is equal to the total electric power input P_t to the receiver less the power rI^2 dissipated as heat, viz.,

$$P = P_t - rI^2$$

or

$$P_t = P + rI^2 \tag{10}$$

Compare this *receiver* equation (10) with the *source* equation (9).

The electric power *output of a source* of electric energy, or the electric power *input to a receiver* of electric energy, *per unit current* through it, is called the **difference of electric potential** between the terminals of this source or receiver, and is usually represented by the symbol V. Difference of electric potential is strictly analogous to the difference of hydraulic pressure in a stream of water.

That terminal of a *source* of electric energy from which the current leaves is said to be at the higher potential. In a *receiver*, on the contrary, that terminal from which the current leaves is at the lower potential. The terminal which is at the higher potential is called the **positive terminal,** whether it be the terminal of a source or of a receiver, and the terminal at the lower potential is called the **negative terminal.**

The *total* electric power developed *within* a source of electric energy *per unit current* through it, is called the **electromotive force** of this source. Similarly, the electric power which is converted in a receiver of electric energy into *any other form than*

rI^2 *heat,* divided by the current through it, is called the electromotive force of the receiver.

In a source of electric energy the electromotive force is said to act in the direction of the current, and in a receiver the electromotive force is said to act in the direction opposite to that of the current. The electromotive force of a receiver is therefore commonly referred to as a **back electromotive force.**

Electromotive force is usually represented by the symbol E.

Using the same notation as in equations (9) and (10), the definitions just given may be expressed as follows:

	Source	Receiver
Difference of potential between terminals..	$V=\frac{P_o}{I}$	$V=\frac{P_i}{I}$
Electromotive force	$E=\frac{P}{I}$	$E=\frac{P}{I}$

Consequently, dividing each of the two equations (9) and (10) by I, there result the two fundamental relations:

For a source of electric energy, $$V=E-rI \tag{11}$$

For a receiver of electric energy, $$V=E+rI \tag{12}$$

That part of the difference of electric potential between the terminals of any portion of an electric circuit which is equal to the product rI, where r is the resistance of this portion of the circuit and I the current in it, is called the **resistance drop** in this portion of the circuit.

The two fundamental relations expressed by equations (11) and (12) may therefore be stated in words as follows:

(*a*) **The difference of electric potential between the terminals of a source of electric energy is equal to the electromotive force of this source LESS the resistance drop through it.**

(*b*) **The difference of electric potential between the terminals of a receiver of electric energy is equal to the back electromotive force of this receiver PLUS the resistance drop through it.**

When the receiver is a pure resistance, i.e., when all the electric energy transferred to the receiver is converted into rI^2 heat, there is no back electromotive force, and equation (12) becomes simply

$$V=rI \tag{13}$$

That is, the difference of potential between the terminals of a pure resistance is equal to the value of this resistance multiplied by the current through it. Or, looked at from another point of view, to force a current I through a pure resistance r requires a difference of potential equal to rI.

The relation expressed by equation (13) is commonly referred to as **Ohm's Law,** and the more general relations expressed by equations (11) and (12) are sometimes referred to as the **Generalized Ohm's Law.**

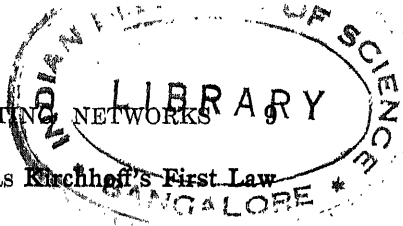
Differences of electric potential and electromotive force are both expressed in the same unit. The practical unit of potential difference and electromotive force is the **volt,** which is defined as that difference of electric potential which is necessary to force a current of one ampere through a pure resistance of one ohm. The c.g.s. electromagnetic unit, sometimes called the abvolt, is equal to 10^{-8} volts, or 1 volt $= 10^8$ abvolts.

Electric potential difference is also called **voltage,** although some purists object to this word. The potential difference between the terminals of a source or of a receiver is then called simply its terminal voltage. In the case of a receiver the terminal voltage is also called the impressed voltage.

Electromotive force is sometimes called the **internal voltage,** or the **generated voltage.**

7. Kirchhoff's Laws for Conducting Networks.—The flow of electricity in an electric circuit, or in any network of electric circuits, is strictly analogous to the flow of water in a system of interconnected pipes which are completely filled with water.

Just as in such a system there is no accumulation of water at any part of the system, so in an electric network carrying direct (non-varying) currents of electricity there is no accumulation of electricity at any part of the circuit. Consequently, at any junction point in such a network the amount of electricity which comes up to this junction in any interval of time must be equal to the amount of electricity which leaves this junction in this same interval. Considering the currents which leave a junction point as *negative* currents, this fact may be conveniently expressed by the statement that:

The algebraic sum of the currents entering any junction point in a network is always equal to zero, provided these currents

do not vary with time. This is known as **Kirchhoff's First Law** for conducting networks.

Calling I_1, I_2, I_3, etc., the currents which enter a junction point, and noting that for any current leaving the junction the corresponding I must be given a negative value, this law may be stated mathematically as follows:

$$I_1+I_2+I_3+\text{etc.}=0 \tag{14}$$

Again, just as in such a system of water pipes as above described, the difference in the pressure of the water at any two points can have but a single value, so also in an electric network the difference of electric potential between any two points can have but a single value. It therefore follows that:

The total drop of electric potential in a given direction around any closed loop in a conducting network is always equal to zero. This is known as **Kirchhoff's Second Law** for conducting networks.

Let I_{12} be the current in any branch 1–2 of a network,* in the direction from 1 to 2, these numbers indicating the two ends of this branch, let r_{12} be the resistance of this branch, and let E_{12} be the electromotive force in this branch in the direction from 1 to 2. Since an electromotive force is considered as positive in the direction in which it produces, or tends to produce a *rise* of electric potential, the total *drop* of electric potential from 1 to 2 is equal to the resistance drop r_1I_1 in this branch *minus* the electromotive force E_1 in this branch, or

$$V_{12}=r_{12}I_{12}-E_{12}$$

Similarly for branches 2–3, 3–4, etc., the potential drops are

$$V_{23}=r_{23}I_{23}-E_{23}$$
$$V_{34}=r_{34}I_{34}-E_{34}$$
$$\text{etc., etc.}$$

When the branches 1–2, 2–3, 3–4, etc., form a closed loop in the network, then from the principle just stated, the sum of the

* In reading a statement of this kind the student should sketch the arrangement described, using a sheet of paper or note-book. "Studying with a lead pencil" should become a habit with every student of engineering.

potential drops around this loop must be equal to zero. Conse quently, adding the above equations for the several branche which form this loop, there results, as a mathematical statemen of Kirchhoff's Second Law, the relation

$$r_{12}I_{12}+r_{23}I_{23}+r_{34}I_{34}+\text{etc.}=E_{12}+E_{23}+E_{34}+\text{etc.} \quad (15$$

Note that in this equation the I's and E's may be either posi tive or negative. For example, I_{23} stands for a positive numbe when the current in the branch 2–3 is actually in the directio from 2 to 3. On the other hand, if the current in this branc is actually in the direction from 3 to 2, then I_{23} stands for a negative number. Similarly, E_{23} stands for a positive numbe when the electromotive force in the branch 2–3 is actually ir the direction from 2 to 3, and for a negative number when the electromotive force in this branch is actually in the directio from 3 to 2.

Equation (15) is then nothing more than a convenient way of stating that *the algebraic sum of the resistance drops around any closed loop in an electric network is equal to the algebraic sum of the electromotive forces in the several branches which constitute this loop.* This is merely another way of stating Kirchhoff's Second Law.

The importance of Kirchhoff's two laws is that they enable one to calculate the current in every branch of an electric net-work, no matter how complicated, provided the resistance and electromotive force in each branch is known. This is done by writing down for each junction point an equation of the form of (14), and for each loop an equation of the form of (15), and then solving this set of simultaneous equations.

It will be found that one junction point equation is superflous, since it can be derived directly from the other junction point equations, and also that one loop equation is superfluous, since it can be derived from the other loop equations. Consequently, the actual number of simultaneous equations which have to be solved is 2 less than the sum of the junction points and loops.

Various expedients can be used to simplify such calculations, and it is usually unnecessary to write down formally all the pos-sible equations; see the author's *Electricity and Magnetism for Engineers*, Vol. I.

8. Series and Parallel Connections.—It follows directly from Kirchhoff's Laws that when two or more conductors are connected in series, so that the same current flows through each, the total resistance of the circuit thus formed is the sum of the resistances of its component parts, and the resultant electromotive force in the circuit is the algebraic sum of the electromotive forces in its component parts. That is, calling r_{12}, r_{23}, r_{34}, etc., the resistances of the various parts of this circuit, and E_{12}, E_{23}, E_{34}, etc., the electromotive forces in these portions of the circuit, then the total resistance of this circuit is

$$R = r_{12} + r_{23} + r_{34} + \text{etc.} \tag{16}$$

and the resultant electromotive force in this circuit is

$$E = E_{12} + E_{23} + E_{34} + \text{etc.} \tag{17}$$

It also follows from Kirchhoff's Laws that when two or more conductors are connected in parallel, that is, in such a manner that the same potential drop is established through each, then, *if the electromotive force in each of these parallel branches has the same value and the same direction*, or if there is no electromotive force in any one of these branches, the several branches in parallel are equivalent to a single branch whose resistance has such a value that its reciprocal is equal to the sum of the reciprocals of the resistances of these branches. That is, calling r_1, r_2, r_3, etc., the resistances of the separate branches, and R the equivalent resistance, then

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \text{etc.} \tag{18}$$

For two branches in parallel this relation becomes

$$R = \frac{r_1 r_2}{r_1 + r_2} \tag{18a}$$

It is particularly important to note that equations (18) and (18*a*) are not applicable to parallel circuits in which there are *unequal* electromotive forces.

9. Magnetic Field of an Electric Current.—Any region in which a freely-suspended magnetic needle is acted upon by a force tending to set it in a definite direction, or in which such a needle would be acted upon by such a force if placed therein, is

called a magnetic field. A magnetic field exists around every conductor carrying an electric current, and also within the substance of this conductor. A magnetic field also exists in and around every magnet.

The **direction of a magnetic field** at any point is defined as the direction in which a small magnetic needle would point * if freely suspended at this point in the field.

Lines drawn in a magnetic field in such a manner that their direction at each point is the same as the direction of the magnetic field at this point are called **magnetic lines of force.**

The magnetic lines of force due to an electric current are always closed loops which link the circuit of the current. An electric current always produces, or tends to produce, lines of force whose direction bear the same relation to the current as that between the direction of advance of a right-handed screw and the direction of rotation of a point in the circumference of the head of the screw. This relation is usually referred to as the **right-handed screw relation.**

10. Electromagnetic Induction and Magnetic Flux.—*During* any change in the magnetic field which threads an electric circuit,† or while a conductor which forms a part of an electric circuit is moving across the lines of force of a magnetic field, an electromotive force is induced in this circuit.

The electromotive force induced in an electric circuit when the magnetic field threading it changes, may be used as a means for measuring one of the fundamental properties of the field. Consider in a magnetic field a surface of area S, and imagine a wire to be bent into a single loop which coincides with the boundary of this area. Now let the magnetic field be completely destroyed. During the disappearance of the magnetic field an electromotive force e will be induced in this loop. This electromotive force will be a variable one, rising from zero to a maximum value and then decreasing to zero again. The integral of this

* A magnetic needle is said to point in the direction of the line drawn through it from its normally south-seeking end to its normally north-seeking end.

† When the change in the interlinkages between the lines of force and the circuit is produced by moving the circuit as a whole, no electromotive force is produced unless at least some of the conductors which form the circuit cut across the lines of force.

electromotive force with respect to time, over the interval required for the field to disappear, is taken as the measure of the magnetic state which existed at the area S before the field was destroyed.

The property of a magnetic field measured in the manner just described is called the flux of magnetic induction, or simply the **magnetic flux,** through the area considered. Magnetic flux is usually represented by the symbol ϕ. The mathematical expression for the magnetic flux through the area S is then

$$\phi = \int_0^t e dt \tag{19}$$

where ϕ is the magnetic flux which exists through the area S before the field is destroyed, e is the electromotive force induced in this loop at any instant during the disappearance of the field, and t is the time taken for the field to disappear.

A magnetic field may be imagined to be made up of closed tubes (each tube forming a closed loop), whose walls are tangent at every point to the direction of the magnetic field at this point. The cross-section of each of these tubes may be chosen of such a value that the flux through each cross-section is unity. Each of these tubes may be considered as represented by a line of magnetic force which coincides with its axis.

When the number of lines of magnetic force in a magnetic field are considered as limited in this manner, then the *total number of lines of force in the field is equal to the total magnetic flux,* and the number of these lines which link any loop in the field (such as an electric circuit) is equal to the total magnetic flux through this loop. This convention in regard to the number of lines of force in a magnetic field is commonly adopted, and the magnetic flux through any area is frequently referred to simply as the number of lines of force through this area.

The magnetic flux per unit area at any point in a magnetic field, the area being taken perpendicular to the direction of the field at this point, is called the density of the magnetic flux at this point, or simply the **flux density** at this point. Magnetic flux density is usually represented by the symbol B.

When the flux density has the same value B at every point of an area S, and this area is perpendicular to the lines of force

through it, the total flux through this area (or total number of lines of force through this area) is

$$\phi = BS \tag{20}$$

The unit of magnetic flux in the c.g.s. electromagnetic system (corresponding to e in abvolts and t in seconds) is called the **maxwell.** The unit of magnetic flux corresponding to the volt and the second may be called the "volt-second," but this unit is not generally adopted. The c.g.s. electromagnetic unit (the maxwell) is almost invariably employed, even in engineering calculations. When e is expressed in volts, t in seconds, and ϕ in maxwells, equation (19) becomes

$$\phi = 10^8 \int_0^t e dt \tag{21}$$

The term "line" is commonly used as equivalent to a maxwell; e.g., by a magnetic flux of 500,000 lines is meant a flux of 500,000 maxwells.

The c.g.s. electromagnetic unit of flux density is called the **gauss,** i.e., a gauss is one maxwell, or one line, per square centimeter. Flux density is also expressed in lines per square inch and in kilo-lines per square inch, a kilo-line being 1000 lines or maxwells.

11. Induced Electromotive Force. Magnetic Flux Linkages.— The electromotive force induced in an electric circuit which forms a single closed loop, when the magnetic field which links this loop changes, is, from equation (21), equal to the *time-rate of change of the magnetic flux* which threads it, viz.,

$$e = 10^{-8}\frac{d\phi}{dt} \qquad \text{volts} \tag{22}$$

where ϕ is the number of lines of force (maxwells of flux) which thread this loop.

When an electric circuit is a coil of N turns, each of which is threaded, or linked, by the same number of lines of force, the total electromotive force induced in the circuit is the sum of the electromotive forces induced in these N turns, viz.,

$$e = 10^{-8}N\frac{d\phi}{dt} \qquad \text{volts} \tag{23}$$

In general, the various turns of a coil in a magnetic field do not all link the same number of lines. Let ϕ_1, ϕ_2, ϕ_3, etc., be the flux which links respectively the first, second, third, etc., turns and put

$$\lambda=\phi_1+\phi_2+\phi_3+\text{etc.} \qquad (24)$$

This quantity λ, which is equal to the sum of the fluxes which link the respective turns of the coil, is called the **flux linkages** of the coil. In general, then, the electromotive force induced in a coil of N turns is

$$e=10^{-8}\frac{d\lambda}{dt} \qquad \text{volts} \qquad (25)$$

When $\phi_1=\phi_2=\phi_3$, etc. $=\phi$, then $\lambda=N\phi$, and (25) reduces to the same form as (24).

Stated in words, the general relation expressed by equation (25) is that **the electromotive force, in volts, induced in a coil when the magnetic field which links this coil is changing, is equal to 10^{-8} times the change per second in the flux linkages, in maxwells, between this coil and the field.**

This relation also holds for a finite length of a single conductor. That is, the electromotive force, in volts, induced in a conductor, when the number of lines of force which link this conductor changes, is equal to 10^{-8} times the change per second in the number of lines of force which link this conductor.

The direction of the electromotive force induced in a coil, when the magnetic field which links this coil changes, is always such that this electromotive force, acting alone, would produce in the coil a current in such a direction as to oppose this change in the field. Since the magnetic field due to a current in a single coil is always in the right-handed screw direction with respect to this current, it follows that the electromotive force induced by an *increase* in the flux which threads the coil, is always in the *left-handed* screw direction with respect to the direction of the lines of force through the coil. Similarly, the electromotive force induced by a *decrease* in the flux which threads the coil is always in the *right-handed* screw direction, with respect to the lines of force through the coil.

Applying equation (22) to a single rectangular loop, placed with its plane perpendicular to the lines of force of a uniform mag-

netic field, and imagining one of the sides of this loop to move parallel to itself, it may be readily shown that the electromotive force induced in a straight conductor l centimeters in length, when this conductor cuts perpendicularly across a uniform magnetic field having a flux density of B gausses, at a velocity of v centimeters per second, is

$$e = 10^{-8} Blv \qquad \text{volts} \tag{26}$$

The direction of the electromotive force given by equation (26) is the direction in which the middle finger of the right hand points when the thumb, forefinger and middle finger of this hand are held mutually perpendicular, and the forefinger is pointed in the direction of the lines of force and the thumb is pointed in the direction of motion of the conductor. This rule in regard to the direction of the induced electromotive force is known as the **right-hand rule.**

12. Magnetomotive Force and Magnetic Reluctance.—The magnetic fields in most machinery and apparatus are due to electric currents. The relation between the intensity of a current and the magnetic flux which it produces is of the same general form as that between electromotive force and current.

Consider in a magnetic field due to a single electric current a closed tube of any size, whose walls are tangent at each point to the direction of the magnetic field at that point. Such a tube may be called a complete magnetic circuit, just as the closed path of an electric current is called a complete electric circuit. Let ϕ be the total number of lines of force inside this tube. Since the walls of the tube are tangent to the lines of force, no lines of force can enter or leave this tube; hence the number of lines of force through each cross-section of the tube (i.e., the magnetic flux through each cross-section) is the same throughout its length.

This closed tube will link the electric current which produces the magnetic field. Let I be this current, and let N be the number of turns in the coil which forms its circuit. The fundamental relation between the flux ϕ and the current I is then

$$\phi = \frac{NI}{\mathcal{R}} \tag{27}$$

where $\mathcal{R}$ is a quantity which represents a definite property of the magnetic circuit formed by the closed tube, just as the resistance of an electric circuit represents a definite property of an electric circuit.

This quantity $\mathcal{R}$ is called the **magnetic reluctance** of the magnetic circuit formed by the closed tube. Its value depends upon the shape, dimensions and material of the magnetic circuit formed by this tube, in exactly the same manner as the resistance of an electric circuit depends upon its shape, dimensions and material.

The c.g.s. electromagnetic system of units is based upon the arbitrary choice of unity as the value of the reluctance of a cubic centimeter of free space (or, what amounts to the same thing practically, a cubic centimeter of air). This arbitrary choice of the value of the reluctance of a cubic centimeter of free space requires the introduction of the factor 4π in equation (27) *. Consequently, when the current is expressed in amperes and the flux and reluctance are expressed in c.g.s. electromagnetic units, equation (27) becomes

$$\phi = \frac{0.4\pi NI}{\mathcal{R}} \tag{28}$$

The c.g.s. electromagnetic unit of reluctance is called the **oersted.**

The product NI in equation (27), or the product $0.4\pi NI$ in (28), is called the **magnetomotive force** of the current. The product NI, being the product of amperes and turns, is called simply the **ampere-turns** of the coil. The unit of magnetomotive force in the c.g.s. electromagnetic system, namely the unit of the quantity $0.4\pi NI$, is called the **gilbert.** Hence

$$1 \text{ ampere-turn} = 1.257 \text{ gilberts.}$$

13. Magnetic Permeability.—The reluctance of a right cylinder or prism of length l and cross-section S, to a uniform magnetic flux which is parallel to its axis, is

$$\mathcal{R} = \frac{l}{\mu S} \tag{29}$$

* In other words, if the value of $\mathcal{R}$ from equation (29) is substituted in equation (27), and the resultant formula applied to a magnetic circuit in air, it is found that the calculated value of the flux differs from the experimentally determined value by 0.4π, when I is expressed in amperes.

where μ is a factor which depends only upon the nature of the material which forms this cylinder and upon the units in which l and S are expressed. This factor μ is called the magnetic permeability of this material.

Permeability is almost invariably expressed in c.g.s. electromagnetic units. This unit is such that the corresponding value of the permeability, when substituted in equation (29), when l is expressed in centimeters and S in square centimeters, will give the value of $\mathcal{R}$ in oersteds.

It should be carefully noted that equation (29) can be used only for calculating the reluctance of a volume with *parallel sides in which the lines of force are straight and uniformly distributed and parallel to the sides of this volume.* It cannot therefore, as a rule, be applied directly to the calculation of the magnetic circuit of an electric machine; see the next Article.

The substance which has the greatest magnetic permeability is pure iron. The various kinds of commercial iron and steel are also highly permeable to a magnetic flux, and, due to their relative cheapness, are invariably used where a path of low reluctance (large flux for a small number of ampere-turns) is desired.

Most substances, such as air and other insulating materials, copper, aluminum, etc., have a permeability practically equal to unity. Such substances are called non-magnetic, as distinguished from those substances which have a permeability of the same order of magnitude as iron, which latter are called magnetic substances. Aside from iron and steel, the only strongly magnetic substances are nickel and cobalt and certain alloys of manganese, known as Heusler alloys.

14. Magnetizing Force.—The magnetic flux density at any point in a magnetic field divided by the magnetic permeability at this point, is called the magnetizing force at this point. Magnetizing force is usually represented by the symbol H.

Magnetizing force is not the same as magnetomotive force, but bears to the latter the same kind of relation that mechanical force bears to work. When a material particle moves over any path in the direction of the mechanical force which acts on it, the work done by this force on the particle is

$$W = \int f dl$$

where f is the force at any element dl of the path, and the integral sign indicates that the products (fdl) for the successive elements of the path between its beginning and end are to be added. Such an integral may be conveniently called a "line integral."

The relation between magnetomotive force and magnetizing force is that **the line integral of the magnetizing force around the closed loop formed by any line of magnetic force is always equal to the resultant magnetomotive force linked by this loop.**

By the resultant magnetomotive force is meant the sum of the magnetomotive forces due to the electric currents which link this line of force in the right-handed screw direction, minus the sum of the magnetomotive forces due to the currents which link this line of force in the left-handed screw direction. Or, considering an electric current positive when it links the line of force in the right-handed screw direction, and negative when it links this line of force in the left-handed screw direction, the resultant magnetomotive force is the algebraic sum of the magnetomotive forces due to all the currents linked by this line of force. The resultant magnetomotive force may then be expressed mathematically

ΣNI ampere-turns

or

$0.4\pi\Sigma NI$ gilberts

The relation between magnetizing force and the resultant magnetomotive force is then

$$\int_0 Hdl = 0.4\pi\Sigma NI \qquad (30)$$

where dl is any elementary length in the line of force, in centimeters; H is the magnetizing force in c.g.s. electromagnetic units, at this elementary length; ΣNI is the algebraic sum of the ampere-turns linked by this line of force, and the integral sign with the subscript "o" indicates that the integral is to be taken completely around the *closed loop* formed by the line of force.

From this relation it follows that magnetizing force has the dimensions of gilberts divided by centimeters. Hence the c.g.s. electromagnetic unit of magnetizing force is called the **gilbert per centimeter.**

By choosing for the unit of magnetizing force a value 0.4π times as large as the c.g.s. electromagnetic unit, the relation between magnetizing force and magnetomotive force may also be written

$$\int_0 H dl = \Sigma NI \qquad (31)$$

The unit of magnetizing force corresponding to this last equation is called the **ampere-turn per centimeter,** when length is expressed in centimeters; or the **ampere-turn per inch,** when length is expressed in inches. In engineering work either the ampere-turn per centimeter or the ampere-turn per inch is usually employed in preference to the gilbert per centimeter.

Equation (31) is the fundamental relation used in all practical calculations of the magnetic circuit of electrical machines; see Chapter III.

In a non-magnetic substance, when the c.g.s. electromagnetic system of units is employed, the magnetizing force at any point is numerically equal to the magnetic flux density at that point. When the flux density B is expressed in gausses (lines per square centimeter) and the magnetizing force H in ampere-turns per centimeter, the relation between B and H for a non-magnetic substance is

$$H = 0.7958\, B \qquad (32)$$

Similarly, when B is expressed in lines per square inch and H in ampere-turns per inch, then for a non-magnetic substance

$$H = 0.3131\, B \qquad (32a)$$

15. Magnetic Poles.—That part of the surface of a magnetic substance, such as iron, from which the lines of magnetic force pass out of the substance into air, is called a **north magnetic pole.** Similarly, that part of the surface of a magnetic substance at which the lines of magnetic force pass into this substance from air is called a **south magnetic pole.**

When the substance in question has a permeability of the order of magnitude of that of iron (1000 c.g.s. units or more), the lines of force leave the north pole practically perpendicularly to the surface, and enter the south pole practically perpendicularly to the surface.

The **strength of the magnetic pole** at a surface S, in c.g.s. electromagnetic units, is

$$m=\frac{\phi}{4\pi}\left(1-\frac{1}{\mu}\right) \tag{33}$$

where ϕ is the number of lines of force which pass from this surface into the air, and μ is the permeability of the substance on which this pole is located. For $\mu=1000$ or greater, this becomes, to a close degree of approximation,

$$m=\frac{\phi}{4\pi} \tag{33a}$$

In engineering work the *flux per pole* ϕ, rather than the *strength of the pole* m, is almost invariably used. The two differ merely by the factor 4π.

16. Magnetic Hysteresis and Hysteresis Loss.—The permeability of iron and other magnetic substances is not a constant quantity, like electric resistivity, but depends both upon the value of the flux density and also upon how this flux density has been established. Starting with a sample of iron completely demagnetized, and then gradually increasing the current in the coil which is used to magnetize it, it is found that the relation between the magnetizing force H established by this current, and the flux density B, is of the form indicated by the ascending curve starting from the origin in Fig. 1. If, when a given value of the magnetizing force is reached, the magnetizing current is gradually decreased, the flux density for any given value of the magnetizing force does not come back to the value it had for this same magnet-

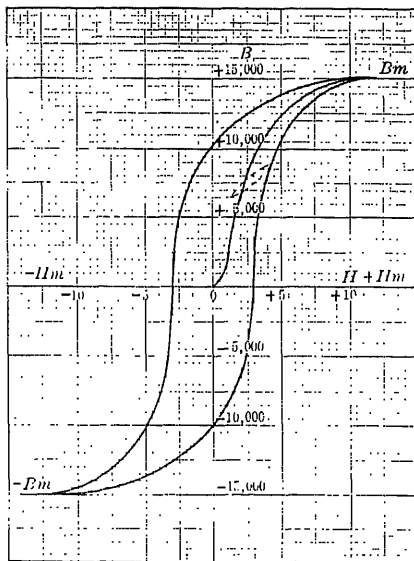



FIG. 1.—Typical Hysteresis Loop.

izing force during the increase in the magnetizing current, but has a higher value, as indicated by the upper branch of the loop in Fig. 1. This phenomenon is called **magnetic hysteresis.**

When the magnetizing force is reduced to zero, the flux density does not fall to zero, but remains at a substantial value. That is, the iron remains magnetized, even when the magnetizing current is completely removed. The flux density for zero magnetizing current is called the **residual magnetism.** The amount of residual magnetism depends upon the quality of the iron, being very small for soft iron, and relatively large for hard steel, especially for the alloy steels used for permanent magnets. Mechanical jarring reduces the residual magnetism, this effect being particularly pronounced in the case of soft iron or steel.

Referring to Fig. 1, when the magnetizing current after being reduced to zero, is re-established in the reverse direction, and increased to the same value in this direction as it had in the first direction, then reduced to zero again, and then brought back to its original maximum value in the first direction, the relation between the flux density and the magnetizing force at the successive points of this cycle is as given by the closed loop in the figure. This cycle may be repeated over and over again, and each time a loop of practically the same size and shape will result. Such a loop is called a **hysteresis loop.** That value of H, for such a loop, which corresponds to zero flux density is called the **coercive force.**

Whenever a piece of iron or other magnetic substance is subjected to a magnetizing force which alternates in direction between equal positive and negative values, an amount of heat energy proportional to the area of the corresponding hysteresis loop is dissipated in each unit volume of this substance per cycle. The power corresponding to the total amount of energy thus dissipated per second in the whole volume of this substance is called the **hysteresis loss** in this substance.

The relation between the hysteresis loss P_h in a volume V of a given material, the maximum flux density B_m established in it during the cycle of variation of the magnetic field, and the number of complete cycles f of variation per second, may be expressed to a close degree of approximation by the formula

$$P_h = KVfB_m^{1.6} \qquad (34)$$

where K is a factor, whose value depends upon the chemical composition of the material and upon the heat treatment to which it has been subjected, and upon the units in which the various quantities in this equation are expressed. This relation is known as **Steinmetz's Law.**

The factor K, when P_h is expressed in ergs, V in cubic centimeters and B in gausses, is usually represented by the symbol η, and is called the **hysteresis coefficient.** For numerical values see any electrical engineer's handbook. Note particularly that equation (34) holds only when the flux density B_m has the same maximum value at each point of the volume V.

17. B-H Curves and Magnetic Saturation.—From Fig. 1 it is evident that for a given value of the flux density in a given magnetic material, the magnetizing force may have any one of several values, depending upon how this value of the flux density has been established. In the calculation of the magnetic circuit of electric machines it is usually the *maximum* value of the flux density during each cycle of variation that is of chief importance.

A curve giving the relation between the maximum value of the flux density and the maximum value of the magnetizing force for successively larger symmetrical hysteresis loops is a single valued function, and has the general form of the curves shown in Fig. 2. Such a curve is called the **normal B-H curve,** or **magnetization curve** for the given material. When the permeability of a magnetic material is stated, what is ordinarily meant, unless otherwise specified, is the permeability corresponding to a point on the normal B-H curve.

When the B-H curves for the various materials which form the magnetic circuit of an electric machine are known, it is possible, by applying equation (31), to calculate with a fair degree of precision the ampere-turns required to produce any desired flux in this circuit; see Article 34.

Both the normal B-H curve and the hysteresis loss for iron and steel depend upon the chemical composition of the material and the heat treatment to which it has been subjected. Very slight impurities have a very marked effect, and slight variations in annealing likewise produce large variations in both permeability and hysteresis loss. Steel used in electric ma-

chinery is made especially for the purpose and is annealed with great care.

As the magnetizing force acting on a magnetic substance is increased, the flux density at first increases relatively slowly for very weak fields (the scale in Fig. 2 is not large enough to show this), then increases much more rapidly (the steep portion of the curves in Fig. 2), then as the field becomes stronger and

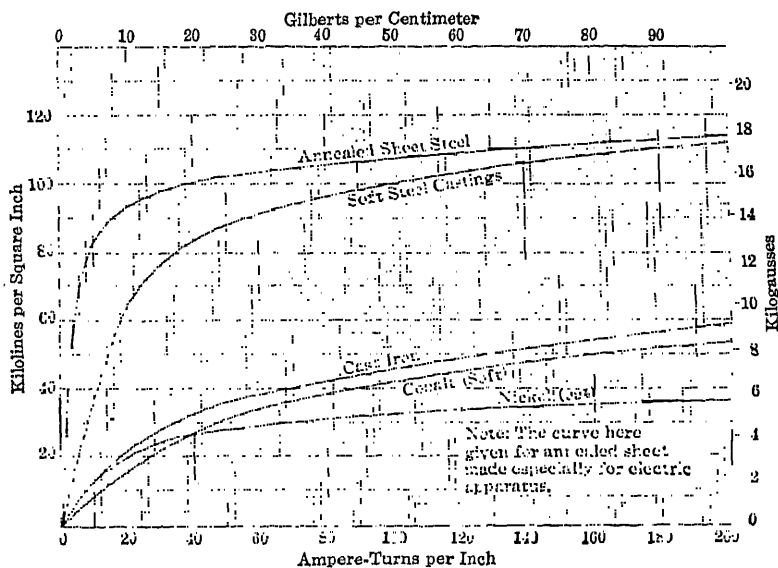



FIG. 2.—Typical *B-H* Curves.

stronger the flux density increases less and less rapidly, and ultimately reaches a value beyond which any further increase in *B* is numerically equal (when c.g.s. electromagnetic units are employed) to the increase in *H*.

In other words, the difference (*B-H*) has a definite maximum value for each kind of material, beyond which it cannot be forced, no matter how strong the magnetizing force may be. When the difference (*B-H*) reaches this maximum value the material is said to be **magnetically saturated.**

The saturation value of (*B-H*) for soft iron and steel is about 20,000 gausses, and for cast iron about 15,000 gausses. In practice, it is usual to speak of the iron or steel parts of a magnetic circuit as being saturated when the flux density has a value

well above the knee of the B-H curve, even though the value of *B-H* may be appreciably below the absolute saturation value.

18. Eddy Currents and Eddy-current Loss.—Any metal which is in a *varying* magnetic field, or which cuts across the lines of force of a constant magnetic field, has an electromotive force induced in it, and this electromotive force will in general set up a current in this piece of metal, irrespective of whether or not it is part of a useful electric circuit. Currents thus set up in metal parts are called **eddy currents.**

Eddy currents are always present in the iron part of a magnetic circuit in which there is a varying flux, and in any iron part (such as the armature core of a direct-current dynamo) which cuts the lines of force of a constant magnetic field. These eddy currents develop heat (rI^2), and the power thus dissipated as heat is called the **eddy-current loss.**

The intensity of an eddy current is equal to the electromotive force induced by the varying flux, divided by the resistance of the path of this current. The power dissipated as heat by an eddy current is equal to the resistance of its path multiplied by the square of the intensity of this current. Hence the eddy-current loss is proportional to the square of the induced electromotive force and inversely to the resistance of the path of these currents.

In a solid core, due to its low resistance, the eddy-current loss would be relatively very large, and would therefore greatly reduce the efficiency of the machine. To reduce this loss, the iron or steel parts of a magnetic circuit subjected to a varying magnetic field, or which rotate in a constant field, are always made up of sheets, or laminations, usually from 0.014 to 0.025 inch thick, with their planes parallel to the lines of force.

The oxide formed on such laminations when they are annealed is a sufficiently good insulator to keep them fairly well insulated from one another. When it is desired to keep the eddy-current loss down to a minimum, these laminations, before being assembled, are coated with an insulating varnish, or thin sheets of paper are placed between them when the core is built up.

The eddy-current loss in a laminated core, subjected to an alternating flux which has a frequency of f cycles per second (i.e., which passes through a complete cycle of values from a

positive maximum to a negative maximum and back to the positive maximum f times per second), is proportional to the square of the frequency f and to the square of the thickness a of the laminations. When the flux varies sinusoidally with time, the eddy-current loss is also proportional to the square of the maximum flux density B_m attained during each cycle. When the flux density attains the same maximum value at each point in a given volume V, the eddy-current loss is also proportional to this volume.

The eddy-current loss in a volume V, under the conditions just stated, may then be written

$$P_e = \epsilon(afB_m)^2 \tag{35}$$

where ϵ is a factor whose value depends upon the chemical composition of the material, upon the heat treatment to which it has been subjected, and upon the units in which the various quantities in the equation are expressed. This factor is called the **eddy-current coefficient** for the material in question. For numerical values see any electrical engineer's handbook.

The eddy-current coefficient ϵ is inversely proportional to the resistivity of the material. Steel containing about 3 per cent of silicon has a resistivity about three times that of ordinary electrical sheet-steel. Hence the eddy-current loss in such steel is about one-third as much as in ordinary electrical sheet-steel. Silicon steel also has a lower hysteresis coefficient than ordinary electrical steel, and is generally used where the total core loss (sum of the hysteresis and eddy-current losses) must be kept small.

It should also be noted that, since the resistivity of iron and steel increases with increase of temperature, the eddy-current coefficient decreases with increase of temperature, and therefore the eddy-current loss likewise diminishes with increase in temperature.

19. Self and Mutual Inductance.—When a current flows in an electric circuit, this current always produces a magnetic field. An electric current is therefore always linked by a certain number of *self-produced* lines of force. The number of linkages λ between the turns in a given portion of an electric circuit and the flux which the current in it produces may always be written

$$\lambda = Li \tag{36}$$

where i is the current and L a factor which depends upon the size and shape of the given portion of the electric circuit and upon the size and shape of the magnetic circuit which it links.

This factor L is called the coefficient of self-induction of the given portion of the circuit, or more briefly, the **self-inductance** of this portion of the circuit. It may be briefly defined as the *flux linkages of the given portion of the circuit per unit current in it,* the magnetic field being due entirely to the current in this particular portion of the circuit.

When the current is expressed in c.g.s. electromagnetic units and the flux linkages in maxwells, the corresponding unit of self-inductance is called the "abhenry." When the current is expressed in amperes and the flux linkages in volt-seconds (see Article 10) the corresponding unit of self-inductance is called the **henry**; this is the practical unit of self-inductance. The relation between the henry and the abhenry is

$$1 \text{ henry} = 10^9 \text{ abhenries}$$

When the magnetic field produced by the current lies wholly in a non-magnetic medium, the self-inductance of the given portion of the circuit is a constant, independent of the value of the current. When the magnetic field lies wholly, or in part, in a magnetic substance of variable permeability, the self-inductance is not a constant, but depends upon the value of the current. However, when the magnetic circuit contains a relatively large air-gap, the variation in the value of the self-inductance is usually negligible, from an engineering point of view.

The self-inductance of an electric circuit bears a relatively simple relation to the reluctance of the magnetic circuit of the flux which the current in this electric circuit produces. For example, in the case of a coil of N turns which are wound so closely together that the same number of lines of force link each turn, i.e., a so-called "concentrated winding," the self-inductance is

$$L = \frac{4\pi N^2}{\mathcal{R}} \qquad (37)$$

where $\mathcal{R}$ is the reluctance of the magnetic circuit linked by the coil.

When the current in a coil increases or decreases, the magnetic flux produced by it increases or decreases, consequently,

an electromotive force is always induced in a circuit when th rent in it varies. The value of this self-induced electron force, in terms of the self-inductance L of the circuit, and th rent i in it, is

$$e=\frac{d}{dt}(Li)$$

When the self-inductance of the circuit is constant this becoı

$$e=L\frac{di}{dt}$$

The direction of the self-induced electromotive force is *op* to that of the current when the current is *increasing*, and i *same direction* as the current when the current is *decreasing*.

In an exactly similar manner as for a single coil, the flux ages of a coil 1 due to a current i_2 in a second coil 2 may be wı

$$\lambda=Mi_2$$

where M is a factor which depends upon the relative positic the two coils, the number of turns in each, the extent and tribution of the magnetic field produced by the current i_2, the magnetic nature of the substances in which this mag field lies. This factor M, which may be briefly defined a *flux linkages of coil* 1 *per unit current in coil* 2, is called the **mı inductance** of coil 1 with respect to coil 2. The mutual in ance of one coil with respect to another is equal to the mı inductance of the second coil with respect to the first.

Mutual inductance is expressed in the same units as inductance.

The electromotive force induced in one coil due to a var current in a second coil is

$$e_1=\frac{d}{dt}(Mi_2)$$

where M is the mutual inductance between the two coils. W the mutual inductance between the two coils is constant may be written

$$e_1=M\frac{di_2}{dt} \quad ($$

20. Mechanical Forces in a Magnetic Field.—A conductor which carries an electric current, when in a magnetic field due to any other agent, e.g., a permanent magnet or another current, is always acted upon by a mechanical force which tends to move this conductor across the lines of force of this field. In the case of a straight wire which has a length of l centimeters and which carries a current of I amperes, this force is

$$f=\frac{BlI}{10} \qquad \text{dynes} \tag{41}$$

where B is the flux density, in gausses, perpendicular to the wire.

The mechanical force given by equation (41) is in the direction in which the thumb of the left hand points when the thumb, forefinger and middle finger of this hand are held mutually perpendicular, and the forefinger is pointed in the direction of the lines of force and the middle finger in the direction of the current. This rule in regard to the direction of the mechanical force is known as the **left-hand rule.** Compare with the right-hand rule for the direction of the induced electromotive force (Article 11).

In the case of an electric circuit of any other form than a straight conductor, such as a coil of wire, the resultant mechanical force exerted on it by a magnetic field may be most conveniently expressed as follows:

Let λ be the flux linkages, in maxwells, between this coil and the magnetic field in which it lies, and imagine the coil to be displaced a distance dx in the direction in which it is desired to determine the mechanical force. Let $d\lambda$ be the corresponding increase in the flux linkages of the coil in the right-handed screw direction with respect to the current in it, and let I be the intensity of this current, in amperes. The mechanical force in the direction of the displacement dx is then

$$f_x=\frac{I}{10}\frac{d\lambda}{dx} \qquad \text{dynes} \tag{42}$$

Similarly, to determine the *torque* exerted by a magnetic field on a coil carrying an electric current, imagine the coil to be turned through an angle of $d\phi$ radians in the direction in which it is desired to determine the torque. Let $d\lambda$ be the corresponding increase in the flux linkages of the coil in the right-handed screw

direction with respect to the current in it, and let I be the intensity of this current in amperes. The resultant torque in the direction of the angular displacement $d\phi$ is then

$$T=\frac{I}{10}\frac{d\lambda}{d\theta} \qquad \text{dyne-centimeters} \qquad (43)$$

From the relations just stated it follows that *the resultant force or torque exerted by a magnetic field on a coil carrying an electric current is always in such a direction as to tend to make the coil move into such a position that it will link the maximum number of lines of force in the right-handed screw direction with respect to the current.*

In the particular case of a coil carrying a constant current I and rotating in a constant magnetic field, the average torque for a complete revolution is zero, since the flux through the coil is first in one direction with respect to the coil and then in the opposite direction. The average torque for half a revolution, starting with the coil embracing the maximum number of lines of force, and ending with the same number of lines threading the coil, but in the opposite direction with respect to the current in it, is (from equation (42))

$$T_{\text{aver.}}=\frac{2N}{10\pi}\phi I \qquad \text{dyne-centimeters} \qquad (44)$$

where N is the number of turns in the coil (assumed to be a concentrated winding), ϕ is the maximum number of lines of force linked by the coil, and I is the current, in amperes.

A magnetic pole (see Article 15) in a magnetic field always has a mechanical force exerted on it by the other poles, and also by any electric currents, which may be in the field. This force, per unit area of the surface on which the given pole is located, when the lines of force are perpendicular to this surface, is proportional to the square of the number of lines of force per unit area, which enter or leave this pole. In c.g.s. electromagnetic units the force is

$$f=\frac{B^2}{8\pi} \qquad \text{dynes per sq. cm.} \qquad (44)$$

where B is the density of the flux at the area considered.

When the flux density is constant over a pole of area of S (in square centimeters), the total force on this area is

$$f = \frac{B^2 S}{8\pi} \qquad \text{dynes} \tag{45}$$

The force exerted on a north pole is always in the direction of the lines of force, and the force exerted on a south pole is always in the direction opposite to that of the lines of force.

PROBLEMS

1. A motor which is running at a speed of 500 r.p.m. develops 10 horsepower at its pulley.

(*a*) What is the torque developed by this motor?

(*b*) The pulley has a diameter of 15 inches. What is the difference in the tension in the two sides of the belt?

(*c*) This motor has an efficiency of 90 per cent. What is the total power input to it, expressed in kilowatts?

(*d*) The electric energy for operating this motor costs 3 cents per kilowatt-hour. The motor operates 8 hours each working-day at a constant load (output) of 10 horsepower. What will be the monthly bill for the energy supplied to it?

(*e*) What is the total power loss in this motor, in kilowatts?

2. A storage battery discharges at a constant rate of 30 amperes for 5 hours. How many coulombs of electricity pass through it in this interval of time?

3. What must be the resistance of a coil of wire in order that a current of 10 amperes flowing through it will develop the heat required to raise a gallon of water from 60° F. to the boiling point in 20 minutes?

4. A wire 0.1 inch in diameter and 1000 feet long has a resistance of 1 ohm at 20° C. What is the resistivity of this conductor at this temperature? Compare with the resistivity of copper (see Handbook).

5. A coil of copper wire at a room temperature of 20° C. has a resistance of 15 ohms. After an electric current has been flowing through this coil for 2 hours, it is found that its resistance is 18 ohms. By how much has the average temperature of the coil increased?

6. The power output of an electric generator is 100 kilowatts, and its internal resistance * is 0.02 ohm. The total current supplied by this generator is 500 amperes.

(*a* What is the total amount of electric power developed within this machine?

(*b*) What is the value of its terminal voltage?

(*c*) What is the value of its electro-motive force (generated voltage)?

7. The electric power input to the rotating member, or armature, of an electric motor is 50 kilowatts. The internal resistance of the armature winding is 0.4 ohm and the current established through it is 80 amperes.

* Of its armature circuit only.

(*a*) How much of the electric power input to the armature is converted into mechanical power?

(*b*) What is the value of the voltage impressed on the armature terminals?

(*c*) What is the value of the back electromotive force developed by this motor?

8. In the motor described in the preceding problem, the field winding (on the stationary member) is in parallel with the armature winding. This field winding has a resistance of 100 ohms. For the same conditions as described in the preceding problem, what will be the current through this field winding, and what will be the total power input to this winding?

9. A single turn of wire which just fits over one pole of an electromagnet is suddenly slipped off and moved completely out of the field of this magnet. The values of the electromotive force induced in this wire at successive instants of time measured from the beginning of this motion are as follows:

Time in Seconds.	Electromotive Force in Millivolts.	Time in Seconds.	Electromotive Force in Millivolts.
0.001	50	0.008	25
0.002	80	0.010	14
0.003	90	0.012	7
0.004	85	0.014	3
0.005	70	0.016	1
0.006	50	0.018	0.05
0.007	35	0.020	0.02

(*a*) Plot electromotive force against time, integrate graphically, and determine therefrom the flux in maxwells in each pole of the electromagnet.

(*b*) The cross-section of each pole of the magnet is 2 square inches. What is the average flux density in each pole, in gausses?

10. At a certain instant a coil in the armature winding of a generator is so located with respect to the field poles that it is threaded by all the lines of force which enter the armature from this pole. When the armature has rotated through an angle corresponding to the distance between the center of this pole and the center of the next pole (which pole will be of opposite sign), the coil is threaded by the same number of lines of force as before, but in the opposite direction.

In a certain 8-pole machine the flux per pole is 2,000,000 maxwells, the armature makes 300 revolutions per minute, and each armature coil has 5 turns. What is the average value of the electromotive force generated in each armature coil, as it moves from a position directly under a north pole to a position directly under a south pole?

11. A given magnetic circuit links two coils, each of 300 turns. The two coils are connected in series, and a current of 5 amperes is sent through them. The coils are so connected that the current in each links the magnetic circuit

in the same direction. The flux produced in the magnetic circuit by this current is 1,000,000 maxwells.

(*a*) What is the total magnetomotive force acting on this magnetic circuit? Give answer in gilberts and in ampere-turns.

(*b*) What is the reluctance of this magnetic circuit, in oersteds?

(*c*) Assuming the reluctance per unit length of the magnetic circuit to be the same at each cross-section, and the total length of this circuit to be 2 feet, what is the magnetizing force at each point of this circuit? Give answer in gilberts per centimeter and in ampere turns per inch.

(*d*) If the cross-section of the given magnetic circuit is 10 square inches, what is the average flux density in it? Give answer in kilolines per square inch and in gausses.

(*e*) Under the conditions stated in (*c*) and (*d*) what is the permeability of this magnetic circuit?

(*f*) Make a sketch, to scale, of a magnetic circuit which will satisfy the conditions stated in (*c*) and (*d*).

12. The average density in the air-gap between the armature and pole face of a certain generator is 50,000 lines per square inch. The length of the air-gap (measured parallel to the lines of force) is $\frac{1}{8}$ inch.

(*a*) What is the magnetizing force in this air-gap, in ampere-turns per inch?

(*b*) How many amperes, flowing in a coil of 200 turns, would be required to produce the given flux density in this air-gap, assuming the rest of the magnetic circuit (*i.e.*, the iron portions) to have a negligible reluctance?

13. The armature core of a certain two-pole generator has a volume of $\frac{1}{4}$ cubic foot. This armature rotates at a speed of 1800 revolutions per minute. As the armature rotates the flux density at each point in it passes through a complete cycle of positive and negative values during each revolution. Assuming the flux density at each point of the core to reach a maximum value of 10,000 gausses during each cycle, calculate the total hysteresis loss in the core. Give answer in watts. Use for K the value given in your Handbook for annealed sheet steel.

14. The armature core described in Problem 13 is built up of annealed steel laminations 0.025 inch thick. Using the value of the eddy-current coefficient given in your Handbook for annealed sheet steel, and assuming the flux density at each point to vary sinusoidally with time as the core rotates, calculate the eddy-current loss in this core for the conditions specified in Problem 13.

15. A current of 100 amperes in a certain coil is reversed in 0.001 second. The average value of the self-induced electromotive force set up in the coil during this interval is 2 volts.

(*a*) What is the self-inductance of the coil? Give answer in henries.

(*b*) Assuming the reluctance of the magnetic circuit linked by this coil to be constant, what will be the average value of the electr motive force induced in the coil when a current of 300 amperes is reversed in 0.0002 second?

(*c*) How does the self-inductance of a coil vary with the current in it?

16. Referring to Problem 10, the armature coil there described carries a current of 50 amperes. What is the average torque, in pound-feet, required to move this coil from a position directly under a north pole to a position directly under a south pole?

CHAPTER II

ELEMENTARY THEORY AND CONSTRUCTION OF DIRECT-CURRENT MACHINES

21. Dynamo, Generator, Motor.—An electric generator is a machine for converting mechanical work into electric energy. An electric motor is a machine for converting electric energy into mechanical work. The term " dynamo " is used to designate either machine.

In their general features of design and construction, a direct-current generator and a direct-current motor are identical. A direct-current generator may be used as a direct-current motor simply by connecting it to a source of electric energy. Similarly, a direct-current motor may be used as a direct-current generator by driving its rotatable member by an engine or other motor, and connecting an electric circuit or circuits to its terminals.

To secure best results, however, each kind of machine should be designed with special reference to its particular function. For example, a 110-volt motor used as a 110-volt generator will not be as satisfactory as a machine designed specifically to be used as a generator.

22. Essential Parts of a Direct-current Dynamo.—The essential parts of a direct-current dynamo are:

(*a*) **A magnetic field** whose strength does not vary appreciably during a complete revolution of the moving member of the machine. This magnetic field is usually produced by a stationary electromagnet (*NS*, Fig. 3), although a permanent magnet is used for this purpose in very small machines, called " magnetos." The term field is also used to designate the structure (winding, cores, etc.) used to produce the magnetic field. The winding (*FF*) of the field magnet is called the **field winding.**

(*b*) One or more coils of insulated wire mounted on a suitable structure (*A*, Fig. 3) so mounted that it can rotate freely in the space between the poles of the field magnet. This rotatable

member of the machine is called the **armature,** and the winding formed by the coils of wire is called the **armature winding.**

(*c*) A cylinder formed of a number of metallic segments (*C*, Fig. 3), insulated from one another and mounted on, but insulated from, the shaft which carries the armature. This segmented cylinder is called the **commutator.** The successive

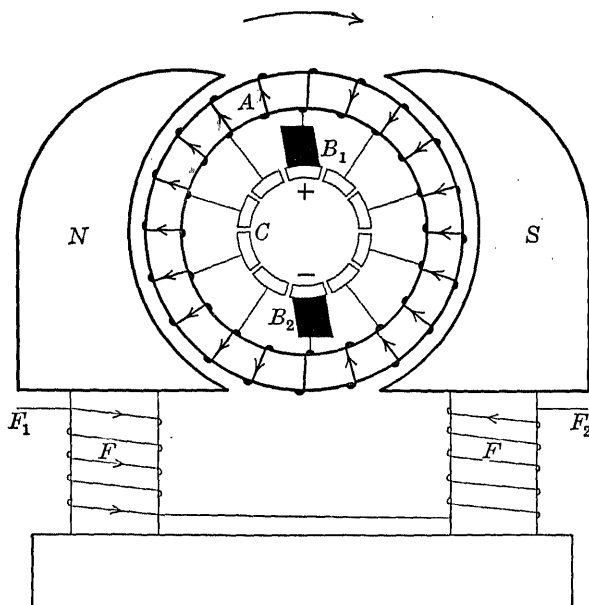



FIG. 3.—Essential Parts of Dynamo.

FF. Field Magnets. *A*. Armature. *C*. Commutator. B_1B_2. Brushes. F_1F_2. Field Terminals.

segments of the commutator are connected to equally spaced points in the armature winding.

(*d*) Two or more **brushes** (*B*, Fig. 3), which are usually small carbon or graphite blocks carried in suitable holders mounted on, but insulated from, the frame of the machine. These brushes are held, by spring pressure, against the cylindrical surface of the commutator.

23. Elementary Theory of Generator.—In Fig. 4 is shown the approximate distribution of the lines of force which are pro-

duced when a current is sent through the field coils *FF* of the machine shown in Fig. 3. The direction of the current in these coils is indicated by the + and − signs in the small circles *FF*, a + sign indicating that the current is in the direction away from the eye of the reader and a − sign that the current is in the direction toward the eye of the reader. The positive sense of

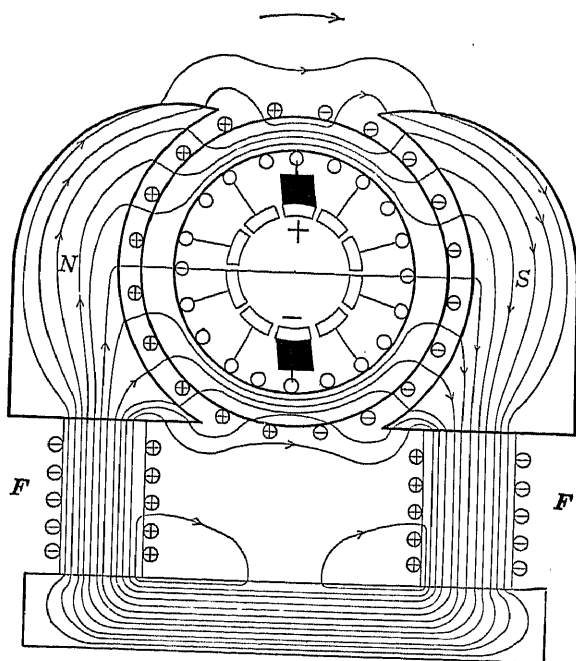



FIG. 4.—Diagrammatic Cross-section of Dynamo.

the lines of force is then as indicated by the arrows on these lines (see Article 9).

Imagine the armature to be rotated by an engine or other source of power in the direction indicated by the arrow at the top of the diagram. All the conductors on the outside of the armature core will then cut lines of force, the rate of cutting of these lines being proportional (1) to the velocity at which the armature is rotated, and (2) to the density of the magnetic field at the point at which the particular conductor is located at the instant under consideration.

An electromotive force is therefore induced in each of these conductors (see Article 11), and its direction is as indicated by the + and − signs in the circles which represent the armature winding. That is, all the conductors on the left-hand side of the armature in Fig. 4 have electromotive forces induced in them in the direction away from the eye of the reader, and all the conductors on the right-hand side of the armature have electromotive forces induced in them in the direction toward the eye of the reader. The direction of the electromotive force in each turn of the armature winding is also shown by the arrows on this winding in Fig. 3.

In the type of machine shown in Figs. 3 and 4, the core on which the armature winding is placed is an iron ring. Such an armature is called a **ring armature.** Due to the high reluctivity of air compared to that of iron, only a few lines of force pass across the hole in the ring, and therefore the wire on the inside of this ring cuts only a few lines of force. The electromotive force induced in these inside conductors is therefore practically zero. Each of these inside conductors may be looked upon merely as an idle connector which joins the front end (commutator end) of an outside conductor to the rear end of the next outside conductor.

Due to the inefficient use of the copper in a ring armature, and also to the difficulty of winding it, this type of armature is now seldom used. However, the ring armature is the simplest form to use in explaining the elementary theory of the dynamo. The ideas developed may then be readily extended to modern drum armatures (see Article 25).

Referring to Fig. 3 it is seen that the armature winding is a continuous closed winding, the outside conductors being all connected in series with one another by means of the inside conductors. The resultant electromotive force acting around this closed winding is therefore the sum of the electromotive forces induced in the outside conductors, assuming the electromotive forces in the inside conductors to be negligible.

When the conductors are uniformly spaced as shown in Fig. 3, then for each conductor under the north pole there will be a corresponding conductor under the south pole, and the electromotive forces induced in these two conductors will be equal and opposite,

as viewed from either end of the machine. Consequently the resultant electromotive force induced in the *whole* armature winding is zero.

However, an inspection of Fig. 3 will show that in all the conductors between the two brushes B_1 and B_2 on the left-hand side of the armature, the electromotive forces are all in the same direction, producing a rise of electric potential from the brush B_2 to the brush B_1. Similarly, the electromotive forces in the conductors on the right-hand side of the armature are also all in the same direction, and likewise produce a rise of potential from the brush B_2 to the brush B_1.

Consequently, with respect to these two brushes, the two halves of the armature winding are in parallel, in exactly the same way that the two batteries in Fig. 5 are in parallel. The resultant electromotive force between the two brushes is therefore equal to the resultant electromotive force in either *half* of the armature conductors, the armature being divided in half by a plane midway between the two pole faces. This dividing plane is called the **neutral plane.**

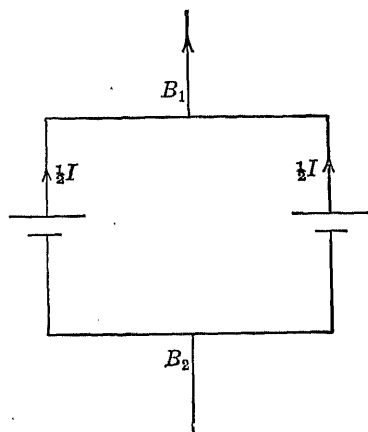



Fig. 5.

As the armature rotates, carrying with it the commutator, the conductors at the top of the armature pass successively from the left-hand side of the neutral plane to the right-hand side, and the conductors at the bottom pass successively from the right-hand side to the left-hand side. As a conductor passes from one side to the other of the neutral plane, the direction of the electromotive force induced in it is reversed, as viewed from the end of the armature, since the direction in which it cuts the magnetic field reverses. However, *with respect to the brushes,* the direction of the electromotive force in this conductor remains unchanged.

The commutator therefore acts as a rectifying device, keeping the resultant electromotive force between the two brushes constantly in the same direction.

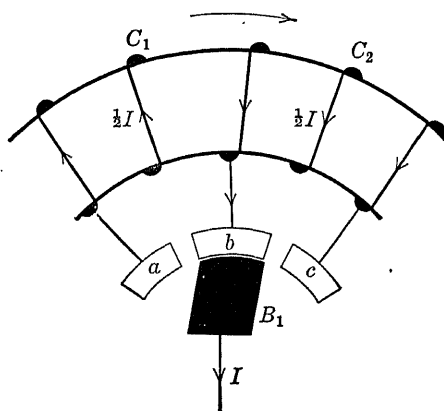
When an external circuit is connected to the two brushes and direct current will therefore flow in this circuit, in the direction 'om the brush which is at the higher potential (positive brush) to ıe brush which is at the lower potential (negative brush), that , from the brush B_1 to the brush B_2. Through the armature inding this current will be from the brush B_2 to the brush B_1, e., in the same direction as that of the electromotive force etween these two brushes. Also, since the two halves of the rmature winding are in parallel with each other, the current ı the conductors forming each half of the winding will be half f the current supplied to the external circuit, or load; compare ith two batteries in parallel, Fig. 5.

An enlarged view of the section of the armature and commuıtor in the immediate vicinity of the positive brush B_1 is shown ı Fig. 6. The arrows indicate the direction of the external urrent and the currents in the two halves of the armature windıg. (In this diagram the brush B_1 is shown inverted simply or clearness in the drawing.)

An inspection of Fig. 6 will show that as the armature rotates he current in the coil C_1 (that portion of the armature winding etween the commutator egments a and b) will e reversed in direction hen this coil C_1 has ıoved into the position ccupied by the coil C_2. ince the two halves of he armature winding re in parallel between he positive and negative rush, the current in ach half of the winding s one-half the external urrent I. Hence every ime a coil passes through he neutral plane the

FIG. 6.

urrent in this coil changes from $\frac{1}{2}I$ in one direction to an equal alue in the opposite direction.

This reversal in the direction of the armature current takes lace while the brush is short-circuiting the two commutator

segments which are connected to the ends of this coil; namely, in Fig. 6 the two segments a and b. In order to allow this reversal to take place, and at the same time not to break the connection of the armature winding to the external circuit, each brush must have a width at least sufficient to bridge the insulation between two adjacent commutator segments. In actual practice the brushes usually have a width several times greater than this.

The reversal of current in the armature coils, as the segments to which these coils are connected pass under the brushes, is usually referred to as **commutation.**

Due to the successive short-circuiting of the armature coil by the brushes, and also to the motion of the armature while each coil is short circuited, the *resultant* electromotive force between brushes, altough constant in direction, it is not strictly constant in value, but pulsates with a frequency proportional to the number of commutator bars and the speed. The greater the number of bars, however, the less is the *magnitude* of these pulsations. When the commutator has 10 or more bars per pole the fluctuation is practically negligible; see Article 32.

24. Elementary Theory of Motor.—The machine just described may also be used as an electric motor, provided a difference of electric potential is established between its brushes. The driving engine is then, of course, removed, and a mechanical load is connected to the shaft of the armature by means of a pulley and belt, or by suitable gearing.

Let the field be excited by a current flowing in the same direction as indicated in Fig. 4, and let the brush B_1 be connected to the positive terminal of the external source which supplies the current to the armature, and let B_2 be connected to the negative terminal of this source. B_1 and B_2 will then be respectively the positive and negative terminals of the armature, just as when the machine was operating as a generator.

However, since electric energy is now being supplied *to* the armature, instead of being supplied *by* it, the current through the armature winding will be in the *opposite* direction to that of the current when the machine is operating as a generator. The directions of the currents in the field and armature windings, when the machine is used as a motor, are shown in Fig. 7. Com-

pare this figure with Fig. 3, which corresponds to generator operation.

As shown in Fig. 7, in the outside conductors on the left-hand side of the armature the current will be in the direction toward the eye of the reader, and in the conductors on the right-hand side of the armature the current will be in the direction away from the eye of the reader. Consequently, on each armature conductor there will be a force exerted by the magnetic field pro-

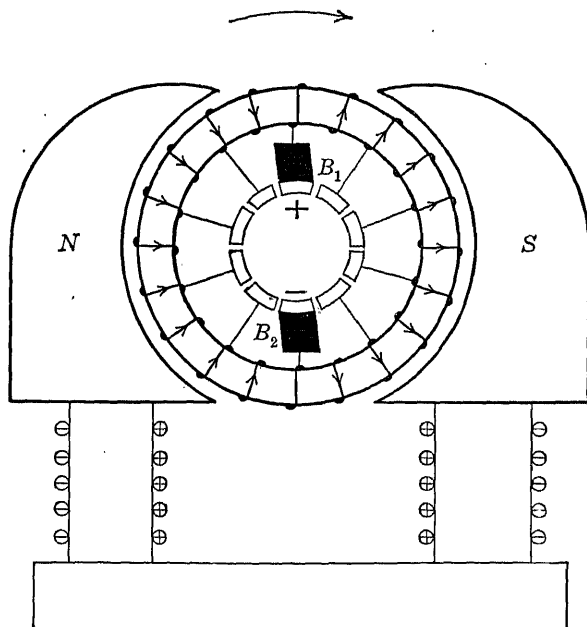



FIG. 7.—Dynamo operating as a Motor.

duced by the field winding, this force tending to move each conductor on the left-hand side *up* (see Article 20), and each conductor on the right-hand side *down*. Hence a torque will be exerted on the armature tending to rotate it in the direction of the arrow at the top of Fig. 7. Note that this is the same direction of rotation as that of the machine when used as a generator with its field excited in the same direction; compare with Fig. 3.

Due to this torque, the armature will begin to rotate (provided the torque developed is greater than the opposing torque of the mechanical load connected to the armature shaft). As soon as the armature begins to rotate, however, the armature conductors begin to cut the lines of force of the magnetic field established by the field winding. Therefore, just as in the case of a generator, an electromotive force is set up in these conductors.

Since the direction of the field and the direction of rotation are both the same as in Fig. 3, this electromotive force will be in the *same* direction as in the generator previously described, and will therefore be in the direction *opposite* to that of the current supplied to the motor. In a motor, therefore, the generated electromotive force is a *back electromotive force.*

As the speed of rotation increases, this back electromotive force increases, since the rate of cutting of the lines of force by the armature conductors increases. The current taken by the motor will therefore decrease, and ultimately become constant at such a value that the product of this current by the voltage impressed across the brushes is equal to the power required to drive the load connected to the armature shaft plus the internal losses.

In comparing the action of a dynamo as a generator and as a motor, it is of interest to note that just as in a motor a back electromotive force is developed, so also in a generator a back, or counter, torque is developed when the generator is supplying current to an external load. This follows from the fact that whenever a conductor carrying an electric current is in a magnetic field due to some other agent (e.g., an electromagnet), a force is always exerted on this conductor.

This mechanical force has nothing to do with the *electromotive force* in the conductor, but depends only upon the strengths of the current and the magnetic field and their relative directions. Consequently, since in the generator shown in Fig. 3 everything is the same as in the motor shown in Fig. 7, except the direction of the current in the armature conductors, there must be exerted on the armature conductors of a generator a force in the direction opposite to that of the force exerted on the armature conductors of a motor, namely, a force in the direction

opposite to that in which the armature is driven. This counter torque is the torque which the driving engine must over come to keep the armature rotating

25. Drum Windings.—As already noted, the particular type of armature shown in Figs. 3, 4 and 7 is called a ring armature, since the core which carries the armature winding is in the form of a ring, or "doughnut." In the early dynamo-electric machines the ring winding was usually employed, but in modern dynamos a different type of winding, called a **drum winding,** is universally used. The ring winding has two serious disdavantages, first, the armature must be wound by hand, and second, the conductors on the inside of the hole in the "doughnut" are practically inactive, since they cut only the few lines of force which leak across this hole.

Referring to Fig. 8, which shows in heavy lines a ring winding for a two-pole machine, it is evident that any inside conductor, such as 2, may be transferred to the outside of the ring, diametrically across from its original position. The electromotive force which is induced in this conductor as it cuts the magnetic field will then be in the *same* direction with respect to the brushes as that which is induced in the outside conductor to which it was originally connected, provided connections across the ends of the armature are made as indicated by the light lines. In Fig. 9 is shown the complete winding transposed in this manner, the end connections at the rear end of the armature being omitted for the sake of clearness. This type of winding is called a "drum winding," since the armature core for such a winding may be a solid * cylinder or drum, in contradistinction to a ring.

In the drum winding shown in Fig. 9 the sequence of the conductors is the same as in the ring winding shown in Fig. 8; that is, starting at the commutator end of conductor No. 1, there is an end connection across this end of the armature to conductor No. 2, then through No. 2 to a similar end connection across the back end of the armature (not shown) to conductor No. 3, then through No. 3, etc. In this drum winding there are the same number of conductors as in the ring winding shown in Fig. 8, but *all* the conductors in the drum winding cut lines of force,

* It must be laminated, however, to reduce eddy currents, see Article 18.

whereas only half of the conductors in the ring winding are active conductors. Consequently, for the same armature speed and same magnetic field, the electromotive force between the brushes

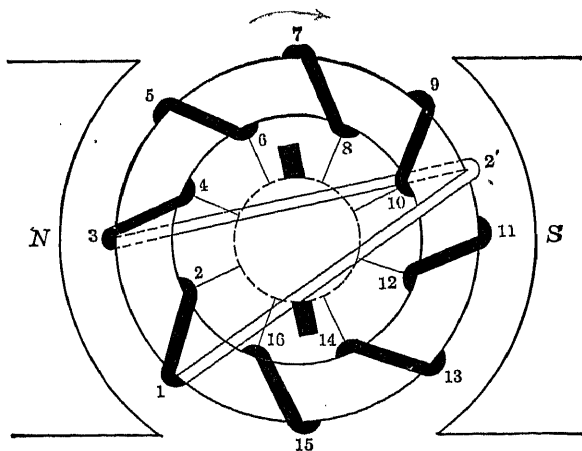



FIG. 8.

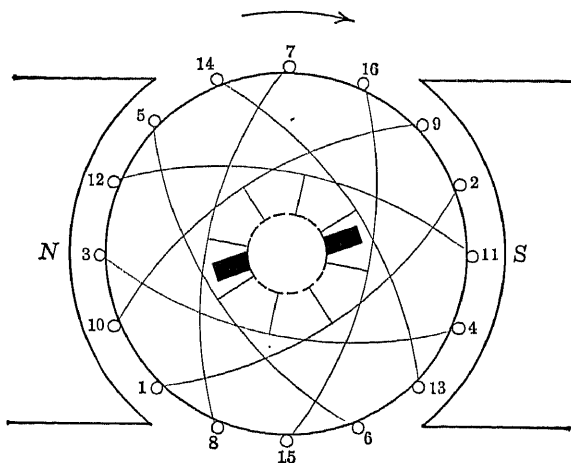



FIG. 9.—Two-pole Drum Winding.

in the drum winding is twice that between the brushes in the ring winding.

The maximum number of commutator segments which could

be used with the ring winding shown in Fig. 8 is 8, the same as the number of turns in the winding, which in the case of a ring winding is the same as the number of active conductors. In the drum winding, there are twice as many active conductors, but just the same number of turns as in the ring winding, since in the drum winding it takes two active conductors to make a turn. The maximum number of commutator segments which can be used with the drum winding is, as in the ring winding, equal to the number of turns, which in the case of a drum winding is half the number of active conductors.

As has already been pointed out, the purpose of the commutator is to reverse the connection between the brushes an each armature coil (turn or group of turns) when the electromotive force induced in this coil changes in direction. To do this, the brushes must be set in such a position that each brush short-circuits a coil as the active conductor or conductors which make up this coil pass from one side to the other of the neutral plane.

Referring to the ring and drum armatures shown in Figs. 8 and 9, it is evident that the positive brush must be set in such a position that, when the armature is in the position shown, this brush will short circuit the coil which contains conductor No. 7, namely the coil 6–7–8. This means that for the ring armature the brushes must be set midway between the poles (see Fig. 3), whereas for the drum armature, with the commutator connected to the armature in the symmetrical (and usual) manner shown in Fig. 9, the brushes must be *opposite the centers* of the two poles.

This is a characteristic feature of a drum winding, but the student should keep in mind that the position of the conductors which form the coil undergoing commutation is the same in the drum winding as in the ring winding, namely, midway between the two poles (or at most a few degrees from this position).

26. Multipolar Dynamos.—The copper in the end connections of an armature winding serves no other purpose than to connect the active conductors which form the sides of this winding. These connections should therefore be as short as possible. In

a two-pole dynamo the armature-end connections must have a length sufficient to span the diameter of the armature. However, by providing 4, 6, 8, or more poles, the distance which the end connections have to span can be reduced, the reduction in their length being greater the greater the number of pairs of poles used.

Fig. 10 shows a diagrammatic cross-section of a 4-pole dynamo, the armature conductors being represented by the small numbered

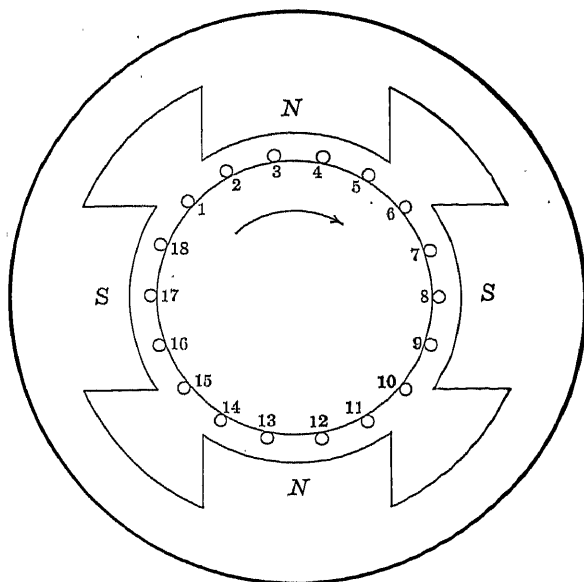



Fig. 10.—Four-pole Drum Winding.

circles. The simplest way to show the end connections is to imagine the cylindrical surface of the armature cut parallel to the shaft and spread out flat, as shown in Fig. 11 and Fig. 12. As shown in these two figures the end connections may be made in two different ways.

The armature winding resulting from the scheme of end connections shown in Fig. 11 is called a **lap winding,** since, as the winding progresses around the armature the successive turns lap back on each other.

The armature winding resulting from the scheme of connections

shown in Fig. 12 is called a **wave winding,** since the conductors and their end connections form a wave which progresses around the armature.

In both types of windings it is seen that the end connections span only the distance between adjacent poles, and not the full diameter of the armature.

In Fig. 10 the armature conductors are shown lying on the surface of the armature core. Actually, in modern machines, the armature conductors are placed in slots between teeth in the armature core (Fig. 25). The teeth serve the double purpose of keeping the armature conductors in place and of reducing the reluctance of the magnetic circuit of the machine, and therefore the field current required for producing the required magnetic field. Usually two conductors, or two coil sides, are placed in each slot, one below the other. In a diagram of the armature winding for a slotted armature the odd numbered conductors correspond to those conductors which lie in the top of the slots and the even numbered conductors to those which lie in the bottom of the slots.

A study of Fig. 11, which represents a **lap winding,** shows that starting at the brush *b* and passing successively through the conductors 7, 12, 9 and 14 the brush *c* is reached, and that the electromotive force induced in each conductor is in the direction of travel of the point which traces out this path. Again, starting at the brush *b* and passing successively through the conductors 10, 5, 8 and 3 the brush *a* is reached, the induced electromotive force in these conductors likewise being in the direction of travel of the point which traces out this path.

The brush *b* is therefore negative, and both the brushes *a* and *c* are positive and at the same (or practically the same) potential above *b*. The two brushes *a* and *c* may then be connected in parallel as shown. Following through the two paths through the armature winding from the brush *c* to the brushes *b* and *d* respectively, it will be seen that the brush *d* is at the same potential as the brush *b*. The brushes *b* and *d* are therefore both negative brushes and may be connected as shown.

For the simple four-pole lap winding shown in Fig. 11 four brushes are required, that is, as many brushes as there are poles.

A simple lap winding, no matter what may be the number of poles, always requires as many brushes as there are poles.

Also, in the simple four-pole lap winding shown in Fig. 11 the armature conductors form 4 parallel paths between the positive ...shes, the number of conductors in series in any ...paths being one-fourth of the total number of arma-...tors (neglecting the conductors which form the coil ... commutation). *In a simple lap winding the number*

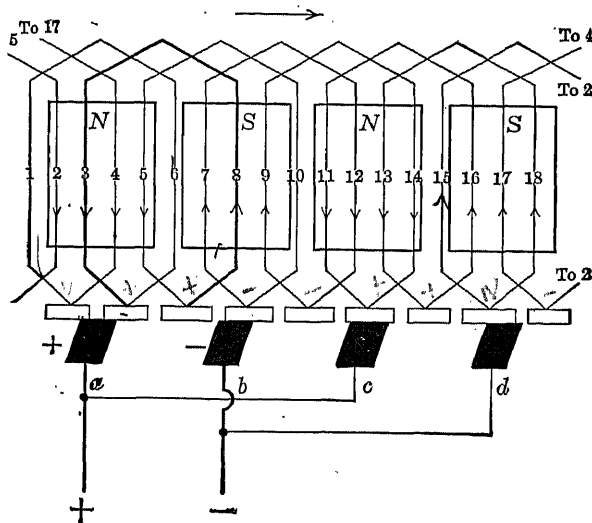



Fig. 11.—Four-pole Lap Winding.

of parallel paths through the armature from the positive to the negative brushes is always equal to the number of poles, no matter how many poles there may be.

Consider next the four-pole **wave winding** shown in Fig. 12. Starting at the brush *b* and passing successively through the conductors 11, 16, 3, 8, 13 and 18 the brush *a* is reached, the electromotive force induced in each of these conductors being opposite in direction to that of the travel of the point which traces out this path. The brush *a* is therefore the negative brush, and *b* is the positive brush. Again, starting at the brush *b* and passing successively through conductors 6, 1, 14, 9, 4, 17, 12 and 7 the brush *a* is reached, this series of conductors forming a second path between the positive and negative brushes.

Although in the particular case of the four-pole machine shown in Fig. 12, there are two more conductors in the second path between the two brushes than in the first, two of these conductors, namely 6 and 1, are inactive, since they are midway between two adjacent poles, where they cut practically no lines of force. The resultant electromotive force in each of the two parallel paths between the brushes is therefore substantially the same. In any actual machine there are usually several times as many conductors

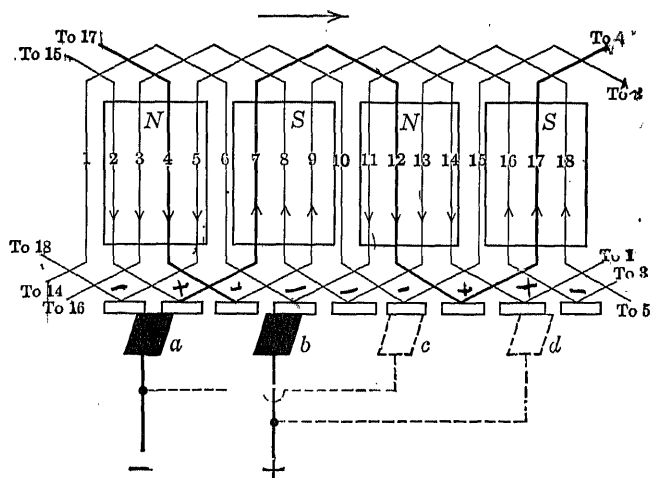



FIG. 12.—Four-pole Wave Winding.

as shown in Fig. 12, so that there is never any appreciable difference in the electromotive forces in the two paths.

In a simple wave winding, no matter how many poles there may be, only two brushes are necessary. However, if it is desired, as many brushes as there are poles may be used. For example, in the machine shown in Fig. 12, a second positive brush *d* may be provided, connected to the positive brush *b*, and also a second negative brush *c* connected to the negative brush *a*. Between the brushes *b* and *d* there are only the two armature conductors 6 and 1, in which there is practically no electromotive force induced, so that the external connection between *b* and *d* does not short-circuit any of the active conductors. Similarly, in the conductors 5 and 10, or 2 and 15, between the brushes *a* and *c* there is practically no electromotive force, or at least would not be in a winding having a large number of conductors.

In a simple wave winding, irrespective of the number of brushes and poles, there are but two parallel paths. Each path contains one-half the total number of armature conductors, all acting in series (neglecting the conductors which form the coils undergoing commutation).

In large machines, in order to improve commutation, two or more armature windings are sometimes sandwiched on a single

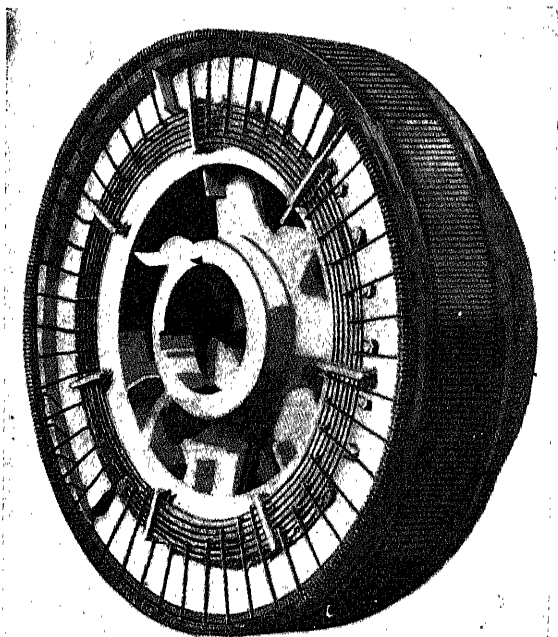



Fig. 13.—Rear View of Armature, showing Equalizer Rings.

armature core. For example, when two such windings are used, alternate armature conductors form one winding and the remaining conductors the other winding. The commutator bars for these two windings are then likewise sandwiched, and the brushes are made wide enough to touch simultaneously three commutator segments.

Windings of this type, i.e., made up of two or more sandwiched windings, are called **multiplex windings,** as distinguished from the simplex windings just described. Multiplex windings may be duplex, triplex, etc., and may be either lap or wave wound.

In a duplex lap winding the number of parallel paths through the armature is equal to twice the number of poles, and in a triplex lap winding the number of parallel paths is equal to three times the number of poles. In a duplex wave winding the number of parallel paths is always 4, and in a triplex wave winding 6.

In a lap winding there are as many parallel paths as there are poles (or a multiple thereof in multiplex windings), and the conductors which form any one path always lie under two adjacent poles. The number of lines of force leaving or entering the various poles in a multipolar machine may not be quite the same for all the poles, due to slight inequalities in the air gap, caused by wearing of the bearings or lack of exact centering of the armature with respect to the pole faces.

Consequently, the electromotive forces induced in the various parallel paths may be slightly different. These unbalanced electromotive forces will cause currents to flow in the armature and in the external connections between the brushes, even when the armature is supplying no current to an external load. This condition is exactly the same as would obtain were several batteries of slightly different electromotive forces connected in parallel. These circulatory currents flowing through the brushes may overload some of them and cause sparking. To prevent this, points of equal potential in the various paths are often connected by heavy insulated copper wires or bars. These so-called **equalizer connections** may be supported on the armature core, as shown in Fig. 13, or fastened to the commutator and tapped into the commutator bars.

In a wave winding each of the paths is subjected simultaneously to the action of all the poles, and any inequality in the magnetic flux will have the same effect upon each path. Consequently, in a wave winding equalizer connections are not necessary.

27. Permissible Number of Conductors in a Drum Winding.—In the simple drum windings shown in Figs. 11 and 12, each armature conductor is connected to a commutator bar. That portion of an armature winding which terminates in two commutator bars (e.g., the heavy loop in Fig. 11) is called a **winding element.** A winding element may consist of one or more armature *coils*. In Figs. 11 and 12 each winding element consists of a coil of one turn. It is often desirable to make each winding element of two or more turns, as

shown diagrammatically in Fig. 14, bringing only the ends of the first and last turn to the commutator. The group of conductors which form one side of a winding element is called an **inductor.**

The number of inductors that the winding advances on the commutator end of the armature is called the **front pitch** of the winding, and the number of inductors that the winding advances on the other end of the armature is called the **back pitch.** By the **average pitch** of a winding is meant the arithmetical average of the front and back pitches. In the lap winding shown in Fig. 11 the front pitch is -3, the back pitch $+5$, and the average pitch 4. In the wave winding shown in Fig. 12, the front pitch and back pitch are both 5 as is also the average pitch.

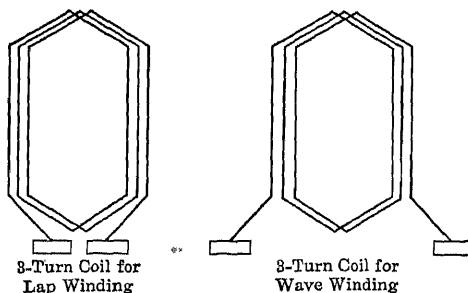



FIG. 14.—Armature Coils.

When the average pitch of an armature coil, measured circumferentially around the air gap, is considerably less than the **pole pitch** (i.e., than the circumferential distance between the centers of adjacent poles), the winding is called a **fractional pitch winding.** As a rule the difference between the average pitch of the winding and the pole pitch is less than 10 per cent.

Let p = number of poles
Z = total number of armature conductors
N = number of turns in each winding element
y = average pitch of winding
m = 1, 2 or 3 accordingly as the winding is simplex, duplex or triplex.

The following relations then hold:

Lap Winding.—The number of inductors is $\frac{Z}{N}$, and this ratio must be an even number. When a slotted armature is used, this ratio must also be a multiple of the number of slots. For good results, in a lap winding the number of inductors should be

a multiple of the number of poles, and the number of slots a multiple of the number of pairs of poles.

The front and back pitches must both be *odd numbers* and differ by $2m$.

Wave Winding.—Front and back pitches must be equal or differ by a multiple of 2. The average pitch y must be an integer, and the number of conductors Z must satisfy the relation

$$Z = N(py \pm 2m) \qquad (1)$$

For a detailed treatment of armature windings see Parshall and Hobart, *Armature Windings*, and E. Arnold, *Die Gleichstrommachine.*

28. Uses of the Various Types of Multipolar Windings.—For a given speed, number of poles and armature conductors, the wave winding gives a higher voltage than a lap winding, since in a wave winding (simplex) half of the conductors are in series between brushes, whereas in a lap winding (simplex) the number of conductors in series between brushes is equal to the total number of conductors divided by the number of poles.

For a given impressed voltage, number of poles and armature conductors, a wave-wound motor runs at a slower speed than a lap-wound motor. This is due to the fact that, since there are more conductors in series in the wave winding, the back electromotive force per conductor is less, and therefore the speed at which this conductor must move does not have to be as great.

For a given speed, terminal voltage and power output a wave-winding requires fewer conductors (but of a larger cross-section) than a lap winding. The smaller the number of conductors, the less is the space occupied by the insulation, which is an important factor, especially in the case of small machines. Also, as noted in Article 26, no equalizer connections are needed on a wave-wound armature.

Wave windings are therefore usually employed except for machines designed for large currents. Except in the case of railway motors, as many brush arms are employed as there are poles.

In the case of railway motors but two brush arms are, as a rule, employed. This is of particular advantage from the point of view of accessibility to the brushes. For example, in the case of a four-pole railway motor, only a single hand-hole (through

the top the casing) is necessary to get at the brushes for adjustment. Were four brush arms used, at least two hand-holes would be required.

The lap-winding, since it provides a large number of parallel paths, is particularly suited for high-current machines, i.e., for generators giving large currents or for motors taking large currents.

29. Field Excitation.—As has already been stated, the magnetic field of a dynamo may be produced either by a permanent magnet or by an electromagnet. Permanent magnets are used only for machines of very small output, such as magnetos for ignition of gas engines and the like. In all other dynamos the magnetic field is produced by an electric current flowing in coils wound on the poles of the machine.

The field current for a generator may be obtained either rom a separate source, such as another generator, or it may be obtained from the armature of the machine itself. When the field is excited from a separate source the machine is called a **separately-excited** machine. Self-excitation may be obtained in three ways, known respectively as shunt excitation, series excitation, and compound excitation.

Shunt Excitation.—In a dynamo designed for shunt excitation the field coils are made of a large number of turns of relatively small wires. These coils are all connected in series, in such a manner that the current through them will make successive poles alternate north and south poles. The two free ends of the group of coils are the field terminals, and are connected directly across the brushes, as shown diagrammatically in Fig. 15. For clearness in the diagram the field winding is shown as a single coil, and the armature and commutator as a circle.

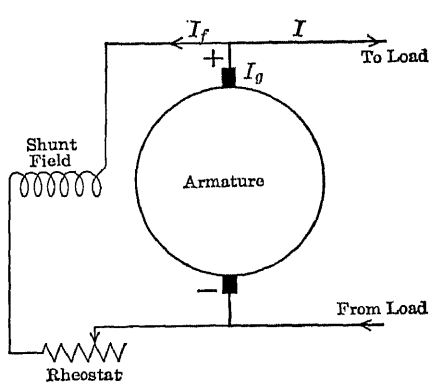



Fig. 15.—Shunt Excitation.

When the armature is at rest there is of course no current in the field coils. There is, however, a small amount of residual

magnetism in the field poles (see Article 16), so that when the armature is rotated a small electromotive force is induced in it, and this electromotive force causes a current to flow through the field coils.

If the residual magnetism in the field poles is in the proper direction with respect to the field connections, this current will increase the magnetic flux through the field and armature, which will cause the induced electromotive force to increase, thus increasing the field, and so on until a stable condition is established. This stable condition is reached when the drop of potential through the field winding (due to its resistance) is equal to the electromotive force induced in the armature, less the armature resistance drop (see Article 63).

Should the voltage of a shunt generator fail to build up, the field should be momentarily excited from a separate source, in order to increase the residual magnetism. This is called "flashing the field." If, after the field has been flashed, the voltage still fails to build up, this is an indication that the field winding is not properly connected. The connections between field and armature must then be reversed.

A variable resistance is usually connected in series with the shunt-field winding, as shown in Fig. 15, to provide means for varying the total resistance of the field circuit, and thereby the field current, so that the voltage between the brushes may be adjusted to whatever value is desired. This resistance is called the **field rheostat**.

A shunt-connected generator gives a terminal voltage which decreases as the load connected to it is increased. This is due to the increase in the resistance drop through the armature with increase in the load current, and also due to the demagnetizing effect of the current in the armature (see Chapter V).

A shunt-connected dynamo used as a motor, connected to constant voltage mains, takes a constant field current, this current being equal to the impressed voltage divided by the resistance of the field circuit. The flux cut by the armature conductors is therefore nearly constant (actually falling off slightly with increase of load, due to the demagnetizing effect of the armature current).

Since the armature must run at such a speed as will develop a

back electromotive force equal to the impressed voltage less the resistance drop in the armature winding, which resistance drop is usually small, it follows that the speed of a shunt motor remains nearly constant with increase or decrease of load.

Series Excitation.—In a dynamo designed for series excitation the field coils are made of heavy wire or bars, since these coils are to carry the total current of the machine. These coils are connected in series in such a manner as to make successive poles alternate north and south poles, the same as in a shunt-connected machine. The two free ends of this group of coils are the field terminals, and are connected in series with the armature as shown diagrammatically in Fig. 16.

When there is no load connected to a series generator, the electromotive force developed by it is due solely to the residual magnetism in its field poles, and is extremely small. When a load (e.g., a bank of lamps), is connected across its terminals this electromotive force builds up in exactly the same way as in a shunt generator, producing a larger and larger current in the load and likewise in the field coils, until the voltage drop across the load reaches a value equal to the generated electromotive force less the combined resistance drop in the armature and field windings. The final value of the electromotive force of a series generator therefore depends upon the resistance (and back electromotive force, if any) of the load, just as the electromotive force of a shunt generator at no load depends upon the total resistance of the field circuit.*

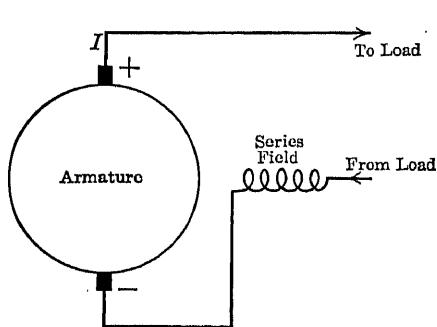



Fig. 16. Series Excitation.

A series generator therefore gives a terminal voltage which increases as the load supplied by it increases.† Such a machine is seldom used, except as a booster (see Article 82) but series

* A shunt generator at no load may be looked upon as a series generator, the load in this case being the shunt-field rheostat.

† Provided the load current does not become large enough to saturate the magnetic circuit of the machine; see Article 62.

fields in conjunction with shunt fields are extensively employed, giving rise to what is known as compound excitation; as explained in the following article.

A series dynamo used as a motor, however, has an extremely useful characteristic, namely, as the load (opposing torque) on it increases (the impressed voltage being kept constant), its speed falls off. (This follows from the fact that as the load increases, tending to make the motor draw more current from the supply mains, this increased current, which also flows through the field coils, produces a stronger magnetic field through which the armature conductors must rotate, and therefore the speed of the armature must fall off to maintain the balance between the impressed voltage and the back electromotive force generated in the armature.)

This characteristic is of special advantage in traction work, where the variations in load due to frequent changes of grade are large. For example, to keep an electric car running at a given speed on a level track may require, say, 20 horsepower, this power being proportional to the speed and the opposing force (friction) which must be overcome. Now suppose that the car begins to ascend a grade, and that due to this grade the opposing force to be overcome is doubled. If the speed remains the same, the power drawn from the power house will likewise double. However, should the speed automatically drop to, say, 70 percent, the power drawn from the power house will increase only 40 percent $(2 \times 70 - 100)$.

Therefore, with series motors the fluctuation in the load on the power house will be much less than it would be were the car equipped with shunt motors. It is also more practicable to obtain with a series motor the high torque required for starting a car.

One disadvantage of a series motor is that when it is disconnected entirely from its mechanical load, its armature may accelerate to such a high speed as to destroy itself by centrifugal action. There is of course no danger of this happening to the driving motors of an electric car.

Compound Excitation.—In a dynamo designed for compound excitation, both shunt and series field are provided. There are two ways in which these fields may be connected, known respectively as the short-shunt connection (Fig. 17) and the long-shunt

connection (Fig. 18). The only difference in these two connections is that in the short-shunt connection the shunt field is connected across the armature only, whereas in the long-shunt connection the shunt field is connected across the armature and series field. The short-shunt connection is ordinarily employed for generators, since it gives a slightly better voltage regulation; see Article 64.

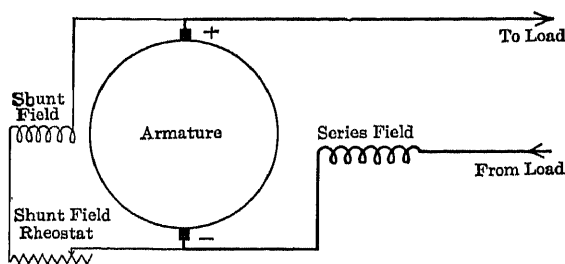



Fig. 17.—Compound Excitation, Short-shunt.

In a compound generator the shunt and series fields are connected in such a manner that the currents through them magnetize the field poles in the same direction. With this connection, the effect of the series field is to compensate for the falling off in the terminal voltage which tends to take place with increase

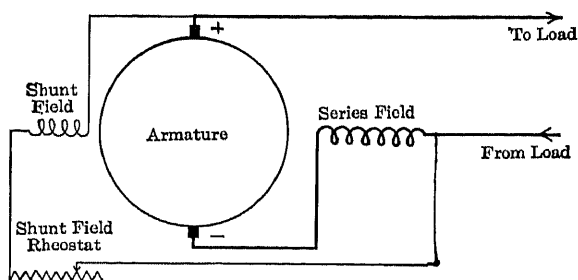



Fig. 18.—Compound Excitation, Long-shunt.

of load, due to armature resistance and to the demagnetizing action of the armature current.

By properly proportioning the shunt and series fields it is possible to make the terminal voltage at full load the same as at no load; the machine is then said to be **flat-compounded.** By using a relatively stronger series field, the terminal voltage at

full load may be made higher than at no load; the machine is then said to be **over-compounded.**

Practically all large generators have compound excitation, since for both electric lamps and electric motors it is desirable to keep the voltage supplied substantially constant, irrespective of the load on the generator.

Compound motors are less extensively used than shunt and series motors. By connecting a series-field winding in such a manner that it tends to magnetize the field poles in the same direction as the shunt-field winding, a motor may be obtained which approximates in its speed characteristics a series motor. Such a motor also has the added feature that the shunt-field current, being always present, produces a sufficiently strong magnetic field to prevent the armature from "running away," should the mechanical load on the motor be entirely removed. When the shunt and series fields are thus connected to aid each other the machine is called a **cumulative compound motor.**

By connecting the shunt and series fields so that they oppose each other, the series field may be made to compensate for the drop in speed which would take place in some motors were a shunt field only used. When the two fields are connected in this manner the machine is called a **differential compound motor.** Differential compound motors are very seldom used, since the speed of a shunt motor is sufficiently constant for most practical purposes.

30. Mechanical Construction of Direct-current Dynamos.— The main parts of the field of a direct-current dynamo are the frame or yoke, pole cores, pole shoes * and windings. The main parts of the armature are the shaft, spider, laminated core, winding, and commutator.

Field Frame.—The field frame or yoke serves as a support for the machine and forms a path of low reluctance connecting the field poles. In small machines the frame is often made of cast iron. In another type of construction the frame is made of a heavy steel plate rolled into the form of a ring, with the joint welded; suitable feet are riveted to this ring. In large machines the frame is usually made of cast steel. The casting is made

* In modern machines the pole shoe is usually a part of the pole-core; see Art. 30.

in two parts which are bolted together at carefully machined joints, as shown in Fig. 19. The purpose of making the frame in two parts is to facilitate dismantling for repairs.

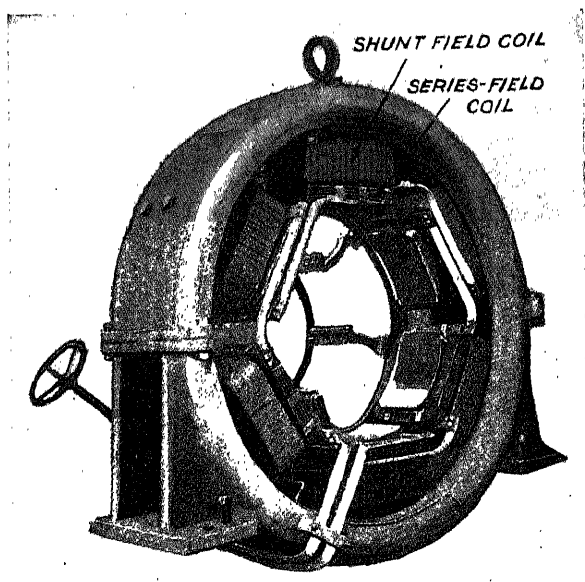



FIG. 19.—Field for Non-Interpole Generator.

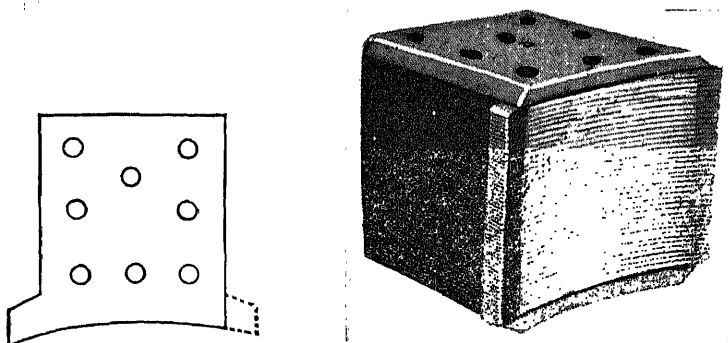



FIG. 20.—Pole Piece of Direct-current Generator.

Field Cores.—Since the field cores carry the field coils, they should be made of as small a cross-section as is consistent with low reluctance, in order to keep the length of the wire required for

the coils down to a minimum. These cores are made of cast steel, forged steel, or steel laminations. When cast or forged steel is used, the cores are made of circular cross-section. When laminated cores are used (which is now almost universal) they are made of rectangular cross-section. A typical laminated field core is shown in Fig. 20.

Pole Shoes.—A flux density of 100,000 lines per square inch, or even higher, may be used economically in the pole cores. In the air-gap between the pole shoe and the armature the practical limit to the flux density is about 60,000 lines per square inch. It is therefore necessary to flare out the flux as it passes from the field cores to the armature, by enlarging the face of the pole opposite the armature core. This enlarged part of the field pole is called the pole shoe, and in modern machines is nearly always laminated.

When solid field cores are used the pole shoe is usually bolted or dove-tailed to the field core. When laminated poles are employed, the laminations are made with a suitable offset to form the necessary flaring out of the pole face, as shown in Fig. 19. To minimize the magnetic leakage between poles, and also to reduce armature reaction (see Chapter V), the offset on each lamination is sometimes made on one side only, and the laminations are assembled with the offsets of the alternate laminations on opposite sides of the pole, as may be seen in Fig. 20. Another type of construction is shown in Fig. 61.

The purpose of laminating the pole shoes is to reduce the eddy currents which are induced in the pole-face by the rapid pulsations produced in the flux in the pole-face by the teeth of the armature core as they move by it. The lines of force crowd into these teeth, so that the flux density opposite a tooth is greater than the flux density opposite the slot between two teeth, with the result that any closed path in the pole-face is subjected alternately to a large flux and then a small flux, and an alternating current is therefore induced in it.

Field Coils.—Shunt-field coils are usually wound with cotton-covered wire, either of round or rectangular cross-section. These coils are not as a rule wound directly on the field cores, but are wound on suitable moulds, then carefully taped and impregnated with insulating compound by a vacuum process. This makes

a solid coil not easily deformed by handling. Sometimes the coils are wound on light metal spools, which give greater rigidity. The appearance of a shunt-field coil is clearly shown in Fig. 19.

In small machines the series-field coils are usually made of heavy round wire. In larger machines these coils are commonly ɔper strap. One type of series coil is shown in Fig. 19.

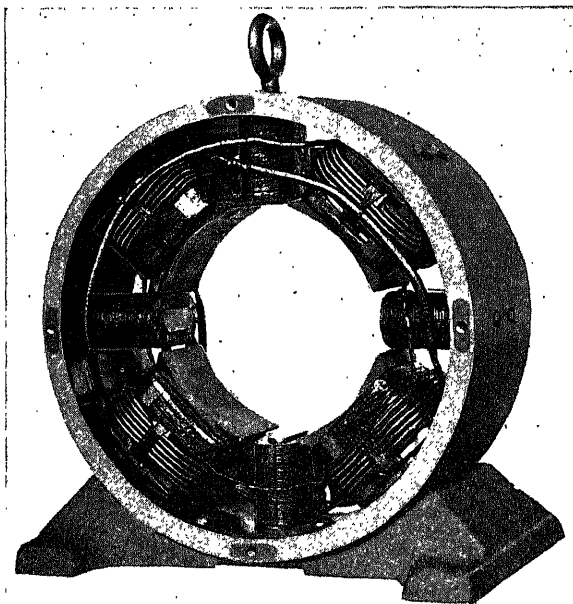



Fig. 21.—Field for Commutating-pole Generator.

Commutating Poles.—Most modern direct-current generators and motors are provided with commutating poles, which are small auxiliary poles between the main poles, and provided with a series winding; see Article 51. The field and frame of a commutating pole direct-current generator are shown in Fig. 21.

Armature Core.—The armature core is always built up of sheets of soft iron or mild steel, 0.014 to 0.025 inch thick. The core is thus laminated in order to cut down the eddy currents which would otherwise be excessive. For armatures having a diameter of not over 30 inches, each lamination usually forms a complete disc, such as shown in Fig. 22. In small machines these discs

are keyed directly to the armature shaft. In larger machines they are keyed to a spider which in turn is keyed to the shaft. The hub and spokes forming the spider are clearly shown in Fig. 13.

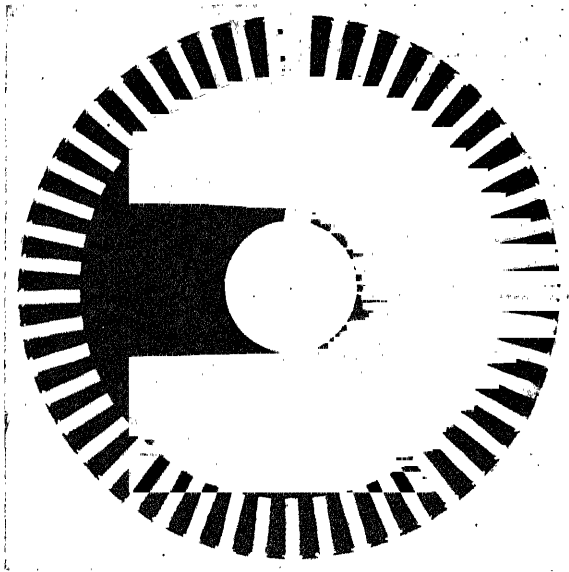



FIG. 22.

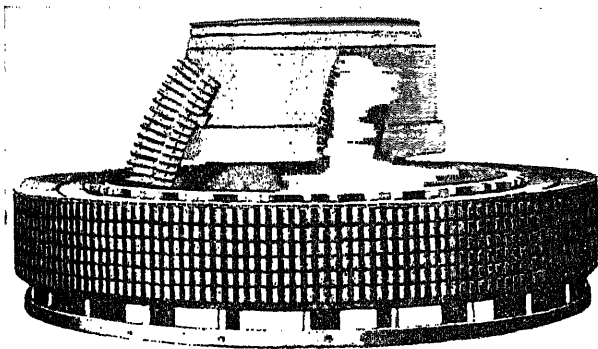



FIG. 23.

In very large armatures the core is built up of laminations which are segments of a complete ring, the segments being laid up with butt joints, with the joints in successive layers staggered.

This type of construction is shown in Fig. 23.

Suitable end plates are provided at each end of the core to clamp the laminations tightly together.

To provide ventilating ducts for keeping the core cool, suitable spacers are inserted after every two or three inches of laminations, as shown in Fig. 23. One of the spacers used in this core is shown

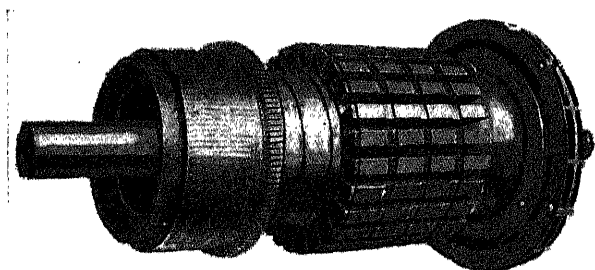



Fig. 24.—Armature of Railway Motor, Showing Ventilating Fan.

to the left in the figure, lying up against the part of the spider on which the commutator is to be mounted. When the laminations are mounted on a spider, the armature, when rotating, acts like a fan, drawing air in through the space between the

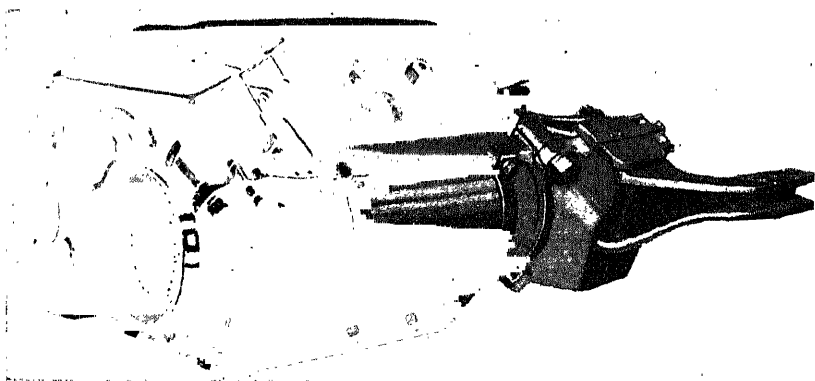



Fig. 25.—Railway Motor.

spokes of the spider and throwing it out through the radial openings formed by the spacers. To assist in the cooling, the laminations are also sometimes provided with holes which form openings in the core parallel to the shaft.

When the service is such as to require a high capacity machine

in a limited space, as, for example, in the case of railway motors, forced ventilation is frequently used. This is accomplished by mounting on the armature shaft, usually at the pinion end, a centrifugal blower, as shown in Fig. 24. The frame of such a motor is so designed as to completely enclose the armature and field, as shown in Fig. 25, except for suitable screened openings through which the ventilating air is drawn in and expelled. The cooling air not only passes over the surface of the field coils, armature, and commutator, but also passes through the commutator spider and through the longitudinal ducts formed by holes in the armature laminations.

The slots in the surface of the core may be either straight-sided, or may have a small notch in each side near the top, as shown in Fig. 26. When a straight-sided slot is used, the conductors are held in place by binding wires. When a notched slot is used, fiber or wood wedges, driven into the notches after the conductors have been placed in the slots, hold the conductors in place. Ample insulation must be provided between the conductors and walls of the slot to prevent grounds, as indicated in Fig. 26.

Fig. 26.

Armature Coils. — The armature coils are made of round wire, square wire, straps or bars. Wire coils are usually wound on collapsible forms, the individual conductors being fastened together by a wrapping of insulating tape. These *formed coils* for any given armature are all exactly alike. As one side of a coil is to go in the top of a slot and the other side in the bottom of a slot, it is necessary to give the ends of the coil a peculiar twist, as shown in Fig. 27. A partly wound armature is shown in Fig. 28.

Commutator.—The commutator is made up of segments of copper which have the form shown in Fig. 29. These pieces are made from hard-drawn copper, or are drop-forged. The segments are laid up with strips of "built-up" mica in between them to form a cylinder, which is then tightly clamped in a heavy external ring. The *V* groove formed by the notches in the two ends of the segments are then accurately turned.

Mica bushings formed to fit into these *V* grooves are then placed

in them, and steel *V* rings are inserted into each end and drawn up tight by means of bolts, see Fig. 30. These *V* rings, fitting into the *V* grooves in the segments, hold them securely in position, but are insulated from the segments by the mica *V* rings.

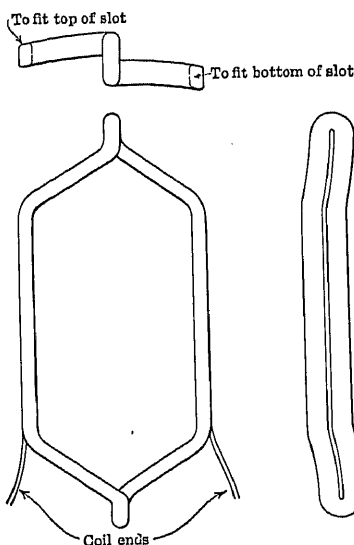



FIG. 27.—Armature Coil.

The external ring used to hold the commutator in shape at the start is then removed, the commutator is placed on a suitable mandrel or on the table of a boring mill, and the outside surface is turned to true cylindrical form.

The upright slotted portions of the commutator bar shown in Fig. 29 is to receive the ends of the commutator coils, which are soldered into them; see Fig. 28.

For very high speed machines, such as turbo-generators, the commutator is given the necessary strength by shrinking onto it heavy steel rings well insulated from the segments by mica bushings.

Brushes and Brush-holders.—The brushes are usually blocks of graphitic carbon. In low-voltage machines brushes of copper gauze, compressed into a rectangular form, or patented metal compounds, are sometimes used. Each brush is held in a suitable holder, one form of which is shown in Fig. 31. These holders are mounted on a stud, as many of them being placed side by side as are necessary to provide sufficient brush area to carry the current (1 sq. in. for every 30 to 50 amperes).

FIG. 28.—Method of Winding.

The studs for all the brushes are mounted, in insulating bushings, on a single ring, called the **rocker ring,** and this ring is suit-

ably mounted on the frame of the machine so that the brushes may all be shifted around the commutator to the position which gives the best commutation. The rocker ring and brushes are clearly shown in Fig. 32.

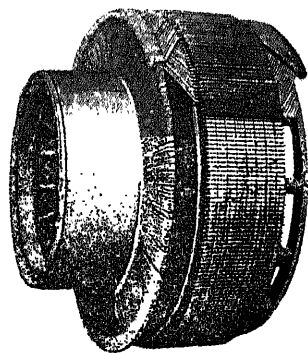
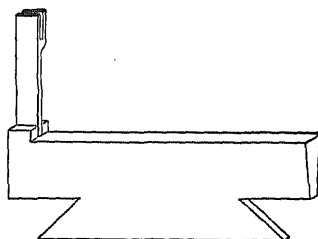



FIG. 29.—Commutator Segment.

The brushes should bear upon the commutator with a pressure of about 1.5 lb. per square inch of contact area. A suitable spring is provided in the brush holder, with means for adjusting its tension; see Fig. 31. The spring is not relied upon to carry the current, but a "pig-tail" of twisted copper wire is usually provided, with one end moulded into the carbon and the other end fastened to the metal of the brush holder.

Another method of securing a good electrical connection between the carbon brush and the brush-holder is to copper-plate the

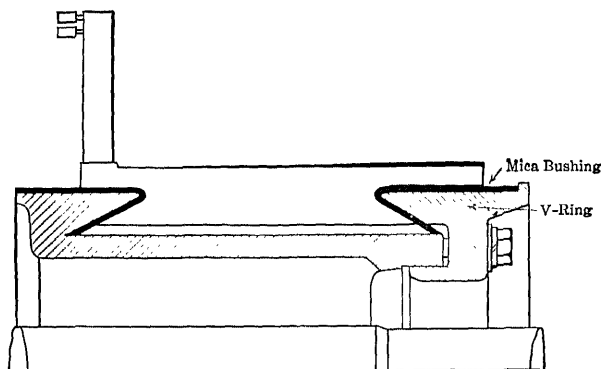



FIG. 30.—Construction of Commutator.

upper end of the carbon, and clamp the brush end of the pig-tail to this copper plating; see Fig. 31.

31. Capacity and Rating of Direct-current Dynamos.—By the capacity of a direct-current dynamo (or more specifically by the *power* capacity, as distinguished from its *electrostatic* capacity) is usually meant the maximum load in killowatts (or horsepower) which the machine can supply under certain specified conditions, (1) without the temperature of any part of its insulation reaching

a value which will injure this insulation, and (2) without causing injurious sparking at the brushes.

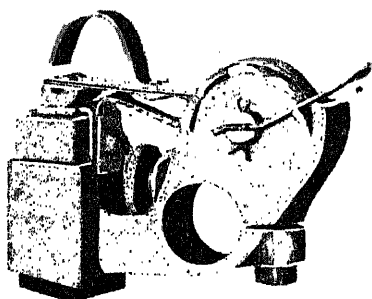



Fig. 31.—Brush-holder.

The continuous rating of a direct-current machine is the load which the manufacturer considers as the safe maximum load to which the machine should be continuously subjected, under certain specified conditions with reference to voltage, speed, ventilation, etc. The rating of a machine is usually less than its capacity, in order to allow a reasonable factor of safety.

Most machines can be overloaded for a short period of time, i.e., supply for a short time a load in excess of their continuous

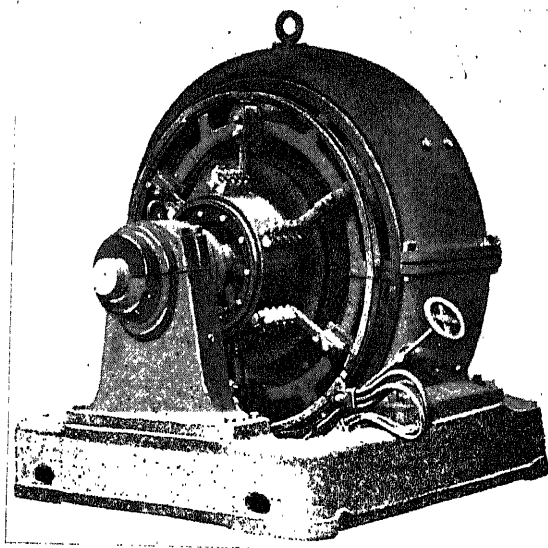



Fig. 32.

rating, without injuring the insulation or commutator. Machines designed for intermittent loads are therefore usually given a

"short-time" rating. For example, a railway motor is usually rated in terms of the number of horsepower which it can supply for *one hour* without injury to the insulation or commutator.

The conditions usually specified in stating the rating of a generator are the terminal voltage and speed to which this rating applies; for example, a 1000 kw., 660 volt, 750 r.p.m. generator. Similarly, the conditions usually specified in stating the rating of a motor are the terminal voltage, and, except in the case of a series motor, the speed to which this rating applies. A machine which is rated at a given voltage and speed may of course be operated at other voltages or speeds, but the load which it can safely carry under these new conditions will in general be different from its rating.

Experience has shown that for each type of insulation there is a fairly definite temperature above which the insulation will deteriorate if this limiting temperature is continuously or repeatedly exceeded. The limiting temperatures for various classes of insulation are as follows:

Cotton, silk, paper and similar materials when neither treated, impregnated or immersed in oil (usually referred to as Class O insulation)	90° C.
Cotton, silk, paper and similar materials when properly treated, impregnated or immersed in oil (usually referred to as Class A insulation)	105° C.
Enamel used as insulation on wires	105° C.
Mica, asbestos and similar materials containing cotton or similar material as a binder (usually referred to as Class B insulation)	125° C.

These are the temperatures which should not be exceeded in *any part* of the insulation, and are from 5 to 15 degrees higher than the *observable* temperature in a finished winding, depending upon the method employed for measuring the temperature; see Chapter IX.

PROBLEMS

1. Draw to scale (as nearly as can be done by external measurements) a cross-sectional view and a longitudinal view of one of the generators in the laboratory, and label the various parts.

2. A 2-pole ring armature is wound with 800 feet of No. 6 A. W. G. copper wire.

(*a*) What is the resistance of this armature winding (between brushes) at 20° C.? (*b*) At 90° C.?

3. The electromotive force generated in each half of the armature winding described in Problem 2 is 120 volts. The machine is separately excited.

(*a*) What is the terminal voltage of this machine when operating as a generator delivering 100 amperes? Assume a drop of potential of 1 volt at each brush contact.

(*b*) What must be the impressed voltage when this machine operates as a motor, taking 100 amperes from the line, the generated electromotive force being 120 volts as before.

(*c*) If the magnetic field in which the armature rotates is kept constant, in which case will the armature run at the higher speed? Why?

(*d*) Compare the value and direction of the torque due to the reaction between the magnetic field and the armature current in the two cases.

4. A 6-pole simplex armature winding has 38 inductors. Show by a diagram similar to Fig. 11 how these conductors may be connected to form a lap winding. Also show the location and external connections of the brushes.

5. Show by a diagram similar to Fig. 12 how the inductors in Problem 4 may be connected to form a 6-pole wave-winding. Locate the brushes.

6. (*a*) What will be the relative values of the armature resistances of the lap and wave windings in Problems 4 and 5.

(*b*) What will be the relative values of the electromotive forces generated in these two windings for the same speed and flux per pole in each case?

(*c*) For the same back electromotive force and same flux per pole in each case, what will be the relative speeds of these two armatures when the machines are used as motors?

(*d*) For the same current per conductor in each case, what will be the relative current outputs of the two armatures?

(*e*) What will be the relative RI^2 losses in the two armatures?

7. A certain machine is rated as a 100 horse-power, 600-volt, series motor. The efficiency at rated load is 88 per cent. The losses in the machine, at rated load, expressed as percentages of the power output, are:

RI^2 loss in armature winding	3.0%
RI^2 at brush contacts	0.4
RI^2 loss in field winding	2.5
Hysteresis loss	2.0
Eddy-current loss	2.5
Friction and windage	1.6
	12.0

Determine:

(*a*) The resistance of each winding;

(*b*) The total drop of potential at the brush contacts;

(*c*) The power loss, in watts, due respectively to hysteresis, eddy-currents, and friction and windage.

8. A 5-H.P. 110-volt, shunt motor has rated voltage impressed across its terminals and the line current is found to be 7.1 amperes when the motor is

running free at 1200 r.p.m. At this speed the resistance of the shunt-field circuit is 81 ohms (including the resistance of the rheostat). The resistance of the armature circuit (including the resistance of the brushes and brush contacts) is 0.2 ohm.

It is observed that when the resistance of the field circuit is decreased to 62 ohms, the speed of the motor drops to 960 r.p.m. when running free, at which speed the motor draws 3.9 amperes from the line.

(*a*) Find the field and armature currents for both speeds.

(*b*) What is the RI^2 loss in the field and in the armature at each speed?

(*c*) Why is the total input at each speed greater than the RI^2 losses in the field and armature.

(*d*) What is the value of this additional loss at each speed?

9. The motor of a motor-generator set takes 60 amperes at 125 volts from the supply mains and delivers 80 per cent of its power input to the generator it drives. This generator in turn delivers 90 per cent of its power to a 60-volt welding circuit. How many amperes flow through this welding circuit?

10. The electromotive force of a shunt generator decreases from 220 volts to 193 volts when the speed is reduced from 1000 to 900 revolutions per minute. The flux per pole at the higher speed is 2,000,000 lines.

(*a*) What is the flux per pole at the lower speed?

(*b*) What would the electromotive force of the generator be at the lower speed if its flux were kept constant at a value of 2,000,000 lines?

11. A certain bipolar shunt generator is rated to give its full-load current of 40 amperes at a terminal voltage of 125 volts. To give 125 volts at zero-load, a field excitation of 9000 ampere-turns is required. At full-load a field excitation of 11,000 ampere-turns is required. How many turns are required in the series-field winding to give flat compounding?

12. A certain short-shunt compound generator has a field-circuit resistance of 100 ohms. The resistance of the armature circuit (including the resistance of the brushes and brush contacts) is 0.1 ohm. The resistance of the series field is 0.04 ohm. The generator delivers 50 amperes to its load at a terminal voltage of 125 volts.

(*a*) What is the value of the shunt-field current?

(*b*) How much power is consumed in the shunt-field winding?

(*c*) What percentage of the power consumed in the shunt field is converted into heat under steady running conditions?

(*d*) What is the value of the generated electromotive force?

(*e*) How much power is lost in the series-field winding?

(*e*) What is the value of the armature current?

(*f*) How much power is lost in the armature circuit due to the resistance of this circuit?

13. A certain shunt dynamo, when operated as a generator at a speed of 1800 r.p.m., has a terminal voltage of 115 volts, when supplying a current of 100 amperes to the load connected to it. The resistance of its shunt-field circuit is 55 ohms and the resistance of its armature circuit (including brushes and brush contacts) is 0.07 ohm.

(*a*) What is the value of the electromotive force generated in the armature of this machine?

(*b*) The machine is next operated as a motor, and 115 volts are impressed across its terminals. The resistance of its field circuit is not changed. The mechanical load is adjusted so that the machine takes from the line the same current (100 amperes) as was supplied to the line when it was operated as a generator. What is the value of the back electromotive force generated in the armature?

(*c*) What will be the value of the field current under each of the two conditions of operation?

(*d*) What will be the speed of the machine when operating as a motor under the conditions specified in (*b*)? (In answering this question the effect of armature reaction is to be neglected.)

14. A certain shunt generator when operated at no-load at a speed of 1500 r.p.m. generates an electromotive force of 120 volts when the total resistance of its field circuit is 60 ohms. How much additional resistance must be inserted in its field circuit in order that, when its speed is increased to 1800 r.p.m., its electromotive force will increase proportionally?

15. A 4-pole shunt generator, which at no load develops a terminal voltage of 600 volts, supplies 100 amperes to a motor located at a distance of 1 mile from the machine. The transmission line between the generator and the load consists of two No. 0000 A. W. G. copper wires.

(*a*) What is the voltage at the motor end of this transmission line when the motor is not connected?

At no load the generator field current is 6.3 amperes. In order to keep the voltage at the motor equal to its no load value, it is found necessary to increase the generator field current to 9.7 amperes when 100 amperes are supplied to the motor. Each field coil of the generator has 2000 turns.

(*b*) What is the terminal voltage of the generator under these conditions.

(*c*) How many series-field turns per pole would be required in order to make it unnecessary to change the setting of the shunt-field rheostat and yet keep the voltage at the motor at the same value as in (*a*)?

(*d*) This would result in a compound generator of what type?

(*e*) Draw a diagram showing the connections between the armature, shunt-field and series-field windings.

CHAPTER III

ARMATURE ELECTROMOTIVE FORCE AND THE MAGNETIC CIRCUIT

32. The Electromotive Force of a Dynamo.—The electromotive force generated in the armature of a direct-current dynamo, whether the machine be operated as a generator or motor, may be readily expressed in terms of (1) the number of armature conductors in series between the negative and positive brush sets, (2) the number of poles, (3) the total magnetic flux per pole, and (4) the speed at which the armature rotates.

The lines of force in the air-gap of a dynamo enter the armature under each north pole of the field magnetic, and leave the armature under each south pole. At some point * between each pair of north and south poles these lines therefore reverse in direction with respect to the surface of the armature. The point at which the lines of force reverse in direction with respect to the surface of the armature is called the **electric** (or magnetic) **neutral.**

The total number of lines of force which enter (or leave) the armature between two successive electric neutrals is called the **useful flux per pole.**

When the magnetic field is due to the field current only (no current in the armature), the electric neutral between each pair of poles will have a definite position, depending upon the shape of the pole faces. This no-load position of the magnetic neutral (i.e., the position corresponding to no current in the armature) is called the **mechanical** (or geometrical) **neutral.** When the pole faces are all alike and each is symmetrical with respect to the axis of its pole, the mechanical neutral is midway between poles.

When the brushes are set so that they short-circuit an armature coil when the conductors which form the sides of this coil

* Strictly, along some line parallel to the shaft of the armature. It is usually more convenient, however, to confine one's attention to a given cross-section of the armature.

are in the mechanical neutral, the brushes are then likewise said to be in the mechanical neutral. For a ring-wound armature the brushes will then be between the poles, see Fig. 7. For a drum armature, however, due to the way in which the conductors are connected across the ends of the armature, the brushes are usually opposite the centers of the poles; see Fig. 8.

In order to obtain sparkless commutation it is usually necessary, unless the machine is equipped with commutating poles, to shift the brushes slightly from the mechanical neutral, in the direction of rotation in the case of a generator, and opposite to the direction of rotation in the case of a motor. When the armature is loaded, the electric neutral is then at some point *between* the mechanical neutral and the actual position of the brushes.

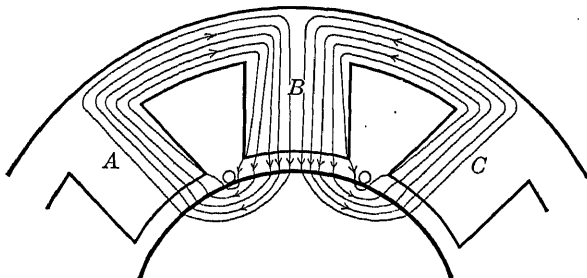



Fig. 33.

Referring to Fig. 33, it is evident that when the armature moves through the angle corresponding to the distance between two successive poles, the armature conductor which was originally in the neutral between two given poles, say *A* and *B*, moves to the next neutral point between the poles *B* and *C*. When the conductor is in the neutral between *A* and *B* it is linked in one direction (right-handed in the figure) by *half* the lines of force from pole *B*, and when it is in the neutral between *B* and *C* it is linked by the other half of the lines of force from the pole *B*, but in the *opposite* direction. Hence as an armature conductor moves from one neutral point to the next the total *change* in the number of lines of force which link it is equal to the useful flux per pole.

The average value of the electromotive force induced in a conductor when it moves through a magnetic field is always equal, in

volts, to 10^{-8} times the *total change* in the number of lines of force which links this conductor, divided by the time, in seconds, required for this motion; see Article 11. Hence the average value of the electromotive force induced in an armature conductor as it moves from one neutral point to the next is equal, in volts, to 10^{-8} times the useful flux per pole, divided by the time, in seconds, required for this conductor to move from one neutral point to the other.

When the armature is exactly centered with respect to the field poles, and the pole faces are all alike, the successive neutral points are always spaced a distance apart equal to the circumference of the armature divided by the number of poles. The time required for a conductor to move from one neutral point to the next is therefore equal to the time of one complete revolution of the armature, divided by the number of poles.

Let Z = total number of armature conductors,
p = number of poles,
ϕ = useful flux per pole, in maxwells, which enters the armature between any two adjacent electric neutrals,
n = number of revolutions of the armature per minute,
a = number of parallel paths formed by the armature conductors between the positive and negative brush sets (see Article 27). In a simple lap winding $a=p$; in a simple wave-winding $a=2$. In a multiplex lap winding $a=mp$ and in a multiplex wave winding $a=2m$, where $m=2$ for a duplex winding and $m=3$ for a triplex winding.

The time required for each conductor to pass entirely around the armature is then $\frac{60}{n}$ seconds. Assuming the brushes to be in the electric neutral, the time taken for it to pass from one neutral point to the next is therefore $\frac{60}{np}$ seconds. Hence the average value of the electromotive force induced in each armature conductor as it passes from one neutral point to the next is $\frac{np\phi}{60 \times 10^8}$ volts. Since the conductors are uniformly distributed around

the surface of the armature, this is also equal to the average value at any instant of the electromotive forces induced in the several conductors which at that instant lie between two successive neutrals points.

Hence, when the armature has a full-pitch winding, the average value of the electromotive force between brushes is equal to this average electromotive force per conductor, multiplied by the number of armature conductors in series between any pair of brushes. Since there are $\frac{Z}{a}$ conductors in series between the positive and negative brush sets, the average value of the total electromotive force between the brushes is therefore

$$E = \frac{10^{-8}}{60} \cdot \frac{pZ}{a} \cdot n\phi \qquad \text{volts} \qquad (1)$$

This electromotive force is usually referred to as the **armature electromotive force,** to distinguish it from the terminal voltage, which is less than this electromotive force by an amount equal to the resistance drop in the armature.

Equation (1) gives the *average* value of the armature electromotive force. Were there only one commutator bar per pole (two bars per pair of poles), the electromotive force between brushes would fall to zero every time a brush short-circuits two bars. However, when the commutator has two or more bars per pole, the electromotive force between brushes will never fall to zero, for there will then always be in circuit between the brushes one or more armature coils which are not short-circuited, and which are moving through a relatively strong magnetic field; see Fig. 3.

The greater the number of commutator bars per pole the less will be the pulsation in the armature electromotive force. When there are 10 or more bars per pole the pulsation in the generated electromotive force of a dynamo is practically negligible.

It should be noted that equation (1) for the value of the generated electromotive force is derived on the assumption that the brushes are so set that they short-circuit a coil when the conductors which form this coil are in the electric neutral. When the brushes have any other position, the electromotive force generated in part of the conductors which form any given path

between the positive and negative brushes is in the opposite direction to that generated in the rest of the conductors, and the resultant electromotive force is therefore less than that given by equation (1). In particular, were the brushes set so that they short-circuited a coil when its sides are under the centers of adjacent poles, the resultant electromotive force between brushes would be zero.

As already noted, in a non-interpole dynamo, in order to secure sparkless commutation, the brushes usually have to be shifted from the neutral. Theoretically, therefore, when equation (1) is applied to such machines the value assigned to ϕ should be slightly less than the total useful flux per pole. Practically, however, this correction is so slight that it is seldom made.

When there is no current in the armature, the flux per pole ϕ is due entirely to the field ampere-turns. However, when there is a current in the armature conductors, i.e., when the machine is loaded, this armature current also produces a magnetic field, which in general both reduces and distorts the magnetic field through which the armature conductors move. This effect of the armature current, usually referred to as "armature reaction," is analyzed in detail in Chapter V. For the present it is sufficient to note that due to the demagnetizing effect of the armature current, the value to be assigned to ϕ in equation (1) in general depends not only upon the field ampere-turns, but also upon the load on the machine.

In general, irrespective of the pitch of the armature winding, setting of the brushes, or load on the machine, the generated electromotive force, whether the machine be operated as a generator or motor, is

$$E_r = \frac{10^{-8}}{60} \cdot \frac{pZ}{a} \cdot n\phi_r \qquad \text{volts} \qquad (1a)$$

where ϕ_r is the change in the *resultant* number of lines of force which thread an armature coil when the commutator bar (either one) to which this coil is connected moves from one brush to the next.

33. Field Ampere-turns.—The flux per pole ϕ, when there is no current in the armature, is directly proportional to the field ampere-turns, and inversely proportional to the reluctance of

the magnetic circuit of the machine (see Article 12). Hence, for a given flux per pole, the required number of field ampere-turns at no load will be directly proportional to the reluctance of the magnetic circuit. In addition to the ampere-turns required to "overcome" the reluctance of the magnetic circuit, an allowance must also be made, in designing a field winding, for the ampere-turns necessary to counteract the demagnetizing effect of the armature current.

A small number of field ampere-turns is desirable for two reasons, viz., the smaller the number of ampere-turns the less space will the field winding occupy (and therefore the smaller the machine), and the less will be the power lost in this winding (and therefore the higher the efficiency of the machine).

The truth of this statement may be seen by considering first a series winding designed to carry a definite *current*, and then a shunt winding designed to operate at a definite *impressed voltage*.

In the case of the series winding (constant current) a reduction of its ampere-turns means a reduction in the number of turns in the winding. Reducing the number of turns in the winding reduces the total length of conductor, and therefore the resistance of the winding. Reduction in the length of conductor means a reduction in volume, and reduction of resistance results in a reduction of power loss (rI^2).

In the case of a shunt winding (constant impressed voltage) a reduction in the number of turns will not reduce the ampere-turns, since, due to the accompanying reduction in the resistance of the winding, the current will increase in the same ratio that the number of turns is decreased, that is, NI will remain constant. A reduction in the cross-section of the wire (keeping the same number of turns) will, however, reduce the ampere-turns, since the accompanying increase in resistance will produce a decrease of current. Reduction in the cross-section of the wire means a reduction in volume, and, since for a constant impressed voltage the power loss $\left(rI^2 = \frac{V^2}{r}\right)$ is inversely proportional to the resistance, the increase of resistance produces a decrease in the power loss.

Hence, whether the field winding be a shunt winding or a series winding, a small number of ampere-turns is desirable, from the

standpoint of both size and efficiency. It is on this account that the magnetic circuit of a dynamo is always so designed that it will have a low reluctance. This is accomplished by making the air-gap between the pole faces and the armature small, and by using, for the rest of the magnetic circuit, iron or steel of high permeability.

When the required number of field ampere-turns has been determined, the size and number of turns of wire required may then be chosen to suit the kind of connection desired. If the winding is to be connected in series with the armature (series field), then the number of turns of wire must be made equal to the number of ampere-turns required at normal load, divided by the normal load current. The wire (or strap) must be of sufficient cross-section to carry this current without over-heating. A series winding will therefore have a relatively few turns of relatively large wire.

If the field winding is to be connected across the armature terminals (shunt field), then the number of turns and size of wire are so chosen that the number of ampere-turns required at normal load, divided by the chosen number of turns of wire, will be somewhat less than the current which this terminal voltage would produce if impressed directly across this winding. Exact adjustment of the field current to give the required number of ampere-turns is then effected by inserting in series with this winding a suitable external resistance, namely, the shunt-field rheostat.

Since the shunt-field current is a current which the armature must supply *in addition* to the load current, it is of course desirable to keep this current as small as possible. Hence the shunt-field winding is always made of a relatively large number of turns, and the wire used is no larger than is necessary to carry the required current without overheating.

34. Calculation of Field Ampere-turns.—Since the permeability of the iron or steel in a magnetic circuit is not constant, but depends upon the flux density, the most convenient way to calculate the field ampere-turns required to produce a given flux is as follows:

(1) Determine the flux density in each portion of the magnetic circuit corresponding to the flux per pole required.

(2) From the B–H curves (see Article 17) for the material used, determine the magnetizing force H for this portion of the circuit.

(3) Apply the fundamental relation that the resultant number of ampere-turns acting on the circuit is equal to the line integral of the magnetizing force for the complete circuit; see Article 14, equation (31).

When there is no current in the armature, or if armature reaction is negligible, the resultant ampere-turns as thus calculated will be the required field ampere-turns per pair of poles. When armature reaction is appreciable, the resultant ampere-turns as thus determined must be increased by the proper amount to allow for armature reaction; see Chapter V.

The flux per pole ϕ in equation (1) for the generated electromotive force is the flux which enters the armature and links the armature conductors. However, since air as well as iron is permeable to magnetic lines of force, the field winding produces lines of force not only in the iron portions of the magnetic circuit and in the narrow air-gap between the pole faces and the armature core, but also in all the surrounding space. That portion of the total flux produced by the field winding which does not link the armature conductors is called the **leakage flux.**

The leakage flux is always appreciable, ranging from 5 to 20 percent of the useful flux, the greater part of it being in the space between the poles. The ratio of the total flux produced by the field winding to the useful flux (i.e., to the flux which links the armature conductors) is called the **leakage coefficient** or **coefficient of dispersion.** The value of this coefficient for modern multipolar machines of the usual type ranges from about 1.15 for small machines to 1.1 for large machines.

In Fig. 34 are shown two poles and part of the armature of a multipolar dynamo. The light lines in the air-gap and in the interpolar space represent lines of force. These lines of course continue on through the iron, forming closed loops similar to the heavy loop; their continuation is omitted in the figure for the sake of clearness. The lines *bb* in this figure are leakage lines.

As shown in Fig. 34, the lines of force in the air-gap are not straight (as they would be were there no teeth in the armature core), but form a tuft at each tooth. Also at the pole tips there

is a spreading out, or "fringing," of the lines of force. There is also a similar fringing at the ends of the armature core, and the ventilating ducts in the armature core (see Fig. 25) distort the lines of force in the same manner as the slots.

Due to the leakage of part of the flux through the interpolar spaces, and to the tufting or fringing of the air-gap flux at the teeth, ventilating ducts and the end surfaces, an exact calculation of the field ampere-turns required for a given armature electromotive force is not possible. However, by applying various correction factors, based partly on calculation and partly

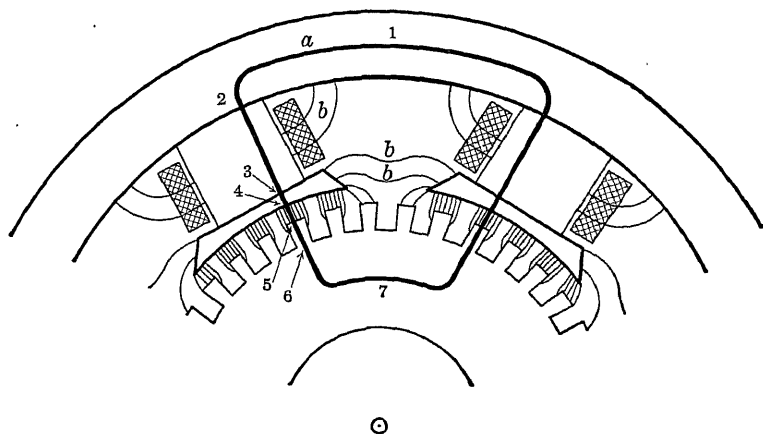



FIG. 34.—Magnetic Circuit of Multipolar Dynamo.

on experience, it is possible to predetermine, with a fair degree of approximation, the ampere-turns required. This is a problem of design, and will not be gone into in detail here; those interested are referred to Langsdorf's "*Principles of Direct-current Machines*," or for a more detailed discussion to Arnold's "*Die Gleichstrommachinen.*"

The procedure in calculating the no-load ampere-turns per pole is briefly as follows:

A cross-section of the machine is laid out to scale as shown in Fig. 34. The mean path of the lines of force is then drawn, as indicated by the heavy loop *a*. Since this loop links two field coils, the line integral of the magnetizing force around this loop is equal to the number of ampere-turns required for *two* poles.

Since this loop is symmetrical with respect to the two poles, the ampere-turn *per pole* is found by integrating the magnetizing force along half the loop, namely, the half 1–2–3–4–5–6–7.

The lengths of the successive portions of this half loop are measured, and their values entered into a table, as indicated in Table I. The symbol l is here used for length, and the subscripts y, c, s, g, t, and a are used respectively for the yoke (1–2), the pole core (2–3), the pole shoe (3–4), the air-gap (4–5), the armature teeth (5–6) and the armature core (6–7). The *effective* cross-sections of each of these portions of the magnetic circuit is then determined, due allowance being made for the tufting at the armature teeth, and at the ventilating ducts, and for the fringing at the sides of the pole shoes. These effective cross-sections are designated by the symbol S, with the proper subscript.

TABLE I

Part of Circuit.	Length of Path.	Cross-section.	Flux Density.	Ampere-turns per unit Length.	Ampere-turns for section.
Yoke..........	l_y	S_y	B_y	H_y	$H_y \times l_y$
Pole core.......	l_c	S_c	B_c	H_c	$H_c \times l_c$
Pole shoe.......	l_s	S_s	B_s	H_s	$H_s \times l_s$
Air-gap........	l_g	S_g	B_g	H_g	$H_g \times l_g$
Teeth..........	l_t	S_t	B_t	H_t	$H_t \times l_t$
Armature core..	l_a	S_a	B_a	H_a	$H_a \times l_a$
				Total Ampere-turns =	Sum

The average flux density in each part of the magnetic circuit is then found by dividing the total flux in this part of the circuit by its effective cross-section. Calling ϕ the total useful flux per pole, and c the leakage coefficient, the total flux in the various parts of the circuit is as follows:

Yoke	$\frac{1}{2}c\phi$
Pole core	$c\phi$
Pole shoe	$c\phi$
Air-gap	ϕ
Teeth	$k\phi$
Armature core,	$\frac{1}{2}\phi$

In the above expression for the flux through the teeth, k is a correction factor (less than unity) which takes into account the fact that some of the lines of force enter the sides of the teeth (see Fig. 34). When the air-gap flux is not sufficient to saturate the teeth, k is practically unity.

From the B–H curves for the materials used in the magnetic circuit, the corresponding value of the magnetizing force H, expressed in ampere-turns per unit length, is determined. Note that for the air-gap, when B_g is expressed in lines per square inch, $H_g=0.313B_g$; and when B is expressed in lines per square centimeters; $H_g=0.796B_g$ (see Article 14).

The ampere-turns required for each portion of the magnetic circuit are then found by multiplying the length of this portion of the circuit by the corresponding value of the magnetizing force. The total number of ampere-turns required per pole is then equal to the sum of these ampere-turns for the successive parts of the circuit, as indicated in Table I.

Since the permeability of the iron portions of a magnetic circuit is usually a 1000 or more times that of air, the major portion of the ampere-turns is required to overcome the reluctance of the air-gap. Consequently, the product H_gl_g for the air-gap, even though the length of the air-gap is relatively small, is usually several times larger than sum of all the rest of the corresponding products. It is not uncommon for the air-gap ampere-turns to amount to as much as 80 percent of the total.

35. Saturation Curve of a Dynamo.—By assuming successive values of the useful flux per pole, and calculating the corresponding number of ampere-turns as just explained, a curve may be plotted with ordinates as flux per pole and ampere-turns as abscissas. Such a curve is called the "saturation curve," or "magnetization curve," of the machine.

The saturation curve has the same general shape as the magnetization curve for iron or steel, but, due to the presence of the air-gap (which by itself would have a straight line as its B-H curve), it is much less steep and is straight for a greater portion of its length. The saturation curve for a 1000 kw., 275 volt, 8 pole, 720 r.p.m. generator is given in Fig. 35.

Note particularly that, except at low flux densities, the flux is not proportional to the field ampere-turns. A given increase

in the field ampere-turns produces less and less increase in the flux, the more nearly saturated the iron parts of the magnetic circuit become.

Although the predetermination of the **saturation curve** of a machine from its dimensions and design constants is tedious, and more or less difficult, this curve for a finished machine may be readily **determined experimentally.** Since for a constant armature speed the generated electromotive force is directly propor-

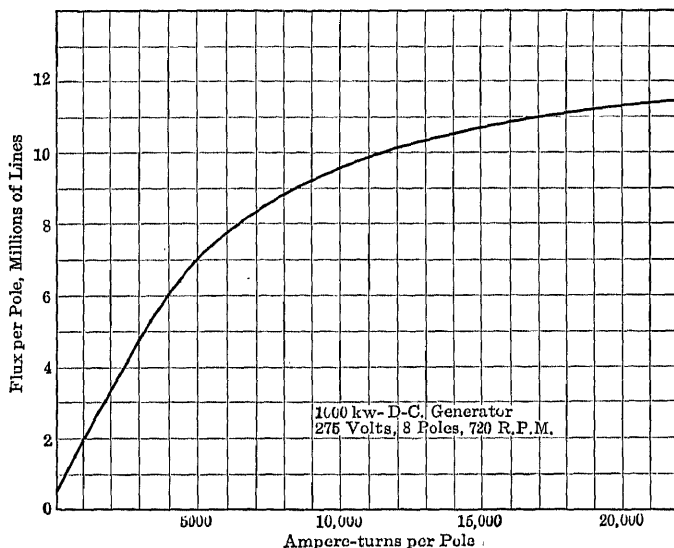



FIG. 35.—Saturation Curve. Flux plotted against Field Ampere-turns.

tional to the flux per pole, equation (1), it is merely necessary to disconnect the field winding from the armature, operate the machine as a separately-excited generator at constant speed, and measure the voltage between the brushes for various values of the field current.

The connections for such a test are shown in Fig. 36. R is a resistance, provided with a movable contact C, the entire resistance being connected directly across the terminals of the source used to supply the field current. This "drop-wire" (or potentiometer) arrangement gives a convenient method for varying the voltage across the field from zero to the full value

of the supply voltage, and thus to vary the field current from zero to its maximum value.

It is the usual practice to drive the machine at rated speed when determining the saturation curve. However, for the reasons pointed out in Article 51, it is sometimes preferable to make this test at a lower speed.

Fig. 36.—Connections for Saturation Tests.

The field current I_f is given by the ammeter A and the armature electromotive force E by the voltmeter V. This voltmeter actually reads the voltage between the brushes, but since the only current in the armature is the extremely small current taken by the voltmeter, this voltage is practically the same as the armature electromotive force.

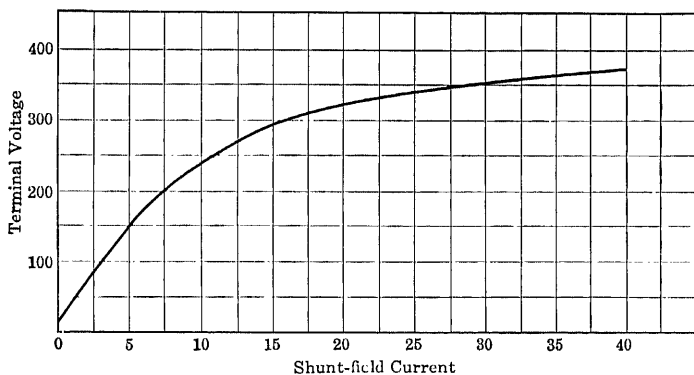



Fig. 37.—Saturation Curve of 1000 Kw., 275 Volt, 8 Pole, 720 r.p.m. Generator.

The saturation curve may also be determined by driving the machine as a motor without load as explained in Article 114.

When the saturation curve is determined experimentally, the electromotive force E is usually taken as the ordinate instead of the flux ϕ, and the field current I_f is taken as the abscissa instead

of the field ampere-turns; see Fig. 37, which applies to the same machine as Fig. 35. When the number of armature conductors and the number of field turns are known, it is a simple matter to calculate from the voltage-current curve the flux ϕ and (see equation 1) the ampere-turns.

As will be shown in Chapters VI and VII, the operating characteristics of a dynamo, whether used as a generator or as a motor, depend upon its saturation curve. This curve is therefore of fundamental importance in the study of the behavior of a dynamo under operating conditions.

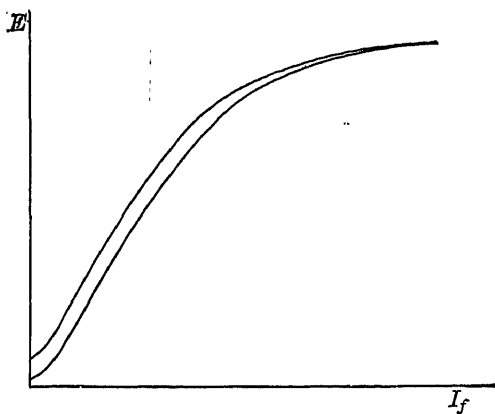



FIG. 38.—Ascending and Descending Saturation Curves.

If, in determining the saturation curve, the field current is increased from zero by successive steps to a maximum value, and then decreased from this maximum value by successive steps to zero, the **ascending and descending saturation curves** will be found to differ slightly, the descending curve being slightly higher than the ascending curve, as shown in Fig. 38.

This double saturation curve is due to hysteresis in the iron portions of the magnetic circuit (see Article 16). Also, on account of the residual magnetism in the iron, the saturation curve starts and ends slightly above the origin. Whenever the experimentally determined saturation curve is referred to, what is usually meant is the curve whose ordinates are the average of the ordinates of these two curves.

In determining the saturation curve by experiment, care should be taken not to decrease the current during the determination of the ascending curve, and not to increase the current during the determination of the descending curve, otherwise minor hysteresis loops will be introduced.

PROBLEMS

1. A certain 6-pole generator has 310 armature conductors. The flux per pole is 4,500,000 lines, and the speed 1000 r.p.m. Calculate the electromotive force of this machine when the conductors are connected to form:

(*a*) a simplex wave winding.

(*b*) a simplex lap winding.

2. The maximum safe current *per conductor* for the machine described in Problem 1 is 80 amperes. What would be the rating of the machine (power, voltage and current) when (*a*) wave-wound, and when (*b*) lap-wound?

3. The armature core of a 4-pole 550-volt railway motor has 57 slots. The series field coil is so designed that the flux per pole is 3×10^6 maxwells when the speed is 1200 r.p.m. What type of winding should be used, and how many armature conductors will be required?

4. (*a*) If the speed of the generator to which the curve in Fig. 37 corresponds were reduced by 20 per cent, by what percentage would its no load terminal voltage be reduced, the shunt-field current being held constant at 15 amperes?

(*b*) If the setting of the shunt-field rheostat is not altered, by what percentage would the voltage change?

5. The effective lengths and cross-sections of the various parts of the magnetic circuit of a certain 4-pole shunt generator are as follows (see Fig. 34 and Table I):

Part.	Material.	Length, Inches.	Cross-section Square Inches.
Yoke (1–2)	Cast steel	14	15
Pole core (2–3)	Laminated steel	5	25
Pole shoe (3–4)	Laminated steel	2	40
Air gap (4–5)	Air	$\frac{3}{16}$	45
Teeth (5–6)	Laminated steel	1.25	18
Armature core (6–7)	Laminated steel	4.5	13

The armature of this generator has a simplex wave winding of 194 conductors. The shunt-field winding has 800 turns per pole. The speed is 900 r.p.m.; leakage coefficient is 1.15. The tooth coefficient k may be taken equal to unity. Determine by calculation the points on the saturation curve corresponding to no-load terminal voltages of 25, 50, 75, 100, 125 and 150 volts. Use the B–H curves given in Fig. 2. Plot the saturation curve (*a*) as flux per

pole against field ampere-turns per pole and (*b*) as terminal voltage against field current.

6. It is desired to obtain experimentally the saturation curve of a 50 h.p., 600-volt railway motor at a speed of 650 r.p.m. The source of power for making this test is at 110 volts. Make a complete wiring diagram of the electric connections required. Indicate on this diagram the location and range of all instruments, and the location, current-carrying capacity and approximate resistance of all rheostats necessary to make this test. Note the size of the auxiliary motor required, and indicate the method of connecting this auxiliary motor to the machine under test. For what values of the field current should readings be taken? Indicate by a curve on cross-section paper, to the proper scale of abscissas and ordinates, the approximate shape of the saturation curve you would expect to obtain.

7. The curves in Fig. 35 and Fig. 37 apply to the same machine, which has a simplex lap winding. From a comparison of these two curves, determine,

(*a*) The number of turns per pole in the shunt-field winding.

(*b*) The total number of armature conductors.

CHAPTER IV

COMMUTATION

36. Introduction.—It has already been pointed out (Article 23) that when a direct-current dynamo is in operation, the current in each armature coil is reversed in direction every time the two commutator segments to which it is connected pass under a brush. Unless certain precautions are taken, this reversal of current will usually be accompanied by sparking at the brushes, which sparking, if pronounced, will seriously interfere with the practical operation of the machine.

A spark between the brush and the commutator can result only from the breaking down of the air-film in immediate contact with the commutator. The rupture of this air-film, even though this film may be exceedingly thin, requires the establishment of an appreciable difference of potential between the brush and the segment of the commutator with which it is in contact. Just what value this potential difference must attain before an appreciable spark will be produced is doubtful, but experience indicates that, for carbon brushes, it is of the order of several volts.

However, irrespective of the exact value of this critical potential difference, it is evident that should the potential difference between either tip of the brush and the commutator segment with which it is in contact approach infinity, a spark will certainly occur. As will be shown in the following paragraphs, this condition is not only possible, but will actually occur unless proper care is exercised in the design of the machine.

The complete analysis of the problem of commutation, taking into account the actual conditions which occur in practice, becomes extremely involved. However, by considering first an ideal case, based on certain simple assumptions, one may readily obtain a fair idea of what factors must be considered and the precautions which must be taken to secure sparkless commutation under actual working conditions. These assumptions are:

1. That the insulation between successive commutator segments is of negligible thickness.

2. That each brush has a width equal to that of a single commutator segment.

3. That the contact resistance between brush and commutator segment at any instant is inversely proportional to the area of the surface of contact between this brush and segment at this instant.

4. That the resistance of the armature coil, the leads and commutator segments connected thereto, and the internal resistance of the brush are negligible in comparison with the *contact resistance* between the brush and the segments which it short-circuits. When carbon brushes are used this assumption is approximately in accord with the actual facts.

Following this preliminary analysis will be pointed out the departure in practice from these assumptions, and the consequent modification which must be made in the conclusions derived therefrom.

37. Elementary Theory of Commutation.—In Figs. 39 to 41 are shown diagrammatically a section of a ring armature and its commutator. The armature is supposed to be moving to the right. Fig. 39 shows the relative position of the armature and brush at the instant when the brush is in contact with one segment only, this segment being designated 2. Fig. 40 shows the relative position of the armature and brush a fraction of a second later, the brush now making contact with both segment 2 and the next segment 1, thus short-circuiting the armature coil C which is connected to these two segments. Fig. 41 shows the relative position of the armature and brush when segment 2 has moved entirely out from under the brush and the brush makes contact with segment 1 only.

Before the short-circuit of the coil C begins, the current in this coil is in the direction indicated in Fig. 39, and is equal to one-half the current which passes from the brush to the load (see Article 23). Calling I the brush current, the current in the coil C just before the short-circuit starts is then $\frac{1}{2}I$. At this instant the current in the lead from the coil to segment 1 is zero, and the current in the lead to segment 2 is I.

The instant the coil is short-circuited by the brush the current in it begins to decrease. At the same time a current i_1, equal in value

to the amount by which the current in the coil C decreases, passes up to the brush through segment 1, and the current which passes up to segment 2 decreases by an equal amount. At the end of the short-circuit, Fig. 41, the entire brush current passes up to the brush through segment 1. Therefore, comparing Figs. 39 and 41, it is evident that at the end of the short-circuit the current in the coil C must be equal to, but in the direction opposite to, the current in this coil at the beginning of the short-circuit.

FIG. 39.

FIG. 40.

At the instant just before the brush makes contact with the commutator segment 1 (Fig. 39) which is approaching it, the voltage between the brush and segment 1 is equal to the resistance drop across the surface of contact between the brush and the segment 2 with which it is in contact, plus the voltage drop in the coil C and lead-wire

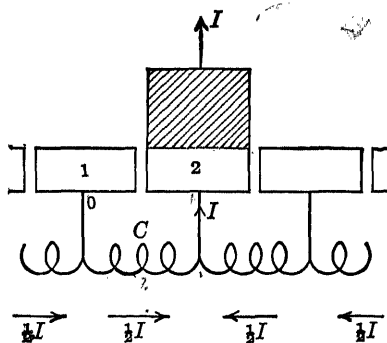
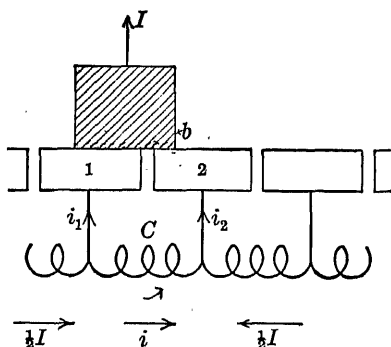
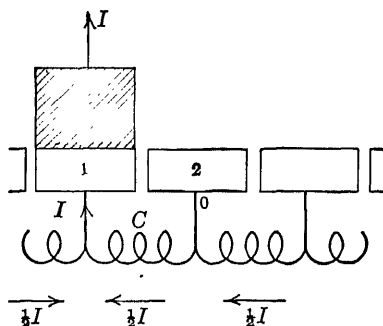



FIG. 41.

connected to segment 2. This potential difference drop is never more than a volt or two, which is not sufficient to produce an appreciable spark.

As a segment leaves the brush (see Fig. 40) the difference of potential between the brush and the segment which is passing out from under it, is equal to the resistance drop across the surface of contact between the brush and this segment. The area of this contact surface gets smaller and smaller as the segment moves out from under the brush, and becomes zero at the instant the segment breaks away entirely from the brush. The resistance at the trailing brush contact (b in Fig. 40) therefore approaches infinity as its limiting value. On the other hand, the current through this diminishing contact surface approaches zero as its limiting value. The difference of potential between the trailing brush tip and the segment which is passing out from under it therefore approaches a value equal to infinity times zero.

This product, infinity times zero, may be zero, a finite quantity, or may approach infinity, depending upon the manner in which the current in the trailing brush tip approaches zero. If the limiting value of this product is less than the voltage required to break down the air between the trailing brush tip and the commutator, no spark will occur (assuming, of course, that there is no chattering of the brush). If, however, the limiting value of this product is sufficiently large, then the voltage between the trailing brush tip and the segment which is passing out from under it may be sufficient to cause a spark to jump through the air from the trailing brush tip to this segment. Therefore, **whether or not sparking will occur depends primarily upon the limiting value of the contact-resistance drop at the trailing brush tip.**

The value of the contact resistance drop at the trailing brush contact may be readily expressed in terms of (1) the current in the short-circuited coil and (2) the time elapsed from the beginning of the short-circuit. From this expression the limiting value of this contact resistance drop, when the variation of current with time is known, may be obtained.

Let i be the current in the short-circuited coil at any instant of time t seconds after the short-circuit begins (e.g., at the instant represented by Fig. 40), and consider i as positive when it is in the direction of the current which exists in this coil just before the brush short-circuits it. Then, at the instant t, the currents

entering the brush from the two segments 1 and 2 are respectively

$$i_1 = 0.5I - i \tag{1}$$

$$i_2 = 0.5I + i \tag{1a}$$

Let A be the area of the contact between the brush and a single segment, and let R be the corresponding contact resistance. Let T be the time taken for a segment to pass completely under the brush, i.e., the time taken for the commutator to move from the position shown in Fig. 39 to that shown in Fig. 41. Then, neglecting the thickness of the insulation between commutator segments, and assuming the brush to have a width equal to that of one commutator segment, the contact areas between the brush and the segments 1 and 2 at the instant t are respectively $A\frac{t}{T}$ and $A\frac{T-t}{T}$. The corresponding contact resistances, assum contact resistance to vary inversely as the contact area, are then

$$R_1 = \frac{T}{t}R \tag{2}$$

$$R_2 = \frac{T}{T-t}R \tag{2a}$$

The variation of R_1 and R_2 with time is shown by the dotted curves in Figs. 43 and 44.

Combining equations (1a) and (2a), the contact resistance drop at the trailing brush contact may be written

$$R_2 i_2 = RT\left(\frac{0.5I + i}{T - t}\right) \tag{3}$$

At the end of the short-circuit ($t = T$), the current i is equal to $-0.5I$, and this expression for $R_2 i_2$ becomes indeterminate. This indeterminate form, however, may be evaluated in the usual manner by differentiating numerator and denominator with respect to t, giving for the contact drop at the trailing brush tip at the end of the short-circuit the value

$$(R_2 i_2)_T = -RT\left(\frac{di}{dt}\right)_T \tag{4}$$

The subscript T is here used to signify the values of the respective quantities at the end of the short-circuit, namely, for $t = T$.

From equation (4) it follows that **the contact resistance drop at the trailing brush tip at the end of the short-circuit is proportional to the rate at which the current in the short-circuited coil is decreasing at this instant.** The more rapidly the current decreases at the end of the short-circuit the greater is this potential difference, and therefore the greater is the tendency for sparking to take place.

In Fig. 42 are shown three possible ways in which the current in the short-circuited coil may vary with time. The straight line A represents a uniform rate of change of this current from its initial value $\frac{1}{2}I$ to its final value $-\frac{1}{2}I$. Curve B represents a relatively slow rate of change at the beginning of the short-circuit and a very rapid change in the current at the end of the short-circuit. Curve C represents a rapid change in the current at the beginning of the short-circuit, an "overshooting" of the current near the end of the short-circuit, followed by a fairly rapid change to its final value $-\frac{1}{2}I$. Any one of these types of commutation is possible in practice.

FIG. 42.—Current in Short-circuited Coil. A. Straight-line Commutation; B. Under-commutation; C. Over-commutation.

Irrespective of the shape of the current-time curve, its slope corresponding to any value of t gives the rate of change of the current at this instant. The limiting value of the brush contact-drop at the trailing brush tip is therefore in each case proportional to the slope of the current-time curve at the end of the short-circuit. Consequently, if the current varies in the manner represented by curve B there will be a greater tendency to spark than if it varied linearly with time as indicated by curve A. Curve C also represents a type of change in the current which is

more likely to result in sparking than would be the case if the current varied according to curve A.

The currents in the trailing and leading brush contacts corresponding to the curves A, B and C in Fig. 42 are shown by the similarly marked curves in Figs. 43 and 44.

FIG. 43.—Current in Trailing Brush Contact. A. Current for Straight-line Commutation; B. Current for Under-commutation; C. Current for Over-commutation; R_2. Contact Resistance.

38. Straight-line, or Linear Commutation. — When the current in the short-circuited coil varies uniformly with respect to time, as indicated by curve A in Fig. 42, the commutation may be conveniently referred to as "straight-line," or "linear," commutation. For the conditions assumed in Article 36, namely, negligible thickness of insulation between commutator segments, brush of same width as one segment, and contact resistance of brush inversely proportional to the area of contact, it may readily be shown that **straight-line commutation will always result, provided there is no resultant electromotive force set up in the armature coil C while it is short-circuited by the brush**; or, what amounts to the same thing, provided the self-induced electromotive force in this coil is exactly

FIG. 44.—Current in Leading Brush Contact. A. Current for Straight-line Commutation; B. Current for Under-commutation; C. Current for Over-commutation; R_1. Contact Resistance.

counterbalanced by an equal and opposite electromotive force produced in some other way (see Article 59).

In contrast to straight-line commutation, represented by curve A in Fig. 42, curve B is typical of the variation of the current in the short-circuited coil when the self-induced electromotive force is not opposed by any other electromotive force, or is only partially neutralized. Commutation under these conditions is conveniently referred to as "under-commutation." On the other hand, when the externally produced electromotive force more than counter-balances the self-induced electromotive force, the current in the short-circuited coil will vary in the manner shown by curve C (although it may not overshoot), and the commutation is conveniently referred to as "over-commutation."

That straight-line commutation will result when the resultant electromotive force in the short-circuited coil is zero, follows from the fact that under these conditions the total resistance drop around the loop formed by the coil, the commutator leads, the two segments and the brush must be zero (see Article 7). When carbon brushes are used, the resistance of the coil, leads and segments and the internal resistance of the brush, are, as a rule, all negligible in comparison with the brush contact-resistance. Using the same notation as in the preceding article, the total resistance drop around this loop in the direction of the current i is then $(R_2i_2-R_1i_1)$, which, put equal to zero, gives

$$R_2i_2-R_1i_1=0 \tag{5}$$

Substituting in this equation the values of the currents and resistances given by equations (1) and (2), and solving for i, there results

$$i=\left(0.5-\frac{t}{T}\right)I \tag{6}$$

This value of i plotted against t gives the straight line A in Fig. 42.

Substituting this value of i in equation (1a), the corresponding value of the current in the trailing brush tip is

$$i_2=\left(1-\frac{t}{T}\right)I=\left(\frac{T-t}{T}\right)I \tag{7}$$

The variation of i_2 with time under the conditions here assumed (no electromotive force in the short-circuited coil) is shown by the

curve A in Fig. 43, and the variation of $i_1 = I - i_2$ is shown by the curve A in Fig. 44.

The substitution of the value of i given by equation (6) into equation (3) gives for the contact resistance drop at the trailing brush tip the value

$$R_2 i_2 = RI \qquad (8)$$

Consequently, on the assumptions stated in Article 56, were there no electromotive force set up in the short-circuited coil, the contact resistance drop at the trailing brush tip would be constant at the value RI throughout the commutation period, as indicated by the horizontal straight line A in Fig. 45.

FIG. 45.—Contact Resistance Drop at Trailing Brush Contact. A. Straight-line Commutation; B. Under-commutation; C. Over-commutation.

The product RI is equal to the voltage drop across the contact between brush and commutator when the brush is directly over a single segment. This drop, which may be called the "normal brush contact drop," is, for carbon brushes, of the order of 1 volt, for the current densities ordinarily employed in practice (30 to 50 amperes per square inch). This potential difference of 1 volt is not sufficient to break down the surrounding air. Consequently, under the conditions assumed in Article 56, **when the self-induced electromotive force in the short-circuited coil is completely neutralized there will be no sparking at the brushes.** Experience shows that this is also true under the conditions of actual practice.

The curve B in Fig. 45 gives the contact resistance drop at the trailing brush contact for the particular case of under-commutation represented by curve B in Fig. 42, and curve C in Fig 45 gives the contact resistance drop at the trailing brush contact for the particular case of over-commutation represented by curve C in Fig. 42. These two curves are found by multiplying the ordinates

of the similarly marked curves in Fig. 43 by the corresponding ordinates of the resistance curve R_2.

As indicated by curve B in Fig. 45, when the self-induced electromotive force in the short-circuited coil is not neutralized or is only partially neutralized, the contact resistance drop at the trailing brush contact may become very large, or even approach infinity, at the end of the short-circuit. Under these conditions sparking will certainly occur. Similarly, should the self-induced electromotive be more than neutralized, corresponding to curve C in Fig. 42, the contact-resistance drop at the trailing brush contact may reverse in sign (Fig. 45) and reach a relatively large value in the opposite direction. Under these conditions sparking may or may not occur, depending upon the degree to which the self-induced electromotive force is over-neutralized.

39. Effect of Self-inductance of the Armature Coils and its Neutralization.—As explained in Article 19, whenever the current in a coil varies with time, the magnetic flux produced by this current likewise varies, producing in the coil an electromotive force proportional to the RATE of variation of this flux. Consequently, when the current in an armature coil is reversed, as the result of the commutator segment to which it is connected passing under a brush, a **self-induced electromotive force** is always set up in it.

As pointed out in Article 19, the self-induced electromotive force is always in such a direction as to oppose the change in current which gives rise to it. Therefore, in the coil undergoing commutation the self-induced electromotive force tends to keep the current flowing in its initial direction, in spite of the increasing resistance at the trailing brush contact. Ultimately, however, the current in the coil must reverse, since at the end of the short-circuit the resistance at the trailing brush tip becomes infinite (see Fig. 43). Hence, unless the self-induced electromotive force is neutralized, the current in the short-circuited coil will reverse in the manner indicated by curve B in Fig. 42, and at the end of the short circuit may have to change so rapidly that sparking will result, as explained in the preceding Article.

Various schemes have been proposed for neutralizing the self-induced electromotive force in the armature coils as they undergo commutation. In practice, however, this is usually done in one of two ways, viz., either by shifting the brushes from the electrical

neutral or by the use of commutating poles; see Article 30. The first method is the simplest, but is effective only for a relatively small range in the load on the machine. On the other hand, although commutating poles add to the cost of a machine, they so greatly extend the range of load which the machine can supply without sparking that the extra cost is usually more than justified. Non-interpole machines are now seldom built except in the smaller sizes (3 h.p. at 1750 r.p.m. or smaller).

Both of these methods of neutralizing (or partially neutralizing) the self-induced electromotive force are based on the same principle, viz., the production of a magnetic field, independent of that produced by the current in the short-circuited coil, which will be CUT by the armature conductors while they are short-circuited by the brush.

When an armature conductor is in the electrical neutral (see Article 32), it is cutting no lines of force. Consequently, if the brushes are set in the electrical neutral, there will be no electromotive force generated in the short-circuited coil as a result of its motion. However, if the brushes are shifted from the electrical neutral, so that the short-circuit occurs either before or after the conductors have passed this point, the conductors which form this coil will be cutting lines of force while it is short-circuited, and therefore an electromotive force will be generated in it.

Similarly, if an auxiliary field pole is placed between each pair of main field poles, as shown in Fig. 20, and each brush is set so that it will short-circuit an armature coil when the conductors which form this coil are passing under this interpole, an electromotive force will likewise be generated in the coil while it is short-circuited.

In either case, this generated electromotive force is caused by the motion of the armature conductors, and may therefore be conveniently referred to as the **rotational electromotive force** set up in the short-circuited coil, as distinguished from the self-induced electromotive force caused by the variation of current in the coil itself.

Dependent upon the direction in which the brushes are shifted from the neutral, in a non-interpole machine, the rotational electromotive force may either aid or oppose the self-induced electromotive force. Similarly, in a commutating-pole machine,

the rotational electromotive force may either aid or oppose the self-induced electromotive force, dependent upon the sign of the interpole. The adjustment in either case should of course always be such that these two electromotive forces oppose each other, which, for non-interpole machines requires that the brushes be set *forward* from the electrical neutral in the case of a *generator*, and *backward* in the case of a *motor*; see Article 50.

Since the rotational electromotive force is produced in order to counteract the effect of the self-induced electromotive, and thereby to prevent or reduce sparking, it is frequently referred to as the **commutating electromotive force.** That is, the term "rotational electromotive force," as above defined, and the term "commutating electromotive force" are used synonymously.

Whether or not sparking will occur at the brushes depends not only upon the value of the commutating electromotive force, but also upon the relation between the duration T of the commutating period (time of short-circuit), the self-inductance L of the coil (or equivalent self-inductance when several coils are simultaneously short-circuited by a wide brush), and the normal brush contact resistance R. The effect of these several factors can best be seen by considering the simple case of a brush having the same width as a single commutator segment, as shown in Figs. 39 to 41.

In addition to the symbols used in the preceding discussion, let e be the commutating electromotive force set up in the short-circuited coil in the direction opposite to that of the current in this coil at the beginning of the short-circuit, and let L be the coefficient of self-induction of this coil (see Article 19).

The self-induced electromotive force in the direction of the initial current is equal to the product of the self-inductance of the coil by the rate of *decrease* of this current (see Article 19), namely, to the product $-L\frac{di}{dt}$. The minus sign is used because di always stands for an *increase* of current, and therefore $-di$ must be used to express a *decrease* of current. The resultant electromotive force, in the direction of the current i, acting around the loop (Fig. 40) formed by the coil, the two commutator leads, two commutator segments and brush, is then

$$-L\frac{di}{dt}-e$$

From Kirchoff's Second Law, this electromotive force must be equal to the total resistance drop around this loop in this same direction. This resultant resistance drop, neglecting the resistance of the coil, leads, segments and internal resistance of the brush, is (see Article 38)

$$R_2i_2-R_1i_1$$

where R_2 and R_1 are respectively the contact resistances between the brush and the two segments 1 and 2 (Fig. 40), and i_2 and i_1 are the currents through these two contacts. Equating the resultant electromotive force to this resultant resistance drop, there results

$$R_2i_2-R_1i_1=-L\frac{di}{dt}-e \qquad (9)$$

By substituting for R_1 and R_2 their values from equations (2) and (2*a*), and making use of the relations (see Fig. 40) that $i_1=I-i_2$ and $i_2=0.5I+i$, this may be reduced to a differential equation involving only the two variables i_2 and t. The solution of this differential equation for i_2 gives for the resistance drop at the trailing brush contact at any time t the value *

$$R_2i_2=RI(T-t)^{n-1}t^{-n}X \qquad (10)$$

where

$$n=\frac{RT}{L} \qquad (11)$$

and

$$X=\int_0^t(T-t)^{-n}t^n\left(n\frac{T}{t}-\frac{eT}{LI}\right)dt \qquad (12)$$

The quantity n defined by equation (11) has a simple physical meaning, as may be seen by multiplying the numerator and denominator of the right-hand side of equation (11) by the brush current I, which gives

$$n=\frac{RTI}{LI}=\frac{RI}{\frac{LI}{T}} \qquad (13)$$

The product RI is the **normal brush contact drop.**

From the definition of self-inductance (Article 19), the flux linkages of the short-circuited coil due to a current i in it are Li. The current in the short-circuited coil changes during the commu-

* The method of obtaining this solution is too complicated to be given here.

tation period from the value $0.5I$ to the value $-0.5I$; see Figs. 39 and 41. Hence the total change of flux linkages of the short-circuited coil during interval T that it is short-circuited is $0.5LI-(-0.5LI)=LI$. The average *rate of change* of the flux linkages of the coil due to the reversal of the current in it is therefore $\frac{LI}{T}$. But the rate of change of the flux linkages of a coil is equal to the electromotive force induced in it. Hence, $\frac{LI}{T}$ is equal to the *average value of the self-induced electromotive force during the short-circuit.* This average self-induced electromotive force is frequently referred to as the average **reactance voltage,** and will be represented by the symbol E_r.

From equation (13) it therefore follows that the ratio n is equal to the ratio of the normal brush contact drop RI to the average reactance voltage E_r, viz.,

$$n=\frac{RI}{E_r} \tag{14}$$

To determine from the above relations the contact resistance drop at the trailing brush tip at the end of the commutating period it is necessary to find the value of the function X for $t=T$. In general, this leads to an indeterminate expression. The evaluation of this indeterminate form by the usual method of the calculus gives the following results:

(1) When the commutating electromotive force e is constant throughout the short-circuit and is *equal* to the average reactance voltage E_r, the contact resistance drop at the trailing brush contact is likewise constant throughout the short-circuit, irrespective of the value of n, and is equal to the normal brush contact drop RI. The commutation is then linear and there will be no sparking; see Article 38. (This condition may be closely approximated in a well-designed commutating-pole machine.)

(2) When the commutating electromotive force is *not* equal to the average reactance voltage, the value to which the contact resistance drop at the trailing brush tip will rise at the end of the short-circuit depends upon the value of the ratio n, that is, upon the relative value of the average reactance voltage E_r and the normal brush contact drop RI.

(2a) For $n>1$, that is, for the average reactance voltage E_r less than the normal brush contact drop RI, equation (10) gives for the contact resistance drop at the trailing brush tip at the end of the short-circuit the value

$$(R_2i_2)_T=RI\left(\frac{RI-e_T}{RI-E_r}\right) \tag{16}$$

Whether or not this potential difference will cause sparking depends upon the relative values of the commutating electromotive force e_T and the average reactance voltage E_r. In practice it is found that when there is a substantial difference between the average reactance voltage E_r and the normal brush contact drop RI, the difference between the commutating electromotive force e_T and the normal brush contact drop may usually be as high as 2 or 2.5 volts without causing sparking.

(2b) For $n<1$, that is, for the average reactance voltage E_r greater than the normal brush contact drop RI, equation (10) gives *infinity* as the limiting value of the contact resistance drop at the trailing brush tip, except for the special case when the function X, equation (12), is equal to zero for $t=T$, in which case the limiting value of the contact resistance drop at the trailing brush tip again reduces to the value given by equation (16). An inspection of equation (12) will show that, to make X equal to zero, the commutating electromotive force must be greater than the average reactance voltage E_r. In other words, when E_r is greater than RI, then e_T must also be greater than E_r, or sparking will result.

The conclusions thus far established may be summarized as follows:

(*a*) If the machine is so designed that the average reactance voltage E_r is considerably less than the normal brush contact drop RI, sparking may not be serious even though there is no commutating electromotive force. See equation (16), and put $e_T=0$.

(*b*) If the machine is so designed that the average reactance voltage E_r is *less* than the normal brush contact drop RI, but only slightly less, sparking can be prevented only by providing a commutating electromotive force *equal to or approximately equal to* the average reactance voltage.

(*c*) If the machine is so designed that the average reactance

voltage E_r is *greater* than the normal brush contact drop RI, sparking can be prevented only by providing a commutating electromotive force *equal to or greater than* the average reactance voltage.

40. Departure in Practice from the Ideal Conditions Assumed in the Elementary Analysis of Commutation.—Although the general conclusions reached in the last Article are substantially in accord with the results actually obtained in practice, the quantitative relations above deduced must be considered as approximations only. This is due to the departure in practice from the ideal conditions assumed in the analysis leading up to the formulas which have been developed.

In the first place, it was assumed that the combined resistance of the coil, leads and segments and the internal resistance of the brush is negligible in comparison with the brush contact resistance. This is only approximately true, and in small machines this resistance may be quite appreciable in comparison with the brush contact resistance.

Again, it was assumed that each brush has a width equal to that of one commutator segment, and the thickness of the mica insulation between segments was neglected. Actually, in order that the brush may have sufficient current-carrying capacity and at the same time permit the use of narrow segments, the brush usually has such a width that it spans two or more segments.

The resistances at the trailing and leading brush contacts were assumed to vary inversely as the area of these contacts. Inverse proportionality between resistance and area of contact is equivalent to direct proportionality between contact resistance drop and current density. As a matter of fact the resistance drop at the contact between carbon (or graphite) and copper is not proportional to the current density, and therefore the resistance is not inversely proportional to the area of contact.

When current is passed from a carbon brush to copper in contact with it (or in the reverse direction) the resistance drop is found to vary with the current density in the manner shown by curve A in Fig. 46. As shown in this figure, only for low-current densities does the proportionality between resistance drop and current density hold. For the particular brush to which this figure applies, the resistance drop at 20 amperes per square inch is

1 volt. Doubling this current density, namely, increasing it to 40 amperes per square inch, increases the resistance drop to only 1.25 volts. When the current density increases to 100 amperes per square inch the resistance drop increases to only 1.4 volts.

The curve A in Fig. 46 gives the relation between the resistance drop and current density *after sufficient time has elapsed for steady temperature conditions to become established* in the brush and copper. If, however, the resistance drop is measured at the instant the current is established, it is found that this instantaneous resistance drop (curve B in Fig. 46) is more nearly proportional to the current density than is the resistance drop (curve A) corresponding to steady temperature conditions. However, even this instantaneous resistance drop is not strictly proportional to the current density, as assumed in the preceding analysis.

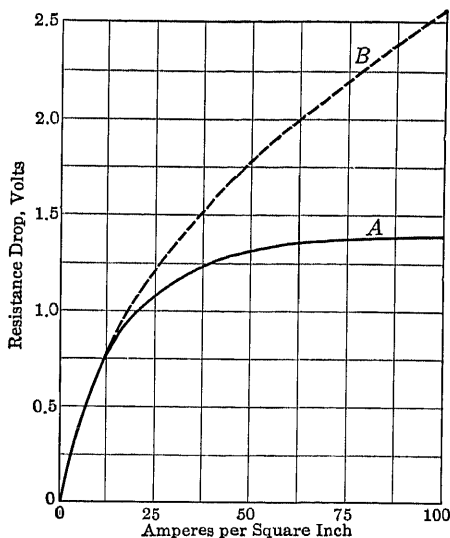



Fig. 46.—Contact Resistance Drop at Carbon Brush.

The decrease in the brush-contact resistance with increase in temperature accounts for the fact that certain machines will commutate perfectly satisfactorily under a *momentary* overload, but will spark badly if this overload is continued. If the overload lasts only for an instant, there will be but a slight change in the brush-contact resistance, and therefore the overload will cause a proportional increase in the normal brush-contact drop RI and the average reactance voltage $E_r = \frac{LI}{T}$, and the necessary relation between RI and E_r will remain unaltered; see Article 39. However, should the overload continue until steady temperature conditions are approached, the brush-contact resistance will decrease, and therefore the normal brush-contact drop will

increase less rapidly than the average reactance voltage, with the result that the normal brush-contact drop RI may become less than the average reactance voltage E_r, and the necessary conditions for satisfactory commutation may no longer obtain.

Another factor which has an important bearing upon commutation in actual practice is that, as has already been pointed out, the flux distribution in the air gap of a machine having a slotted armature is not a smooth curve, but contains ripples, a depression for every slot and an elevation for every tooth. Moreover, as the armature rotates the position of these ripples with respect to the main poles oscillates, moving back and forth with a frequency proportional to the number of slots. Consequently, when a slotted armature is used, which is standard practice in modern machines, it is not practicable to produce a constant commutating electromotive force, particularly when the brush spans more than one commutator segment. This is true whether the commutating electromotive force is produced by shifting the brush, or by interpoles, or by a compensating winding.

Consequently, the ideal condition of a constant commutating electromotive force directly proportional to the load, or brush, current can never be perfectly realized in practice, although it may be closely approximated when commutating poles are employed. Moreover, the ripples in the flux distribution will always be reflected in the current in the short-circuited coil (or coils), so that the actual relation between this current and time will not be a smooth curve, such as those shown in Fig. 42, but will contain high-frequency ripples which are often of considerable magnitude, e.g., as much as 50 per cent of the maximum current in the coil.

The net result of these practical variations from the ideal conditions assumed in the above analysis is that the quantitative relations there derived are approximate only. In particular, although it would appear from the deductions in the preceding article that an exact relation must hold between the value of the commutating electromotive force and the average reactance voltage, in order to avoid sparking when this latter voltage is greater than the normal brush-contact drop, yet actually it is found that this relation need be only approximated within a limit easily realized in practice.

41. Importance of Low Self-inductance and High Brush Contact Resistance.—In non-interpole machines the commutating electromotive force is produced by shifting the brushes from the magnetic neutral. In such machines it is impossible to keep the commutating electromotive force even approximately equal to, or proportional to, the armature current, unless the brushes are shifted every time the load changes, which is impracticable (see Article 50), Consequently such machines must be so designed that the ratio n is greater than unity, that is, the average reactance voltage must be less than the normal brush contact drop RI, viz.,

$$E_r < RI \tag{17}$$

or

$$L < RT \tag{17a}$$

The time of short-circuit, or commutating period T, depends upon the peripheral speed of the commutator and the width of the brush. The higher the speed and the thinner the brush, the shorter will the commutating period be.

The self-inductance L of the short-circuited coil varies directly as the square of the number of turns in the coil undergoing commutation, and varies inversely as the reluctance of the magnetic circuit of the flux produced by the current in this coil. In order to satisfy the condition expressed by equation (17) or (17a) it is necessary to keep the inductance L of the short-circuited coil down to a low value. This means few turns in the coil undergoing short-circuit and a high reluctance for the magnetic circuit of the flux which the current in this coil produces. This requirement is one of the reasons why a multi-polar machine with short armature is preferable to a bi-polar machine with a long armature.

The condition expressed by equation (17a) brings out clearly the desirability of a brush having a high contact resistance R, namely, a carbon or graphite brush in preference to a copper brush. The greater this contact resistance the smaller may be the commutating period T, and the larger the self-inductance L, and yet keep within the limit required by equation (17a).

Note that it is the *contact* resistance between the brush and commutator, not the *internal* resistance of the brush, which is of primary importance in commutation.

When the ratio $\frac{L}{T}$ differs considerably from R, the contact

resistance drop at the trailing brush tip, equation (16), may be sufficiently small for satisfactory commutation even when there is no commutating electromotive force e_T. For example, if $\frac{RT}{L}=2$, and there is no commutating electromotive force, the resistance drop at the trailing brush tip at the end of the commutating period would be equal to $2RI$. As already noted the normal brush-contact drop RI is usually of the order of 1 volt. Hence, the contact drop at the trailing brush tip at the end of the short-circuit would under these conditions be only 2 volts, and this would not be sufficient to produce sparking.

These numerical relations are, of course, approximate only, but nevertheless indicate that it should be possible, by making the self-inductance of the armature coils small, to design a machine which will commutate satisfactorily without introducing a commutating electromotive force. This is actually done in non-interpole motors designed for rotation in either direction. In such machines the brushes are set permanently in the *mechanical* neutral, in which case a small rotational electromotive force is set up in the short-circuit coils, but in a direction *opposite* to that which would aid in commutation (see Article 50). When the brush-contact resistance is thus relied upon entirely for producing satisfactory commutation, the commutation is sometimes referred to as **resistance commutation.**

42. Heating of the Brushes.—Equation (16) indicates that when $E_r < RI$ and the commutating electromotive force e_T is greater than the normal brush-contact drop RI, then $(R_2 i_2)$ will be negative. This means that i_2 must decrease from its initial value I to zero before the short-circuit is completed, rise to some other value in the opposite direction, and then fall to zero again at the end of the short-circuit, as indicated by the curve C in Fig. 43. This condition, known as over-commutation, is to be avoided, for it always causes unnecessary heating of the brushes.

In fact, any variation from straight-line commutation will result in a greater power loss at the brush contact than would result were the commutation strictly linear. This is clearly shown by Fig. 49, which gives the total power lost in the brush at each instant during the short-circuit (1) when the current i in the short-circuited coil varies linearly with time as shown by curve A, Fig. 42 (straight-line commutation); (2) when the current varies

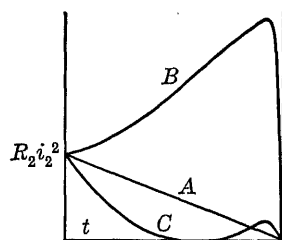



FIG. 47.—Power Loss in Trailing Contact.

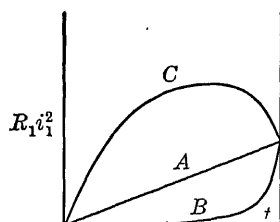



FIG. 48.—Power Loss in Leading Contact.

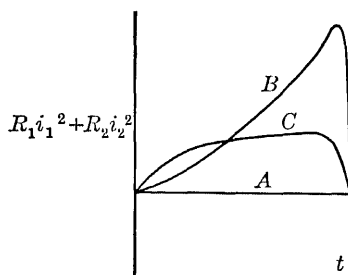



FIG. 49.—Total Power Loss in Brush Contact.

A. Straight-line Commutation
B. Under-commutation
C Over-commutation

with time in the manner shown by curve *B*, Fig. 42 (under-commutation); and (3) when the current varies with time as shown by the curve *C*, Fig. 42 (over-commutation). As shown in Fig. 49 the energy lost in the brush contact (i.e., the area under the respective curves) is greater for both under-commutation (*B*) and for over-commutation (*C*) than it is for straight-line commutation.

The curves in Fig. 49 are derived from those in Figs. 43 and 44 by squaring the ordinates of the current curves and multiplying by the corresponding ordinates of the resistance curves. In this way the power lost in the trailing contact (Fig. 47) and the power lost in the leading contact (Fig. 48) are obtained. The ordinates of the curves in Fig. 49 are the sum of the corresponding ordinates in Figs. 47 and 48.

From Figs. 47 and 48 it is evident that for under-commutation the trailing contact is overheated, whereas for over-commutation the leading contact is over-heated. Although these curves are for the ideal conditions stated in Article 36, they are nevertheless typical of the results actually obtained in practice. In particular, it is found in practice that when the brushes are not properly set, they not only tend to heat up as a whole, but the heat developed at one tip or the other may be sufficient to cause this brush tip to become red hot.

When the current in the short-circuited coil has ripples in it, due to the armature teeth or to any other high-frequency pulsation in the flux which threads the coil, the energy lost in the brush contact is still further increased.

The extra power loss at the brush contacts when there is a departure from the ideal condition of straight-line commutation is not particularly serious as far as the efficiency of the machine is concerned. However, due to the fact that the brush-contact resistance *decreases* with increase of temperature, this extra power loss may lead to commutation difficulties.

43. Wearing Down of Commutator.—The contact between a brush and moving commutator is different from that between the brush and commutator when the commutator is at rest, for there is always more or less movement of the brush with respect to the commutator as a whole. This slight motion of the brush causes the formation of minute sparks between the wearing

surface of the brush and the segment (or segments) with which it is in contact. In the case of a properly designed machine with brushes properly set, these minute sparks are not perceptible to the eye, but they are nevertheless of sufficient strength to cause a gradual vaporization of the copper of the commutator.

This vaporization of the copper is an extremely slow process, for a commutator may operate for months without showing any appreciable loss of metal which can be attributed to this cause, rather than to the gradual abrading action of the brushes. However, when a graphite or soft carbon brush is used, it frequently happens that after a time the surface of the segments is found to be below that of the edges of the mica between the segments, as shown (exaggerated) in Fig. 50.

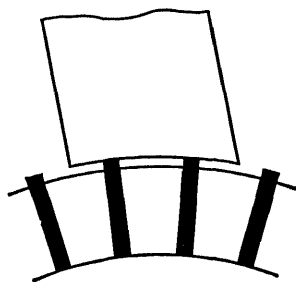



Fig. 50.—High Mica.

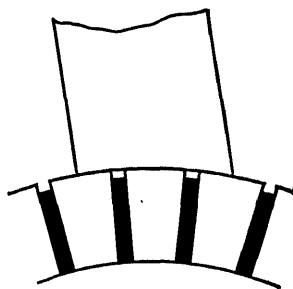



Fig. 51.—Undercut Mica.

When this condition of "high mica" once starts, it develops rapidly, since the higher the mica above the copper surface, the greater will be the vibration of the brush and therefore the greater the tendency for the formation of sparks and consequent vaporization of the copper. The cure for high mica is either to under-cut the mica, as shown in Fig. 51, or to use a brush which has the proper abrasive qualities to wear down the mica at the same rate as the copper is vaporized. Too hard a brush will of course also wear the copper away too rapidly. Consequently the proper type of brush to give the best results is a matter which must be given careful consideration.

When soft brushes are used the mica should always be undercut (see Fig. 51), to a depth of from $\frac{3}{64}$ to $\frac{1}{16}$ inch. This is preferably done by a small revolving cutter similar to small cir-

cular saw. A properly shaped tool should be used for this purpose, and care should be taken to leave no copper particles between the segments.

From the practical standpoint, the criterion of satisfactory commutation is a relatively dark but highly polished commutator surface. If, in a properly designed machine, the commutator surface becomes scratched or roughened, or shows high mica, the trouble is usually due to the grade of brush used. Slight roughness may be removed by sandpapering, but when the commutator has become badly scratched, the armature should be put in a lathe and the commutator turned down to a true cylinder, and the mica under-cut. *Emery cloth should never be used on a commutator*, for some of the grains of emery will embed themselves in the copper, thereby preventing uniform wear of the surface under action of the brushes.

44. Ring-fire and Flashing. Permissible Volts per Bar.—Oil or grease on the commutator will also cause trouble. Such substances gradually soak into the binder used in forming the "built-up" mica insulation of the commutator, become carbonized, and ultimately short-circuit the commutator bars, or cause a "ground." Before a serious short-circuit occurs, this surface-layer of carbonized binder is usually heated to incandescence by the current which leaks through it, producing what is commonly known as "ring-fire." Should ring-fire begin to develop the commutator should be carefully cleaned.

If grease or dirt is permitted to collect on the commutator a destructive arc may ultimately form between bars, resulting in what is known as "flashing." The intense heat developed by a serious flash may actually melt part of the commutator.

Even with a perfectly clean commutator, flashing is likely to occur unless the machine is so designed that the difference of potential between adjacent commutator bars never exceeds a relatively very low value. This limiting value of the "volts per bar" is very much less than one would expect.

For example, in ordinary air, the difference of potential between two needle points $\frac{1}{32}$ inch apart must be raised to about 1500 volts before a spark will pass. Yet experience shows that if the potential difference between adjacent commutator bars, separated by $\frac{1}{32}$ inch of mica, reaches a value of about 30

volts,* an arc will form between the adjacent edges of these two bars, thus short-circuiting the commutator.†

This low limit to the permissible volts per bar (i.e., potential difference between adjacent bars) is the chief limitation to the voltage for which it is practicable to build direct-current dynamos. Direct-current dynamos are seldom built for terminal voltages in excess of 750 volts, although 5000-volt d-c. machines have been constructed for special applications.

Even when a machine is so designed that the *average* voltage between commutator segments is well below the permissible volts per bar, a heavy over-load may cause the voltage between one or more pair of commutator segments to rise above this limit, and thus start a flash at the commutator (see Article 49).

PROBLEMS

1. The current per brush in a certain generator is 100 amperes, and the period of commutation (i.e., the interval of time during which an armature coil is short-circuited by a brush) is 0.002 second. For a given setting of the brushes the current in the coil undergoing commutation varies according to the formula

$$i = 50 \cos (90{,}000t)^\circ, \qquad (A)$$

where t is in seconds and the angle $(90{,}000t)$ is in degrees.

For another setting of the brushes the current during commutation varies according to the formula

$$i = 50 - \frac{500t}{1.01 - 500t}. \qquad (B)$$

For a third setting of the brushes the current during commutation varies according to the formula

$$i = 250(500t - 0.7)^2 - 72.5 \qquad (C)$$

(*a*) Plot, on the same sheet of cross-section paper, the current represented by each of these three equations. Take 1 inch = 20 amperes and 8 inches = period of commutation.

(*b*) On a second sheet of cross-section paper plot the corresponding values of the current in the trailing brush contact, and on a third sheet of cross-section paper the corresponding values of the current in the leading brush contact.

* The exact value depends on the size of the machine, the smaller the machine the higher this limiting voltage per bar.

† A possible explanation of the low dielectric strength of the air-layer in contact with a commutator is that this film of air becomes ionized by the minute sparks which always form between the wearing surface of the brush and the commutator.

e three settings of the brushes is there over-commuta-
er-commutation.

›f Problem 1 assume the normal brush contact resistance
ual to 0.01 ohm, and each brush to have a width equal to
ment.

of cross-section paper (1) the resistance of the trailing brush contact and (2) the resistance of the leading brush contact. For each curve take 1 inch = 0.04 ohm and 8 inches = period of commutation.

3. Making the same assumptions in regard to the contact resistance of the brushes as in Problem 2, determine, by applying equation (4), the resistance drop at the trailing brush contact at the end of the commutation period, for each of the three settings of the brushes referred to in Problem 1.

(*b*) Plot curves, similar to those in Fig. 45, showing the variation of the resistance drop at the trailing brush contact throughout the period of commutation, for each of these brush settings. In each case take for the ordinate scale 1 inch = 2 volts.

(*c*) What would be the resistance drop at the trailing brush contact were the commutation linear? Indicate this value on the curve-sheet used for answer to preceding question.

4. The self-inductance of each armature coil of the generator referred to in Problems 1 and 2 is 0.019 millihenry.

(*a*) What is the average reactance voltage induced in each coil as it undergoes commutation?

(*b*) What is the normal brush contact drop for a total brush current of 100 amperes?

(*c*) Calculate the factor n, defined in Article (39), for the given generator.

(*d*) Combining equations (4) and (9), prove that when the rate of change of the current in the short-circuited coil at the end of the period of commutation is *not infinite*, the value of the commutating electromotive force e, at the end of the short-circuit is

$$e_T = RI + (RT - L)\left(\frac{di}{dt}\right)_T,$$

where $\left(\frac{di}{dt}\right)_T$ is the rate of change of the current in the coil at the end of the short-circuit.

(*e*) From the formula just deduced determine, for each of the three brush settings referred to in Problem 1, the value of the commutating electromotive force at the end of the commutation period, and its direction relative to the current in the coil at the beginning of the short-circuit.

(*f*) For which of the three brush settings will the commutation be the most satisfactory?

5. (*a*) From the formula derived in Problem 4*d* prove that the resistance drop at the trailing brush contact at the end of the period of commutation, when it is not infinite, has the value given by equation (16), Article 39.

(*b*) What step in this deduction of equation (16) is not permissible when the rate of change of the current at the end of the short circuit is infinite?

(*c*) From the formula given in section (*d*) of Problem 4 it would appear that

when there is no commutating electromotive force, and $RT=L$, the rate of change of the current at the end of the short-circuit would be infinite. Why is such a conclusion incorrect?

6. (*a*) Plot curves, similar to those in Fig. 49, showing the power loss at the brush contact at successive instants during the period of commutation, for each of the three brush settings referred to in Problem 1. The normal brush contact resistance is 0.01 ohm and the period of commutation 0.02 second.

(*b*) From the curves plotted in (*a*) determine the average power loss for each of the three brush settings, and compare with the power loss which would occur were the commutation linear.

7. For the particular case of no commutating electromotive force set up in the coil undergoing commutation and $RT=\frac{1}{2}L$ (that is, for $e=o$ and $n=\frac{1}{2}$), the solution of equation (9) for the current in the short-circuited coil is

$$i=\frac{I}{2}\left\{\sqrt{\frac{T-t}{t}}\left[\sin^{-1}\left(\frac{2t-T}{T}\right)+\frac{\pi}{2}\right]-1\right\}.$$

(*a*) Plot this current i as ordinates against the time t as abscissas, taking $I=100$ amperes and $T=0.002$ second. For the scale of abscissas take 4 inches $=0.001$ second and for the scale of ordinates take 1 inch $=20$ amperes. Compare this curve with those plotted in Problem 1.

(*b*) Prove by applying equation (4) to this particular case, that the corresponding resistance drop at the trailing brush contact approaches infinity at the end of the commutation period. (This is a special case of the general rule, that when RT is less than L sparking will always occur unless a commutating electromotive force of proper value is present.)

(*c*) For a normal brush contact drop of 1 volt, what is the value of the average reactance voltage which would make $n=\frac{1}{2}$, and what is the minimum value of the commutating electromotive force necessary to prevent sparking.

CHAPTER V

ARMATURE REACTION

45. Armature Reaction.—As shown in Article 32, the electromotive force generated in the armature of a direct-current dynamo, for a given setting of the brushes, is proportional to the flux per pole. When there is no current in the armature this flux is due solely to the field current, and its value is readily calculated by the method outlined in Chapter III.

A current in the armature conductors, however, also tends to set up a flux through the magnetic circuit of the machine. Hence, when the machine is loaded, there are two magneto-motive forces acting on the magnetic circuit, one due to the field current and the other due to the armature current. Consequently, whenever a generator or motor is supplying a load, the resultant flux per pole, and therefore also the electromotive force generated in the armature, depend upon both the field current and the armature current.

The effect of the armature ampere-turns is in general both to decrease the total flux per pole and to distort this flux, increasing it under one pole tip and decreasing it under the other. On account of the decrease in the total flux per pole, the armature electromotive force, for a given field excitation, decreases as the load on the machine increases. Due to the distortion of the flux, particularly under heavy overload, serious sparking at the brushes may occur, or even a severe flash between brushes.

The reduction and distortion of the flux by the armature current is known as **armature reaction.**

46. Flux Distribution Due to Field Current Alone.—In the upper part of Fig. 52 is shown the general shape and distribution of the lines of force in two poles of the magnetic circuit of a smooth-cored, multipolar dynamo when there is **current in the field winding only.** The density of these lines at the surface of the armature

is a maximum under the central portion of each pole, decreases gradually toward the pole tips, then rapidly beyond the pole tips, and becomes zero midway between the poles.

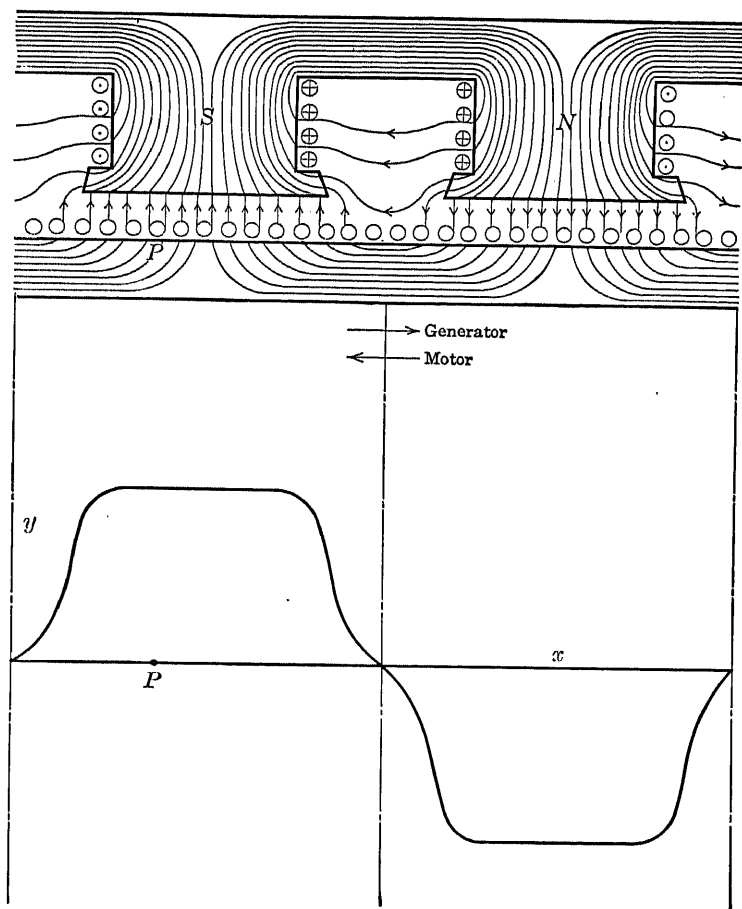



Fig. 52.—Flux Distribution Due to Field Current Only.

Although such a "map" of the magnetic field of the machine gives a good idea of the distribution of the lines of force throughout the magnetic circuit, the distribution of the useful flux at the surface of the armature may be more conveniently represented by a **flux-distribution curve,** as shown in the lower part of Fig. 52. The flux-distribution curve is constructed as follows:

Imagine the surface of the armature to be made up of strips parallel to its axis. Let x be the distance (measured circumferentially around the armature) of any one of these strips from the point midway between a north and south pole, and let dx be the width of this strip. Let $d\phi$ be the number of lines of force which pass out of the armature through this strip, and put

$$y=\frac{d\phi}{dx} \tag{1}$$

That is, put y equal to the number of lines of force which pass out of the armature *per unit distance measured around its surface.* This ordinate y may be called the *lineal flux density.* When there is no current in the armature, the variation of the lineal flux density from point to point around the air-gap is as shown by the curve in the lower part of Fig. 52.

From the way in which the ordinate y of the flux-distribution curve is defined, it follows that this ordinate at any point P (see Fig. 52) is proportional to the average flux density along the line in the surface of the armature which passes through P parallel to the axis of the armature. Note also that this ordinate y is proportional to the electromotive force generated in an armature conductor as it moves past this point, provided the armature is driven at a constant speed.

This follows from the fact that when the conductor moves a distance dx from a given point, the change in the number of lines of force which link this conductor is ydx. Hence, calling dt the time required for the conductor to move this distance, the electromotive force generated in it at this instant is $y\frac{dx}{dt}$. But $\frac{dx}{dt}$ is equal to the peripheral speed of the armature, which by assumption, is constant. Hence the electromotive force generated in a single armature conductor at any instant is proportional to the ordinate of the flux-distribution curve at the point occupied by this conductor at this instant.

The relation just stated leads to a relatively simple method of determining the flux-distribution curve experimentally, provided the armature has a full-pitch winding. Two narrow brushes, b_1 and b_2, insulated from each other, and set at a distance apart equal to the width of a commutator bar plus the thickness of the

mica insulation between bars, are so mounted that they may be placed in any position with respect to the main brushes B_1 and B_2, as indicated in Fig. 53. The two auxiliary brushes are connected to a low-reading voltmeter, as shown in the figure. The main brushes are set in the mechanical neutral, the field is excited, and the armature driven at constant speed by means of an auxiliary motor.

The voltmeter reading will then be proportional to the average ordinate of the flux-distribution curve over the distance on the armature surface corresponding to the distance between the two auxiliary brushes b_1 and b_2. By taking voltmeter readings for successive positions of the two auxiliary brushes, and plotting these readings against the distance of these two brushes from one of the main brushes, a curve will be obtained having the same

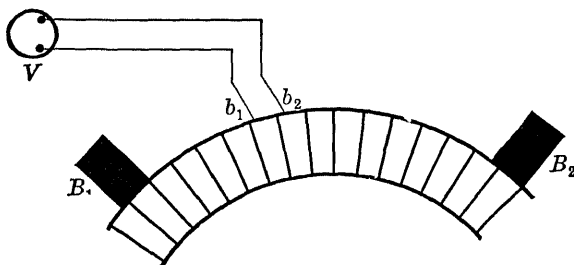



FIG. 53.

shape as the flux-distribution curve. If the number of turns per armature coil is known and the speed of the armature is noted, the lineal flux density y may then be calculated.

From the way in which the ordinates of the flux-distribution curves are defined, equation (1), it follows that the *total useful flux per pole* is equal to either half of this curve, the area of the positive half giving the number of lines of force which pass out of the armature into a south pole, and the area of the negative half giving the number of lines of force which enter the armature from a north pole.

In the case of a slotted armature, the flux-distribution curve will contain ripples, a depression for each slot and an elevation for each tooth. However, except for these ripples, the general shape of the no-load flux-distribution curve will be as shown in Fig. 52.

47. Flux Distribution due to Armature Current Alone—Brushes in Mechanical Neutral.—When the *brushes* of a multipolar machine are set in the *mechanical neutral*,* and a **current is established through the armature only** (no field current), the magnetic field produced by this current is as shown by the line-of-force diagram in the upper part of Fig. 54. The lines of force are all symmetrical with the centers of the poles, since in all the armature conductors between any pair of adjacent brushes the current is in the same direction, and each such group, or band, of conductors is symmetrical with respect to the center of a pole.

In this and the subsequent figures the direction of the armature current in each conductor is taken as that of the current which would exist in these conductors were the armature moving in the magnetic field shown in Fig. 52, to the right in the case of a generator, and to the left in the case of a motor.

The flux-distribution curve corresponding to the magnetic field set up by the armature current, when the brushes are in the mechanical neutral, is shown by the solid-line curve in the lower part of Fig. 54. This curve is drawn on the assumption that the reluctance of the iron part of the magnetic circuit is small in comparison with the reluctance of the air, which is usually the case unless the iron is magnetically saturated (see Article 17). Note that half of the band of conductors under one pole and half of the band of conductors under the next pole may be thought of as forming a "pancake" coil with its center under the brush which is between these two poles.

Were it not for the relatively large air-space between poles, the flux-distribution curve would be as indicated by the dotted lines, with a maximum immediately over the brush. However, on account of the high reluctance of the large air-space between poles, compared with the reluctance of the air-gap under the pole faces, the flux distribution is actually as shown by the solid-line curve, with maxima under each pole tip,

An inspection of Fig. 54 will show that when the brushes are set in the mechanical neutral the effect of the armature current is to make half of each pole of the field structure a north pole

* As noted in Article 32, the brushes are said to be in the mechanical neutral when they are so set that they will short-circuit a full-pitch coil when the conductors which form this coil are midway between poles.

and the other half an equal south pole. Hence, the resultant armature reaction flux which enters or leaves a field pole is zero, for there will be as many lines leaving a field pole as enter it.

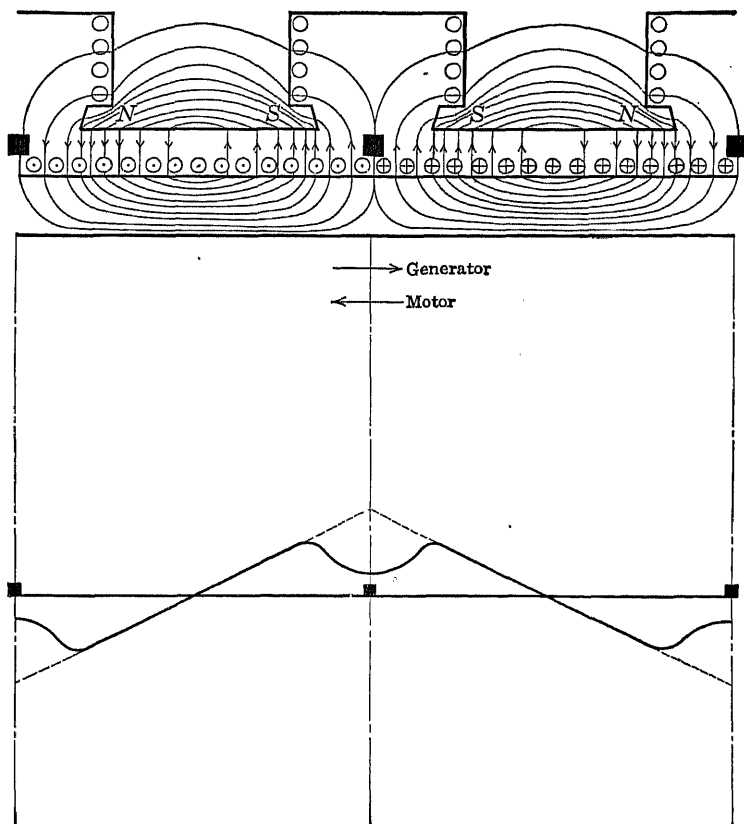



Fig. 54.—Flux Distribution Due to Armature Current Only. Brushes in Mechanical Neutral.

48. Flux Distribution Due to Armature Current Alone—Brushes Shifted from Mechanical Neutral.—When the brushes are set ahead of the neutral in the case of a generator, or behind the neutral in the case of a motor, as indicated in Fig. 55, the band of conductors between any two brushes is no longer symmetrical with the center of the field pole under which it lies. The map of

the magnetic field due to each of these bands therefore assumes an unsymmetrical shape. However, this unsymmetrical field may be resolved into two symmetrical fields by considering each of the band of conductors between a pair of brushes as made up of

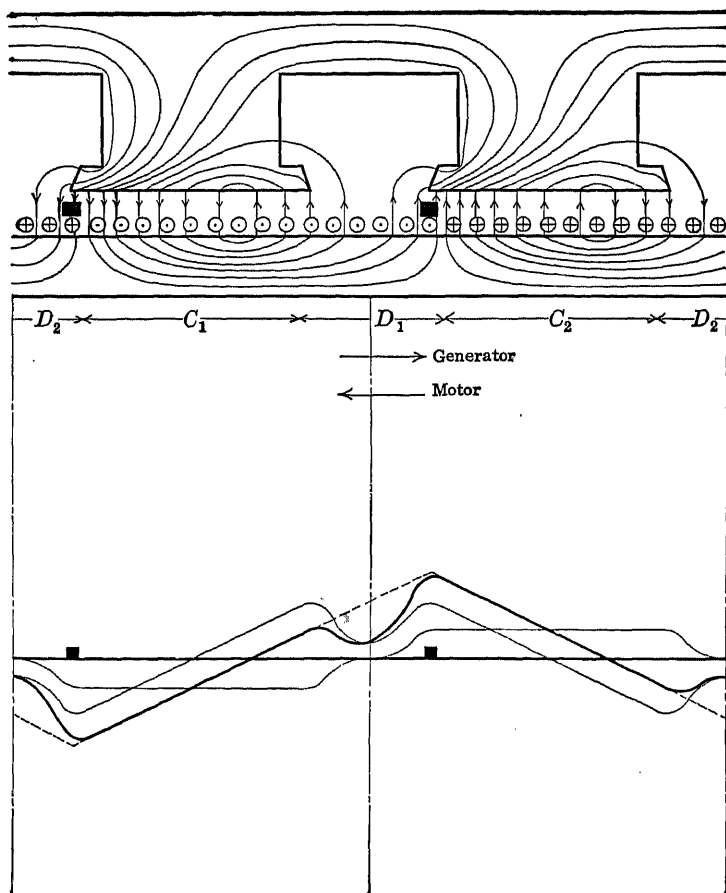



FIG. 55.—Flux Distribution Due to Armature Current Only. Brushes Not in Mechanical Neutral.

two parts (see Fig. 55), one part, C_1, symmetrical with respect to the center of the pole, and the other part, D_2, symmetrical with respect to the point midway between two poles.

In the upper part of Fig. 56 is shown the map of the magnetic field due to those armature conductors which constitute the

bands which are symmetrical with respect to the centers of the poles, namely, the part C of each band. The corresponding flux-distribution curve is shown in the lower part of this figure. The magnetic field due to this portion of the armature conductors

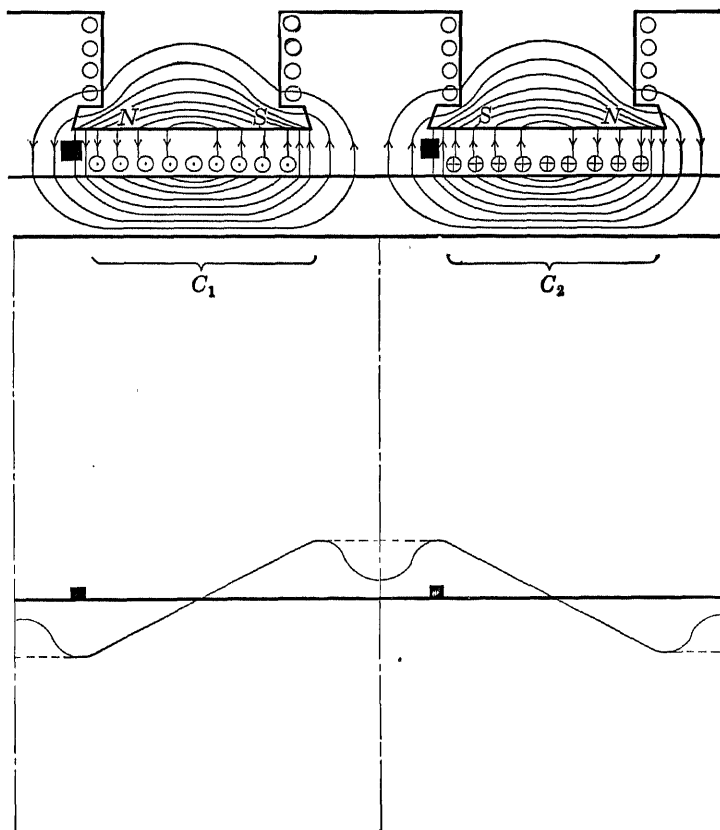



Fig. 56.—Flux Distribution Due to Cross Magnetizing Armature Ampere-turns.

is similar to that due to all the armature conductors when the brushes are set in the mechanical neutral, the only difference being that the strength of the field in the region between the poles is somewhat less, owing to the omission of the conductors in each band which are symmetrically located with respect to this space.

An inspection of Fig. 56 will show that each line of force due to the conductors C, which passes from the air-gap into a given pole on one side of its center, comes out into the air-gap on the other side of the pole center. The ampere-turns corresponding to these conductors therefore tend to magnetize the poles *transversely*, and for this reason they are usually referred to as the **cross ampere-turns.**

The resultant flux per pole due to the cross ampere-turns, when there is no current in the field winding, is zero, since each of the corresponding lines of force passes twice through the air-gap under one and the same pole, once in one direction and once in the opposite direction. The primary effect of the cross ampere-turns is therefore to distort the main magnetic field, increasing the flux under one-half the pole-face and decreasing it under the other half. When the brushes are set in the mechanical neutral (Fig. 54), all the armature turns are cross-magnetizing turns.

The effect of the remainder of the armature conductors, namely, the portion D (see Fig. 55) which form groups symmetrical about the mechanical neutral, is shown in Fig. 57. An inspection of this figure will show that half of the group D_1 between any two poles, taken with half the group D_2 in the next interpolar space, for example, the groups D_1'' and D_2' in Fig. 57 have practically the same effect as would be produced by a coil of the same number of ampere-turns located on the field pole itself. However, comparing the magnetic field produced by the current in this group of armature conductors with the main field produced by the field current (Fig. 52), it is seen that the field due to these armature conductors is in the *opposite direction* to that of the main field. Hence, the turns in the armature winding formed by these armature conductors are called the **demagnetizing armature turns.**

From Fig. 57 the number of demagnetizing turns per pole is equal to the number of armature conductors between a given brush and the mechanical neutral. When the brushes are set in the mechanical neutral (Fig. 54) there are no demagnetizing turns.

The heavy-line flux-distribution curve in Fig. 55, which shows the combined effect of the cross-magnetizing and demagnetizing armature ampere-turns, is found by adding the corresponding ordinates of the separate flux-distribution curves due to these

two groups of armature conductors. As shown by the dotted line in Fig. 55, were the iron of the field structure continuous all the way around the air-gap (no projecting poles), this resultant curve would have a triangular shape, but would be shifted from the

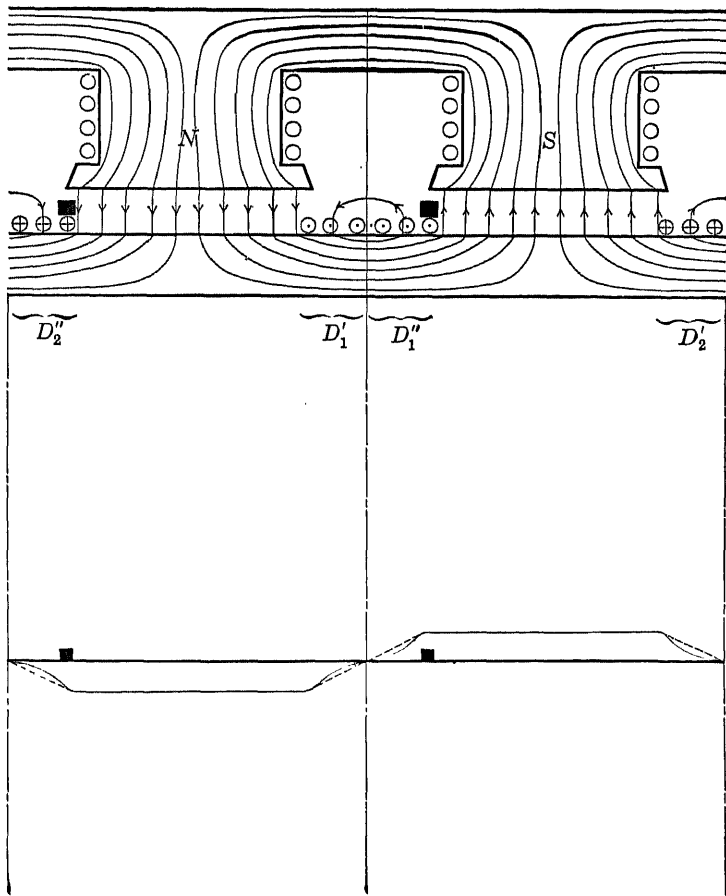



FIG. 57.—Flux Distribution Due to Demagnetizing Armature Ampere-turns.

position shown in Fig. 54 by an amount equal to the shift of the brushes. Actually, the air-space between the poles causes a depression in this triangular wave-shape, this depression being always over the mechanical neutral, irrespective of the position of the brushes.

49. Effects of Armature Reaction when Brushes are Set in Mechanical Neutral.—Were the reluctance of the magnetic circuit of a dynamo constant, the magnetic field due to the combined action of the field current and armature current could be found by

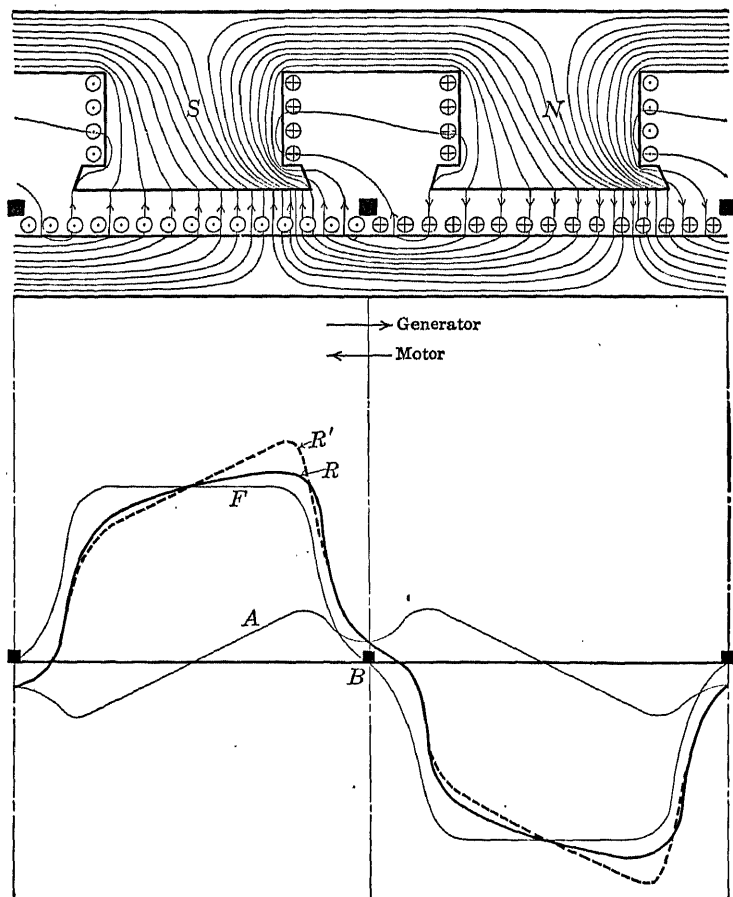



Fig. 58.—Flux Distribution Due to Combined Action of Field and Armature Currents. Brushes in the Mechanical Neutral.

superimposing upon the field due to the field current by itself, the field due to the armature current by itself. The resultant flux distribution at the surface of the armature would then be given by the curve whose ordinates are the algebraic sum of the ordinates of the flux-distribution curves due to the field current alone

and to the armature current alone. The "apparent" resultant flux-distribution curve determined in this manner, when the brushes are in the mechanical neutral, is shown by the dotted curve R' in Fig. 58. The curve F is taken from Fig, 52 and the curve A from Fig. 54.

However, as pointed out in Article 35, the reluctance of the magnetic circuit is not constant, but increases as the flux in this circuit increases, giving for the relation between the flux and the resultant ampere-turns acting on the circuit a curve of the form shown in Fig. 59. From this curve it is evident that the change in the flux produced by a given change in the ampere-turns which

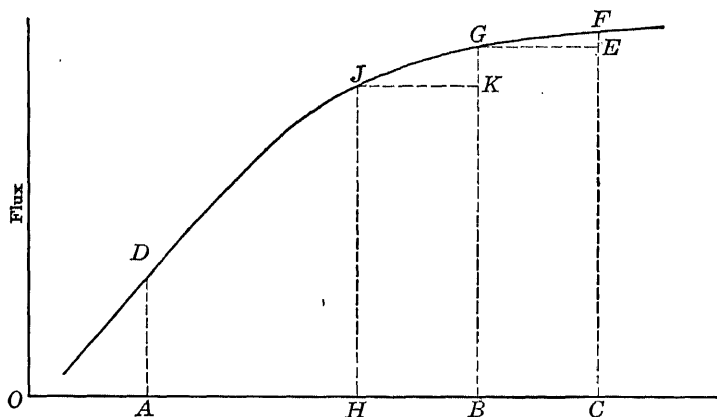



FIG. 59.—Ampere-turns.

link its circuit becomes less and less the greater the initial value of the flux. For example, a change in the ampere-turns from zero to a value AO in Fig. 59 changes the flux from zero to AD, whereas if the flux has an initial value BG, then an equal change $BC(=OA)$ in the ampere-turns changes the flux by a much smaller amount EF.

Consequently, if in Fig. 58 curve F represents the flux distribution at the surface of the armature due to a given field current by itself, and curve A represents the flux distribution due to a given armature current by itself, and R' the sum of these two curves, then the actual resultant flux distribution due to the joint action of these two currents will be as shown by the full-line curve R, which differs less from the initial flux distribution F than does the apparent resultant R'. The difference between

the two curves R and R' will depend upon the degree of magnetic saturation produced by the field current alone, i.e., the higher the initial saturation of the magnetic circuit, the less effect will the armature current have on the resultant field.

As shown by the curve R' in Fig. 58, when the brushes are set in the mechanical neutral, the effect of the armature current is always to distort the magnetic field, increasing the flux density under one pole tip (the trailing pole tip for a generator and the leading pole tip for a motor) and decreasing it under the other pole tip. This is also shown by the map of the lines of force in the upper part of Fig. 58. As shown by this map, the armature current causes the lines of force to crowd into one pole tip and to spread out in the other pole tip.

As a consequence of the shape of the saturation curve (Fig. 59), it also follows that a given *decrease* in the ampere-turns acting upon a magnetic circuit will, except at low flux densities, cause a *greater* change in the flux than an equal *increase* in the ampere-turns. Compare the lengths of FE and GK in Fig. 59, noting that BH and BC are equal. Therefore, even when the brushes are in the mechanical neutral, the armature current decreases the flux at one pole tip more than it increases it at the other, with the result that there is a net decrease in the total flux which enters the armature between each pair of brushes, and therefore a net decrease in the generated voltage. This effect, which is a secondary effect of the distorting action of the cross-magnetizing turns, is sometimes referred to as the **demagnetizing effect of the cross-magnetizing turns.**

Due to the reduction in the total flux which enters the armature between any pair of brushes, this demagnetizing action of the cross-magnetizing turns causes a proportional decrease in the armature electromotive force (i.e., in the resultant electromotive force between brushes). This effect becomes more pronounced the greater the value of the armature current. It is usually inappreciable at loads less than about one-third full-load, but produces a substantial decrease in the generated electromotive force at higher loads.

Another effect of the distortion of the flux by the armature current is to cause an increase in the maximum voltage between commutator bars. An inspection of Fig. 60 will make this evident,

when it is remembered that the instantaneous value of the electromotive force generated in an armature conductor is proportional to the ordinate of the flux-distribution curve at the point corresponding to the position of the conductor at this instant. The greater the peak in the flux-distribution curve, the greater the electromotive force per conductor, and therefore the greater the

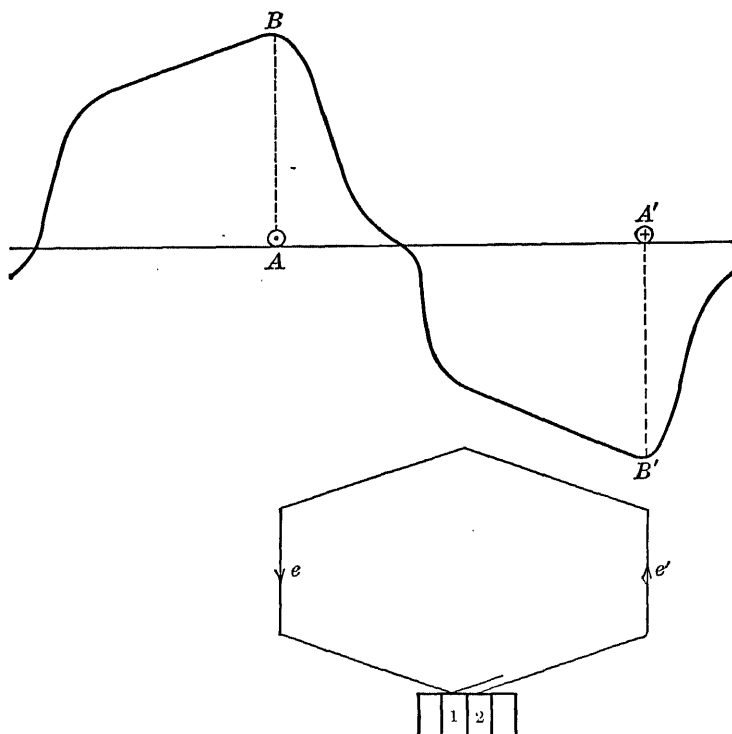



FIG. 60.

voltage between the commutator bars (1 and 2 in Fig. 60) which form the terminals of an armature coil. Under heavy overloads (large armature current) the distortion may be sufficient to raise the voltage between bars to such a value as to cause an arc to form between bars, thus causing a flash at the commutator (see Article 44). Fortunately, the tendency to flash under heavy overloads is partly counteracted by the saturation of the pole tip by the armature current, particularly if the field ampere-turns are

sufficient to produce initially a high degree of magnetization in the pole faces.

An inspection of Figs. 54 and 58 shows that the effect of the armature ampere-turns, when the brushes are in the mechanical neutral, is confined almost entirely to the pole faces. In the outer portion of each pole and in the yoke the armature turns, which in this case are all cross-magnetizing turns, produce but little, if any, effect; compare Fig. 58 with Fig. 52. Hence, the cross-magnetizing effect of the armature turns can be reduced by saturating the pole faces only, which will increase the reluctance of the total magnetic circuit only slightly, but will produce a relatively large increase in the reluctance to the cross flux which the armature current tends to produce (see Fig. 54).

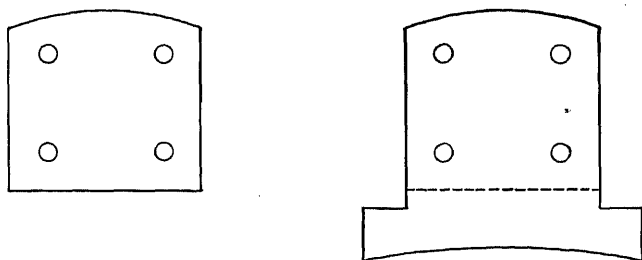



Fig. 61.

The saturation of the pole face is usually secured by cutting away a portion of each of the laminations of which the poles are built up, as shown in Fig. 20, or by building up the pole of alternately short and long laminations, as shown in Fig. 61. A more effective, but more expensive, way of eliminating the effects of armature reaction is to employ a compensating winding (see Article 52).

Remembering that the ordinate of the flux-distribution curve at any point is proportional to the electromotive force generated in an armature conductor as it moves past this point, it is evident from Figs. 54 and 58 that the magnetic field due to the armature current, when the brushes are set in the mechanical neutral, is in such a direction as to generate in an armature coil, as it is short-circuited by a brush, an electromotive force in the *same* direction as that of the current in this coil just before the short-circuit.

Consequently, when the brushes are set in the mechanical neutral, the magnetic field due to the armature current sets up

in each coil, as it undergoes commutation, an electromotive force which tends to prevent the reversal of the current in this coil. This electromotive force is therefore in the opposite direction to that required to overcome the tendency to spark (see Article 39). To prevent sparking under these conditions it is therefore necessary either (1) that the armature coils have a very low self-inductance (e.g., few turns) and that the brushes have a high-contact resistance, or (2) that some auxiliary means, such as commutating poles or a compensating winding, be provided to counteract this effect of the armature current, and to produce a commutating electromotive force in the proper direction.

In the older types of railway motors and other motors designed to run in either direction, with a fixed setting of the brushes, low self-inductance of the armature coils and high brush-contact resistance were relied upon entirely to secure sparkless commutation. In modern machines, however, except those of small capacities, commutating poles or compensating windings are almost universally employed, since these auxiliary means greatly extend the range of load which may be satisfactorily commutated. Commutating poles and compensating windings are considered in detail in Articles 51 and 52.

49a. Effect of Armature Reaction when Brushes are Shifted from the Mechanical Neutral.—In most machines which have no auxiliary means for rendering commutation sparkless, a commutating electromotive force in the proper direction is usually secured by shifting the brushes from the mechanical neutral. As has already been pointed out (Article 39), this is accomplished by shifting the brushes forward in the direction of rotation in the case of a generator, and by shifting them backward in the case of a motor.

In Fig. 62 is shown the effect of the armature current on the field of the machine when the brushes are shifted in this manner. This figure applies either to a generator or to a motor, provided that in the case of a generator the armature is thought of as moving to the right, and in the case of a motor to the left. In the figure, then, the brushes have a forward lead for a generator and a backward lead for a motor.

The resultant flux-distribution curve R in this figure is plotted in the same manner as in Fig. 58, by adding the flux-distribution

curve A due to the armature current alone (Fig. 55), to the flux-distribution curve F due to the field current alone (Fig. 52), and making an allowance for the saturation of the iron, as explained in the preceding article.

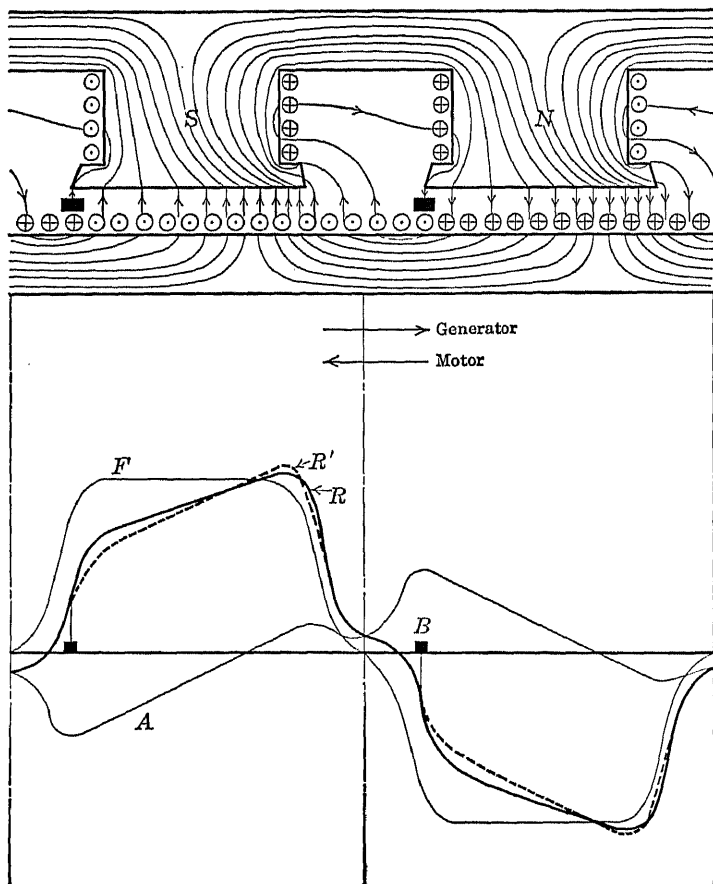



FIG. 62.—Flux Distribution Due to Combined Action of Field and Armature Currents. Brushes Shifted from Neutral.

Comparing Fig. 62 and Fig. 58 it is seen that, whereas in Fig. 58 (brushes in mechanical neutral) the resultant magnetic field over the brush B is positive, in Fig. 62 the resultant field over the brush B is negative. This change in the direction of the field has been secured by moving the brush B forward (for a generator),

or backward (for a motor), to a position where the field due to the field current is stronger than that due to the armature current, and is in the reverse direction. Remembering that the electromotive force generated in an armature conductor at any instant is proportional to the ordinate of the flux-distribution curve at the point corresponding to the position of this conductor at this instant, it is evident that, when the brushes are set as shown in Fig. 62, the electromotive force generated in the armature conductors when they are short-circuited by a brush is in the proper direction to aid commutation.

The resultant field at the conductors which are undergoing short-circuit is the *difference* between that due to the field current and that due to the armature current. Consequently, the value of the commutating electromotive force obtained in this manner is not proportional to the armature current, as it should be to produce the best results (see Article 39), but *decreases* as the armature current *increases.* Of course it is possible, by shifting the brushes as the load changes, to keep an exact balance between this electromotive force and the reactance voltage of the short-circuited coil, but shifting the brushes with change of load is not practicable. However, when the machine is properly designed, and brushes having a high contact-resistance are employed, it is possible to secure satisfactory commutation over a relatively wide range of load without shifting the brushes after they have once been set.

To prevent too great a decrease in, or possible reversal of, the commutating electromotive force as the armature current increases, various expedients may be used to increase the reluctance of the path of the cross flux (Fig. 56) which the armature current tends to produce. In large machines, the reluctance of this path is increased by cutting away part of the laminations in the pole face, as shown in Fig. 20 or 61. In small and medium size machines, the laminations are frequently all alike, but are shaped to form a chamfered pole face (Fig. 63) resulting in an air-gap of greater length at the pole tips than at the center of the poles.

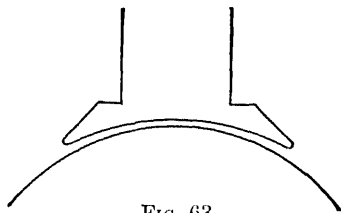



Fig. 63.

As explained in Article 47, when the brushes are given a forward lead for a generator or backward lead for a motor, the armature ampere-turns produce not only a cross-magnetizing action (Fig. 56) but also a direct demagnetizing action (Fig. 57). Hence, when the brushes are shifted to secure sparkless commutation, the resultant flux which enters the armature between any pair of brushes is not only distorted, but is decreased by an amount dependent upon the angle through which the brushes are shifted. The greater this angle the greater the number of demagnetizing armature turns (see Fig. 57). This demagnetizing effect is the same as would be produced by decreasing the field ampere-turns per pole by an amount equal to the demagnetizing ampere-turns per pole of the armature, i.e., by the ampere-turns formed by the conductors D_1'' and D_2' in Fig. 57. In addition to this direct demagnetizing action of the armature current, there is the further reduction in the resultant flux due to the demagnetizing action of the cross-magnetizing turns, caused by the unequal degrees of saturation in the two halves of each pole (see Article 49).

A reduction in the flux which enters the armature between any pair of brushes means a reduction in the armature electromotive force (see Article 32). Consequently, in a machine which must be operated with shifted brushes armature reaction always produces a marked decrease in the generated electromotive force with increase of load, this decrease in voltage being greater the greater the angle through which the brushes are shifted.

With shifted brushes the distortion of the magnetic field by the armature current is somewhat less than when the brushes are in the mechanical neutral. However, just as in the case of machines designed to be operated with brushes in the mechanical neutral, the distortion of the flux may be sufficient, under heavy overloads, to cause a flash-over at the commutator.

50. Reaction Due to Currents in the Armature Coils Short-circuited by the Brushes.—While an armature coil is being short-circuited by a brush, the center of this coil is practically under the center of a pole. An inspection of Fig. 10 will make this clear; the coil short-circuited by the brush a, consisting of the conductors 1 and 6, is substantially equivalent to a coil on the field pole itself.

Since successive armature coils occupy this position as they are

short-circuited, the resultant effect of the short-circuit currents in these coils is practically equivalent to that which would be produced by an alternating current flowing in a fixed coil on each field pole, this current passing repeatedly through the same cycle of values as the actual short-circuit current.

In Fig. 42 are shown some of the possible ways in which the current in an armature coil may vary during the commutation period. An inspection of this figure will show that the *average value* of this current over the full time of short-circuit may be either positive, negative or zero, depending upon whether the conditions of commutation are such as to give under-commutation, over-commutation or straight-line commutation.

Irrespective of their average value, the currents in the short-circuited coils always produce pulsations in the flux in the main magnetic circuit of the machine. The frequency of these pulsations is inversely proportional to the time required for a commutator bar. to move past a brush. These pulsations are, therefore, very rapid, and since their amplitude is usually small, they are not, as a rule, noticeable in the normal operation of the machine.

If the variation of the current in the short-circuited coil is such that the *average* value of this current during the period of commutation is different from zero (i.e., if the areas of the positive and negative portions of the current-time curve in Fig. 42 are unequal), then the average value of the resultant flux per pole will be either increased or decreased, depending upon whether this average value is positive or negative and upon whether the machine is operating as a motor or generator. The currents in the short-circuited coils may, therefore, on the average, produce either a magnetizing or demagnetizing effect, depending upon the conditions of commutation. This magnetizing or demagnetizing effect is usually slight, but may, under certain conditions, become appreciable.

Even when no current is being taken from or supplied to the armature through the brushes, there will in general be a current induced in these short-circuited coils, for during the small interval of time required for a commutator segment to pass by a brush there is usually at least a small change in the magnetic flux which links this coil, and therefore a small electromotive force is induced

in it. Although this electromotive force is very small, the resistance of the coil and brush is likewise very small, and therefore the current in this short-circuited coil may be of appreciable magnitude. This current starts from zero, rises to a positive maximum, falls to zero, decreases to a negative maximum, and comes back to zero again. Its variation with respect to time will be somewhat as shown in Fig. 64. The exact shape of this current-time curve depends upon the self-inductance of the armature coils and upon the setting of the brushes with respect to the magnetic neutral. Either the positive or negative half of the curve may be the greater.

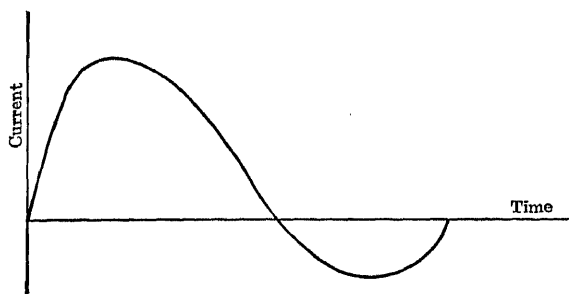



Fig. 64.

The value of the no-load short-circuit current is proportional to the electromotive generated in the armature coil while it is short-circuited by a brush, and is therefore proportional to the speed at which the armature is driven. Hence, at no-load the effect of the currents induced in the short-circuited coils will be less, the lower the speed at which the armature is driven. On this account, in determining the magnetization curve experimentally, the armature is sometimes driven at a very low speed, instead of at rated speed. The flux per pole calculated from the results of such a test will be more nearly equal to that which the field current alone would produce than would be the case were the test carried out at rated speed.

51. Commutating Poles.—As pointed out in the preceding article, the commutating electromotive force required to overcome the self-induced electromotive force in the short-circuited coil may be secured by shifting the brushes, forward in the case of a generator and backward in the case of a motor. An inspection of Fig. 62 will show that shifting the brushes from the mechanical

neutral moves the short-circuited coil from this neutral toward one of the pole tips. Another way of securing the same effect is to leave the brushes in the mechanical neutral, and to move a portion of each pole to a position directly over the mechanical neutral. This results in the construction shown in Fig. 65, which

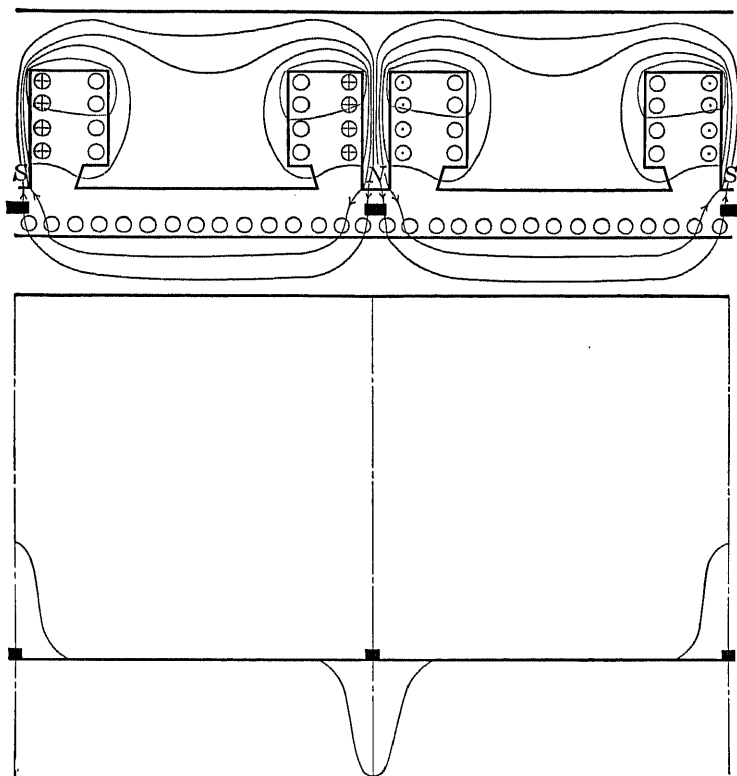



Fig. 65.—Flux Distribution Due to Current in Commutating-pole Winding only.

shows a small auxiliary pole between each of the main poles. This auxiliary pole is called a *commutating pole*, or *interpole*.

The winding on the commutating pole is connected in series with the brushes, so that the ampere-turns of this winding are directly proportional to the load current. The coil is so connected to the brushes that in the case of a generator the commutating pole has the same sign as that of the main pole immediately ahead of it, and in the case of a motor the same sign as the main pole immediately behind it.

When commutating poles are used, the resultant magnetic field at any point in the air-gap is then made up of three component fields, viz., (1) that due to the field current, Fig. 52; (2) that due to the armature current, which will be the same as in Fig. 54 except that on account of the low reluctance of the interpole the flux density due to the armature current alone will be a maximum directly under the interpole as shown by curve *C*, Fig. 66; and (3) that due to the current in the commutating-pole winding, Fig. 65. These three component fields and their resultant are shown by the curves *F*, *A*, *C* and *R* in Fig. 66. In plotting the resultant *R* allowance has been made for saturation, as explained in Article 49.

By giving the commutating-pole winding the proper number of turns it is possible to make it produce at the short-circuited conductor a field which not only counteracts the field at this point due to the armature ampere-turns, but in addition give a resultant field in the opposite direction, as shown in Fig. 66. Since the magnetomotive force due to the armature ampere-turns and that due to the commutating-pole winding are each proportional to the load current, and since at the mechanical neutral the field due to the main field is zero, it follows that, were there no magnetic saturation, the resultant field at the short-circuited conductor would be proportional to the load current at all loads. By properly choosing the number of turns in the commutating-pole winding it would then be possible to produce in the short-circuited coil a commutating electromotive force equal and opposite to the self-induced electromotive force at all loads, which is the ideal condition for commutation (see Article 39).

Comparing Fig. 66 with Fig. 62 it is evident that when commutating poles are used there is no necessity for shifting the brushes from the mechanical neutral. As pointed out in Article 49, when the brushes are in the mechanical neutral there is no direct demagnetizing action of the armature current. Interpoles, however, are employed not for the purpose of reducing armature reaction, but to provide a commutating electromotive force which, instead of decreasing with the load, increases as the load increases, thereby greatly extending the range of load which can be successfully commutated. On account of the increased capacity which can be obtained by their use, practically all modern dynamos having a capacity of 5 kilowatts or over are built with commutating poles.

Were there no saturation of the commutating poles they would render commutation perfectly satisfactory at all loads up to the heaviest overload which might be taken from the machine without

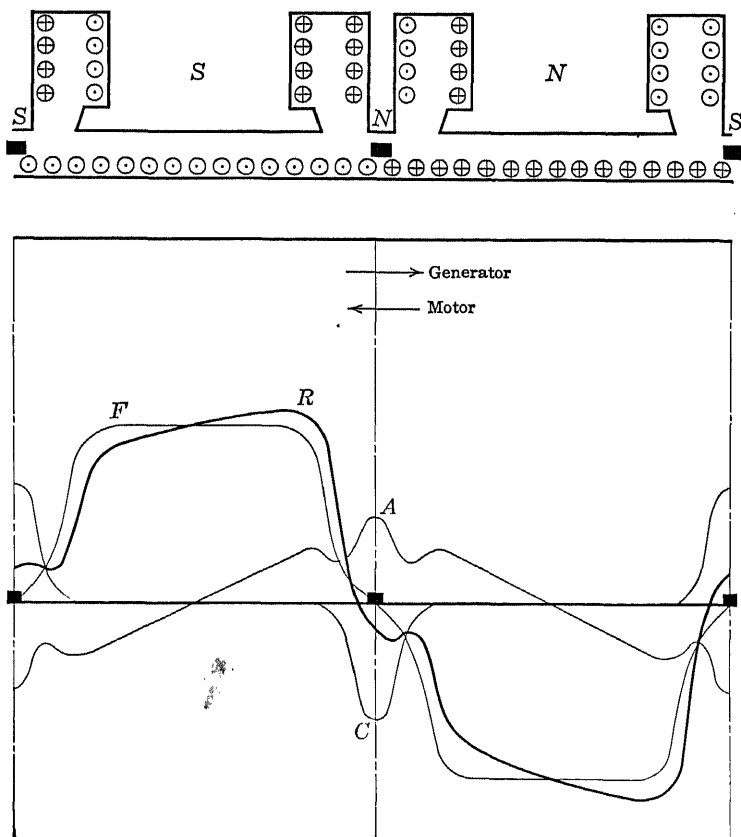



FIG. 66.—Flux Distribution Due to Combined Action of Currents in Field, Armature and Commutating-pole Windings.

overheating it. However, it is usually impracticable to give the commutating poles sufficient cross-section to prevent the effect of saturation (i.e., the increase of reluctance) from becoming appreciable under overload. As saturation is approached the flux entering the armature from the commutating pole increases less rapidly than the load current, and ultimately a load will be reached

for which the commutating electromotive force produced by this flux is no longer sufficient to overcome the self-induced electromotive force in the short-circuited coil, and sparking will result.

The winding of a commutating pole should preferably be designed to carry the total armature current and have such a number of turns that the ampere-turns of this winding is in excess of the armature ampere-turns per pole by exactly the right amount to produce the required commutating electromotive force. As it is difficult to predetermine exactly the number of turns required to obtain this result, the number of turns is sometimes made slightly greater than the calculated value and an adjustable shunt is connected across the terminals of the commutating-pole winding. The proper proportion of the total armature current may then be diverted through this shunt, by changing its resistance, to give the required current through the commutating-pole windings.

When a shunt of high-resistivity material is used to obtain the proper number of ampere-turns in the commutating-pole windings, the commutating poles may fail to prevent sparking under sudden changes in load. This is due to the fact that for a sudden change in the load the shunt takes more than its normal proportion of the total current, since the commutating-pole windings have a relatively high self-inductance, whereas that of the shunt is practically negligible.

To overcome this difficulty an inductive shunt is sometimes placed in series with the resistance shunt. This inductive shunt is merely a coil of copper wire wound on an iron core in which there is a suitable air-gap to make the self-inductance of this shunt equal to, or usually slightly greater than, the self-inductance of the commutating-pole winding.

A more common method of adjusting the commutating poles, and the one usually employed for small and medium size machines, is to adjust the air-gap under the pole by means of shims at its base.

In a commutating-pole machine the brushes are usually set exactly in the mechanical neutral. Under these conditions the resultant flux due to the commutating poles which enter the armature between any pair of brushes is zero (see Fig. 65), and the only change in the generated electromotive force between brushes is that due to the demagnetizing action of the cross-magnetizing armature ampere-turns (see Article 49) and the

magnetizing or demagnetizing action of the currents in the coils undergoing commutation (see Article 50).

If the brushes are given a backward lead in the case of a generator (or forward lead in the case of a motor), commutation may still be satisfactory, provided the ampere-turns of the commutating-pole winding are properly adjusted. In this case, however, the commutating-pole winding will produce the same effect as a series winding on the main pole, i.e., will increase the flux entering the armature between any pair of brushes. In addition the armature ampere-turns will also produce a magnetizing action (just the opposite to the effect produced in a generator when the brushes have a forward lead). Consequently the resultant electromotive force between brushes for a given speed will then increase as the load on the machine increases.

If the brushes are given a forward lead in the case of a generator (or backward lead in the case of a motor), the commutating-pole ampere-turns will produce a demagnetizing action and will therefore reduce the resultant electromotive force between brushes at any given speed.

In order to prevent such a compounding or differential action by the current in the commutating-pole winding, it is essential that the brushes of an interpole machine be set exactly in the mechanical neutral (see Article 90).

In the machine shown in Figs. 65 and 66 there are as many commutating poles as there are main poles. For 2-pole and 4-pole machines, on account of space limitations, it is usual practice to employ only one interpole per *pair* of main poles. In this type of construction the commutating poles are placed between alternate pairs of main poles, and are all of the same polarity (i.e., their faces are all north poles or all south poles).

That one commutating pole per pair of main poles will produce the necessary commutating electromotive force in the coil short-circuited by *each* brush may be seen by referring to Fig. 11, and imagining a commutating pole between the first and second main poles and one between the third and fourth main poles. At the instant shown the brush a short-circuits the coil formed by conductors 1 and 6, and the brush b is about to short-circuit the coil formed by conductors 5 and 10. One conductor in each of these coils is therefore subjected to the action of the commutating pole

between the first and second main poles, and each coil will therefore have an electromotive force generated in it by the flux from this commutating pole. Similarly, the interpole placed between the third and fourth main poles will be sufficient to produce the required commutating electromotive force in the armature coils as they are short-circuited by the brushes *c* and *d*. Hence for a four-pole machine only two commutating poles are required, although four may be used.

52. Compensating Winding.—As explained in the last Article, a machine provided with commutating poles will commutate satisfactorily over a much greater range in load than a non-interpole machine with brushes set in a fixed position. However, due to the saturation of the commutating poles, there is a definite limit to the overload which can be commutated without sparking. Moreover, as the magnetomotive force of the commutating-pole winding neutralizes the effect of armature reaction only at the conductors which are undergoing short-circuit, there still remains the distortion of the magnetic field under the main poles (see Fig. 66), and, as explained in Article 49, this distortion may, under heavy overload, cause flashing at the commutator.

Fig. 67.—D-C. Generator Stator, Showing Compensating Winding.

Both the saturation of the commutating poles and the distortion of the main flux may be practically completely eliminated by the use of a so-called "compensating winding." A compensating winding is a winding placed in slots in the faces of the main poles,

as shown in Fig. 67, and is connected in series with the brushes and commutating-pole windings. The connection to the brushes is such that the current through the conductors which form this winding is opposite to that of the current in the armature con-

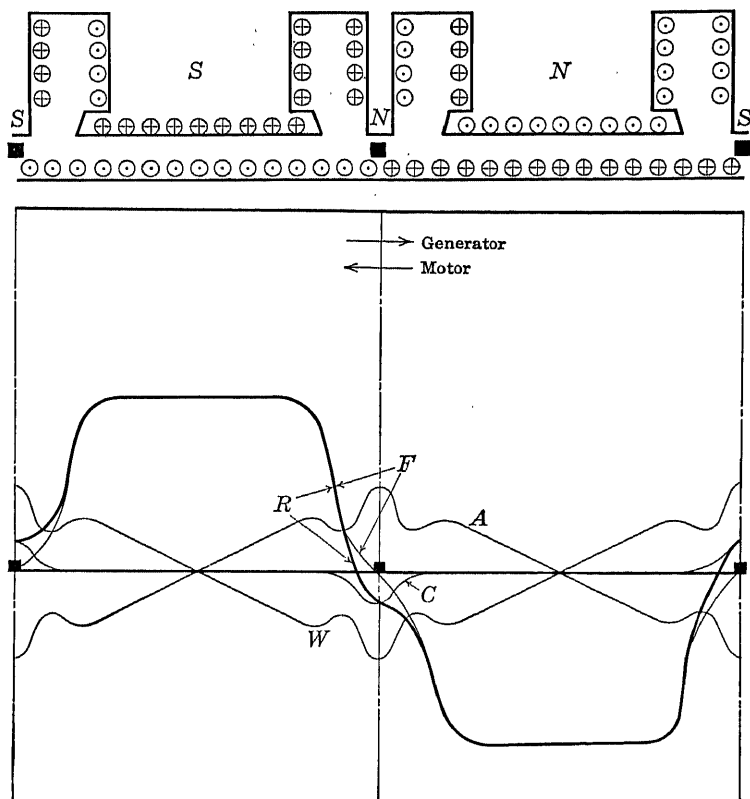



FIG. 68.—Flux Distribution Due to Combined Action of Currents in Field, Armature, Commutating-pole and Compensating Windings.

ductors which are immediately opposite, as shown in Fig. 68. By making the compensating winding of the proper number of turns, it is possible to neutralize almost completely the effect of armature reaction.

In the lower part of Fig. 68 are shown the flux-distribution curve due to the field current alone, curve F; that due to the armature current only, curve A; that due to the current in the compensating

winding only, curve W; that due to the current in the commutating-pole winding only, curve C; and the resultant flux distribution due to the currents in all four windings, curve R. Since the current in the compensating winding neutralizes the effect of the armature current, the ampere-turns of the commutating-pole winding need be only a fraction of the value required in a machine without a compensating winding; compare curve C in Fig. 68 with curve C in Fig. 66. This means that the magnetomotive force tending to produce leakage from the commutating pole to the main poles will be much less, and therefore the tendency to saturate the base of the commutating pole is greatly reduced.

A compensating winding therefore improves commutation in two ways. First, due to the elimination of the distortion of the main field, it reduces to the likelihood of flashing occurring at heavy overloads. Secondly, due to the reduction in the flux through the commutating pole, a much greater current may be taken from (or supplied to) the machine without causing sparking at the brushes.

The compensating winding of course adds materially to the cost of a machine and its use is justified only under special conditions, as, for example, in rolling-mill motors, which must be capable of withstanding momentarily extremely heavy overloads.

PROBLEMS

NOTE.—The following problems all refer to the same shunt generator, to which the following data apply:

Number of poles	4
Speed	1000 r.p.m.
Field turns per pole	800
Field current held at same value at no load and full load	2 amperes
Number of armature conductors	400
Type of armature winding	simplex lap
Total full-load armature current	100 amperes

The lineal flux density in the air-gap at no load, in maxwells per electrical degree,* is given in Table A, and at full load in Table B. The angles in these tables are measured from a point in the air-gap midway between a north and a south pole.

* By an "electrical degree" is meant $\frac{1}{360}$th of the angle subtended by the arc which extends from the center of one pole face to the center of the next pole face of the *same polarity*. One geometrical (or mechanical) degree in the circumference of the armature of a machine which has p poles is therefore equal to $\frac{p}{2}$ electrical degrees.

TABLE A.
No-load Flux Distribution

Electrical Degrees x.	Lineal Flux Density y.
0	0
10	1,000
20	3,000
25	7,000
30	11,000
45	14,000
90	15,000
135	14,000
150	11,000
155	7,000
160	3,000
170	1,000
180	0

TABLE B.
Full-load Flux Distribution.

Electrical Degrees x	Lineal Flux Density y.	Electrical Degrees x	Lineal Flux Density y.
0	1,000	120	15,200
10	400	130	16,000
20	400	140	16,400
25	1,000	150	15,000
30	2,000	155	10,800
40	4,400	160	6,200
50	6,600	170	2,400
60	8,300	180	1,000
90	12,000		

1. (*a*) Plot the no-load and full-load flux distribution curves corresponding to Tables A and B. Put each curve on a separate sheet of cross-section paper. Show the distribution in each case for 360 electrical degrees, and indicate the position of the poles. Scales: 1 inch = 40 electrical degrees and 1 inch = 8000 maxwells.

(*b*) Determine from the no-load flux distribution curve the total useful flux per pole. (*c*) What would be the generated electromotive force at no load were the brushes set in the mechanical neutral?

2. The flux distribution given in Table B is that which occurs when the brushes are shifted from the neutral.

(*a*) By how many electrical degrees is the electrical neutral corresponding to this flux distribution (curve B) displaced from the mechanical neutral?

(*b*) Assuming the permeability of the magnetic circuit to be constant, plot on the same sheet as curve B the flux distribution which this armature current alone would produce for this particular setting of the brushes.

3. In order to produce the necessary commutating electromotive force, the brushes must be so set that the armature conductors which form the sides of the coil undergoing short-circuit are in a field whose lineal flux density is 1000 maxwells per electrical degree.

(*a*) Indicate, on the curve sheet showing the full-load flux distribution, the position of the conductors which are undergoing short-circuit.

(*b*) By how many electrical degrees are the brushes shifted from the mechanical neutral and in what direction? From the electrical neutral?

4. (*a*) For the given setting of the brushes, what proportion of the armature conductors constitute *demagnetizing* turns and what proportion *cross-magnetizing* turns? (Neglect the demagnetizing effect of the cross-magnetizing turns.)

(*b*) What is the number of armature demagnetizing turns per pole?

(*c*) What is the current in each turn?

(*d*) What is the number of demagnetizing *ampere-turns* per pole?

(*e*) If there were no armature reaction, by how much would the field current have to be decreased in order to give the same resultant useful flux per pole as under actual conditions with armature reaction? (Neglect the demagnetizing effect of the cross-magnetizing turns.) Give answer in amperes and also in percentage of the actual field current.

(*f*) By what percentage is the resultant magnetomotive force acting on the magnetic circuit of the given machine reduced by armature reaction?

5. When the armature is supplying full-load current, by how much will the maximum voltage between the commutator bars of this generator exceed the maximum voltage between bars when there is no current in the armature? Indicate on curve sheet B the location of the two commutator bars between which the voltage is a maximum under load.

6. Locate on both curve sheets A and B the positions of the armature conductors which are short-circuited by the brushes when the brushes are set as specified in Problem 3.

(*a*) Assuming a full pitch armature winding, what is the value, at no load and at full load, of the resultant useful flux per pole, namely, the flux ϕ_r defined at the end of Article 32?

(*b*) What is the value of the generated voltage at no load and at full load? Compare with the no-load electromotive force when the brushes are set in the mechanical neutral; see Problem 1(*c*).

(*c*) For the actual setting of the brushes, by how much does the armature reaction reduce the resultant flux ϕ_r, and by how much does it reduce the generated voltage? Give answers as percentages of the no load values. Compare with the reduction in the resultant magnetomotive force; see Problem 4(*f*).

(*e*) Why is the reduction in the useful flux less than the reduction in the magnetomotive force?

7. (*a*) Replot the no-load flux distribution curve of the generator considered in the preceding problems. On this same sheet plot the armature flux distribution which the armature current alone would produce were the brushes set in the mechanical neutral. Draw this curve to scale, as nearly as you can estimate.

(*b*) Show the position and polarity of the commutating poles which would produce the proper commutating electromotive force.

(*c*) What lineal flux density must the current in the commutating-pole winding produce at the short-circuited conductors in order to produce the same commutating electromotive force as in Problem 3?

(*d*) Draw the flux distribution curve which the current in the commutating-pole winding alone would produce. Draw this curve to scale, as nearly as you can estimate.

(*e*) Draw the resultant flux distribution curve corresponding to the combined action of the field, armature and commutating-pole currents.

8. How many conductors per pole would be required in a compensating winding for the generator here under consideration, in order to completely neutralize the total magnetomotive force produced by the current in the armature conductors? Assume this winding to have no shunt.

CHAPTER VI

VOLTAGE REGULATION OF GENERATORS

53. Introduction.—From a practical standpoint, one of the most important characteristics of a direct-current generator is the way in which its *terminal voltage* varies with the *current* which it supplies to the line, or load, connected to it. This particular characteristic of a generator is commonly referred to as its "voltage regulation."

In this Chapter will be considered in some detail (1) the requirements, as regards voltage regulation, of the various kinds of service in which direct-current generators are employed, (2) the factors upon which the voltage regulation of a given generator depends, (3) the characteristic variation in voltage of each type of generator (separately excited, series, shunt and compound), (4) the use of automatic voltage regulators, and (5) the operation of direct-current generators in parallel.

54. Kinds of Service—Systems of Distribution.—Experience has shown that, with the exception of street lighting, the most satisfactory way of distributing direct-current energy from a generator or group of generators is to run from each terminal of the generator (or group) one or more wires, called "mains" or "feeders," and to connect the various individual loads in parallel across these mains, as shown in Fig. 69. This type of distribution is known as **parallel** or **multiple** distribution.

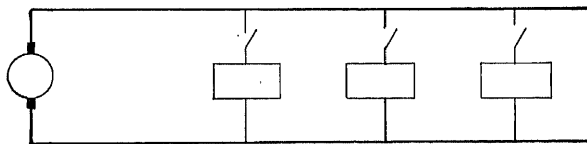



FIG. 69.—Parallel Distribution.

Contrasted with this multiple system is the **series system** of distribution, in which the individual loads (usually lamps) are all connected in series with each other and with the generator, as

shown in Fig. 70. A combination of the two systems, consisting of groups of individual loads connected in series, and these groups connected in parallel, or vice versa, is called a *series-parallel system.*

In a series system the current in every translating device * in the circuit (whether this device be a lamp, motor or any other electric apparatus) must be the same. The opening of the circuit in any one of these devices interrupts the current in all the others. Hence in a series system a device can be taken out the circuit without affecting the remaining devices only by first short-circuiting the line terminals to which it is connected, and then removing it from the circuit. When electric lamps are connected in series, which is common practice in street lighting, an automatic short-

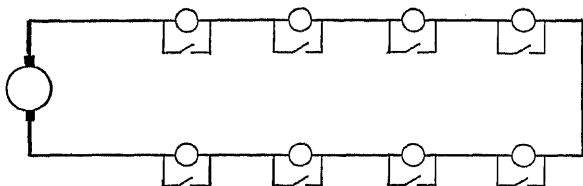



Fig. 70.—Series Distribution.

circuiting device is placed in the socket or base which holds the lamp. Should a lamp burn out, this short-circuiting device closes and provides a by-pass for the line current.

Within certain limits an incandescent lamp may be operated at any current. However, when the current falls below the normal value for which the lamp is designed, there is a relatively very large decrease in its candle-power. On the other hand, if a current only slightly greater than the normal value is maintained through the lamp continuously, the useful life of the lamp is greatly reduced. Consequently, in a series system the current must be kept practically constant, irrespective of the number of lamps in the circuit. The series system of distribution is therefore frequently referred to as **constant-current distribution.**

Formerly direct-current series generators, with brush-shifting devices or other means for maintaining constant current at all loads, were used to supply series systems. In modern direct-current series systems, however, the direct current is usually

* By a translating device is meant any device in which electric energy is transformed, or translated, into some other form of energy.

obtained from a mercury-arc rectifier, in conjunction with an alternating-current supply.

The current in a series system being constant, the voltage supplied by the generator must increase as the load (number of lamps) on the system is increased. For series street-lighting voltages as high as 5000 volts are used. This is the voltage at the generator. The voltage across each lamp is usually about 75 volts for arc lamps, and ranges from about 10 to 50 volts for incandescent lamps.

The chief advantage of the series system is that the current in the supply wires (i.e., in the wires which connect the lamps to one another and to the generator) is small, being only that required for a single lamp. Hence, the supply wires may be small, and therefore their cost be kept low. For any large amount of power, however, a relatively high voltage is required. On account of the danger of high voltages, the series system is not applicable to interior lighting.

55. Voltage Requirements for Parallel Distribution.—In the parallel system of distribution each translating device which is connected to the mains is substantially independent of every other load on the system, unless the loss in voltage in the mains, due to their resistance, is excessive. A single lamp, group of lamps, or a motor, may be connected to the mains, or disconnected therefrom, without interfering with any of the other loads on the system, except indirectly due to the change in the resistance drop in the mains. Another advantage of the parallel system of distribution is that the supply voltage may be kept low, differing from the voltage at the individual loads only by an amount equal to the resistance drop in the supply mains.

For interior lighting the supply voltage is usually in the neighborhood of 125 volts for 2-wire distribution, or 250 volts for 3-wire distribution (see Article 83). The supply voltage for direct-current motors in factories and mills is usually in the neighborhood of 115, 230 or 550 volts, although supply voltages as high as 750 volts are sometimes used in large mills, such as steel rolling mills. The supply voltage for ordinary city trolley cars is usually 600 volts. For interurban systems a supply voltage of 1500 or even 3000 volts, is sometimes employed. Direct-current dynamos may be built to operate at terminal voltages of 5000 volts or even higher, but, on account of the refinements necessary to secure satisfactory commutation at these high voltages, their cost becomes excessive.

As already noted, to secure the maximum of light from an incandescent lamp consistent with a reasonable life, it is necessary that the current through the lamp be maintained substantially constant. In order to keep the current constant through each lamp it is necessary, in a parallel system of distribution, to keep the voltage between the mains constant, irrespective of the total load being supplied by these mains. To do this it is necessary that the generator or generators supplying the system be so designed that their terminal voltage remain practically constant, or preferably increase slightly, as the load on the system increases, in order to compensate for the resistance drop in the supply mains.

A substantially constant voltage is necessary not only for the operation of electric lamps, but also for the satisfactory operation of motors. A decrease in the voltage across the terminals of a motor always produces a decrease in its speed. It is obvious, therefore, that if the driven machine is to be operated at a constant speed, the impressed voltage must be kept constant. A decrease in voltage, if sufficiently large, may also cause overheating of the motor and sparking at the commutator under heavy loads. (This is due to the fact that for a given power output of the motor, the current taken by it is inversely proportional to the impressed voltage.)

Since the voltage supplied to a parallel system is usually constant, or at least approximately constant, this system of distribution is frequently referred to as **constant-voltage distribution.** It should be kept in mind, however, that the term "constant voltage" as here used does not necessarily imply a strictly constant voltage, but only that the voltage is approximately constant.

A generator for supplying a parallel system on which the load fluctuates must be so designed that it gives a terminal voltage which is substantially constant irrespective of the load, or preferably a terminal voltage which increases with the load by an amount equal to the increase in the resistance drop in the line wires, or mains. A simple shunt generator is not suitable for this purpose, since, due to the resistance drop in its armature, and also to armature reaction, its terminal voltage falls off as the load increases. (The compound generator is therefore usually employed for this purpose, and may be built to give either an approximately constant terminal voltage or a terminal voltage

which increases with the load, depending upon the number of turns in the series-field winding.)

By providing an automatic regulator to control the shunt-field current, a shunt generator may also be made to give a substantially constant terminal voltage, or even a terminal voltage which increases automatically as the load increases. When so operated a shunt generator is well adapted to supply a fluctuating load of either lamps or motors. The most satisfactory form of automatic voltage regulator is the Tirrill regulator, described in detail in Article 67.

The chief application of the simple-shunt generator, without a voltage regulator, is to supply a load which remains constant, or which varies so slowly that the terminal voltage may be controlled manually by operating the shunt-field rheostat, as, for example, in charging storage batteries and in other electrolytic processes.

56. Percent Voltage Regulation.—From the foregoing discussion it is obvious that, in selecting a generator for a given service, the degree to which its terminal voltage varies with the load supplied must be given careful consideration. In comparing in this respect the performance of two or more generators it is frequently convenient to express the voltage regulation of each machine as the *percentage* change in its terminal voltage for a change in load from no-load to some specified load (usually full load). This percentage change is called the "percent voltage regulation" at the specified load. More specifically, the percent voltage regulation of a generator at any given load is defined as one hundred times the difference between its no-load terminal voltage* and its terminal voltage at the given load, divided by the terminal voltage at this given load, the speed and the setting of the shunt-field rheostat and series-field shunt remaining unaltered. That is, calling V the terminal voltage at a given load, speed, and the setting of the field rheostat and series shunt, and V_0 the corresponding terminal voltage at no load, then

$$\text{Per cent voltage regulation} = 100\frac{V_0 - V}{V} \qquad (1)$$

* By the no-load terminal voltage is meant the terminal voltage when the machine is supplying no load-current (i.e., no current other than its shunt-field current).

The reader should note that the greater the percent voltage regulation, the greater is the variation of the terminal voltage for a given fluctuation in the load. One naturally uses the term "poor regulation" to signify lack of constancy. Hence, a machine which has *poor* voltage regulation is one which has a *high* percent voltage regulation. One must be careful not to be confused by this double meaning of the word regulation.

In a shunt generator the effect of the load current is in general to decrease the flux per pole, as pointed out in the preceding chapter on armature reaction. This decrease in flux causes the voltage generated in the armature of a shunt machine to decrease as the load increases. The terminal voltage likewise decreases, and, to a greater extent, due to resistance drop in the armature. The current in the shunt-field winding therefore also diminishes as the load increases, since this winding is connected directly across the terminals of the machine. This produces a further reduction in the magnetic field and in the generated voltage. A condition of equilibrium is reached when the generated electromotive force, less the armature resistance drop, is equal to the resistance drop in the shunt-field winding (see Article 63). The voltage regulation of a shunt machine is therefore always positive.

In a compound generator, which has both a shunt and a series field winding, it is possible, by the proper choice of turns in the series field, to make the ampere-turns of this winding compensate for armature resistance and armature reaction and give an approximately constant terminal voltage, or even to over-compensate and give a terminal voltage which increases with the load. The voltage regulation of a compound generator may therefore be either positive, zero or negative.

Since in a series generator the magnetic field is produced by the load current, the flux per pole, and therefore the generated electromotive force, increase as the load on the machine increases; see equation (1), Article 32. The terminal voltage of such a machine likewise increases with increase of load, provided the load current does not become so large that the flux per pole increases less rapidly than the resistance drop in the armature and field windings, i.e., provided the magnetic circuit does not become saturated. The voltage regulation of a series generator with unsaturated magnetic circuit is therefore negative. Practically, the percentage voltage regulation of a series generator is of no

significance, although a curve showing the actual value of the terminal voltage for various values of the load current is of considerable interest (see Article 62).

57. Factors upon which Voltage Regulation Depends.—The terminal voltage of a given generator depends upon a number of factors, each of which must be taken into account in order to get a clear idea of the performance of the machine. These factors are:

1. The speed at which the armature is driven.

2. The line current. In the following discussion this current will be represented by the symbol I.

3. The resistance of the armature winding, the brush-contact resistance and the resistance of the brushes. The equivalent * resistance of these several elements is usually referred to as the armature resistance, and will be represented by the symbol R_a.

4. The resistance of the series field winding (if any) and the resistance of its shunt. The equivalent * resistance of this winding and shunt in parallel will be represented by the symbol R_s.

5. The resistances of the commutating-pole winding (if any) and the compensating winding (if any). The equivalent resistance * of these two windings and their shunts (if any) will be represented by the symbol R_c.

6. The resistance of the shunt-field winding (if any) and the resistance of the shunt-field rheostat. The equivalent resistance of this winding and rheostat in series will be represented by the symbol R_f.

7. The reluctance of the magnetic circuit of the machine.

8. The demagnetizing effect of the armature current (see Chapter V).

Without going into a detailed analysis, certain relations are obvious. For example, since the electromotive force developed in the armature of a generator is proportional to its speed, the terminal voltage of the machine will likewise increase with increase of speed.

The shunt-field resistance affects the terminal voltage of the machine in that the value of the shunt-field current depends

* By the equivalent resistance of two or more resistances in series or parallel is meant a single resistance equivalent, with respect to voltage drop and power loss, to these several resistances; see Article 8.

upon the value of this resistance. The various resistances in the armature circuit affect the terminal voltage in that they give rise to an internal voltage drop in the machine, which drop is proportional to the current supplied by the armature.

The reluctance of the magnetic circuit affects the terminal voltage in that it is one of the factors which determine the useful flux per pole, and therefore the generated electromotive force; see Chapter III. The reluctance is not a constant, but depends upon the flux per pole. The variation in reluctance, however, may be taken into account by making use of the saturation curve of the machine, as will be explained in the following Articles.

The effect of armature reaction is in general both to reduce and to distort the useful flux per pole, and thereby to decrease the generated electromotive force, with a consequent reduction in the terminal voltage. The reduction and distortion of the flux has already been considered in detail in the preceding chapter. The effect on the terminal voltage produced by this reduction and distortion of flux may be taken into account in a relatively simple manner, as explained in the following Article.

The variation of the terminal voltage of a generator produced by these various factors can usually best be shown by means of a curve, the ordinates of which give the terminal voltage and the abscissas the line current (i.e., the current supplied to load). Such a curve is called the **voltage-regulation curve,** or the **external characteristic,** of the machine.

In the following Articles will be shown how the voltage-regulation curve of each type of machine may be derived from its saturation curve and the resistances of its armature and field windings. This analysis will also bring out clearly the particular effect produced by each of the several factors enumerated at the beginning of this article.

58. Armature Demagnetizing Factor and Effective Field Current.—As shown in Chapter V, the current in the armature winding of a generator always tends to reduce the magnetic field in which the armature rotates.

Were this demagnetizing effect due solely to the demagnetizing armature turns (see Article 48), it would be the same as would be produced were there no armature reaction, but instead a reduction

in the field current by an amount proportional to the armature current. That is, for a given field current I_f and given armature current I_a, the reduction in the flux per pole due to armature reaction would be the same as would be produced were there no armature reaction, but the field current reduced from its actual value I_f to a new value $(I_f - AI_a)$, where A is a constant.

In the general case of armature reaction due both to the demagnetizing armature turns and the demagnetizing effect of the cross turns, the reduction in the flux per pole may likewise be considered as equivalent to a reduction in the actual field current by an amount AI_a, where A is a factor, not necessarily a constant, which takes into account the reduction in the flux per pole due to both the demagnetizing and cross turns. This factor A will be called the **armature demagnetizing factor,** and the current $(I_f - AI_a)$ will be called the **effective field current.**

When the flux density in the magnetic circuit is well below the saturation value, the demagnetizing factor A is simply the ratio of the effective number of demagnetizing turns to the number of turns in the field winding. By the *effective* number of demagnetizing turns is meant the actual number of these turns divided by the number of parallel paths through the armature winding. This takes into account the difference between the current in the armature and the total arma ure current I_a (see Problem 4 at end of Chapter V).

For flux densities approaching the saturation value, the armature demagnetizing factor is greater than this ratio and increases as the armature current increases.

In the following discussion the armature demagnetizing factor will, as a first approximation, be assumed constant, but it should be kept in mind, that the value of this factor will, as a rule, increase as the armature current increases, the amount by which it increases depending upon the degree of saturation of the pole tips.

59. Experimental Determination of the Armature Demagnetizing Factor.—When the dimensions of the magnetic circuit, number of field turns, number of armature turns, brush lead and degree of saturation are known, it is possible to calculate the value of the armature demagnetizing factor A. For a completed machine,

however, it is much simpler to determine the value of this factor experimentally.

Referring to Fig. 71, let $QCBM$ be the descending saturation curve, determined as explained in Article 35. For any given value $\overline{OF}$ of the field current I_f the electromotive force generated in the armature when there is no armature current (and therefore no armature reaction and no armature-resistance drop) is equal to the corresponding ordinate $\overline{FB}$.

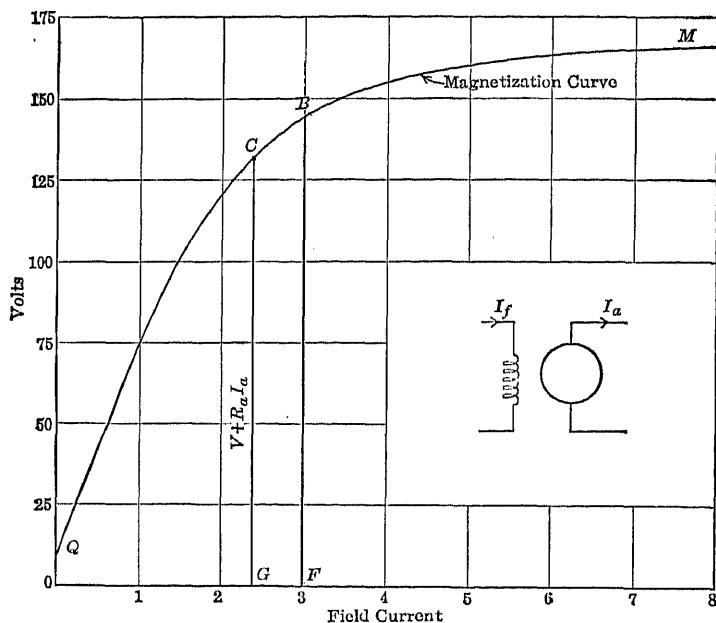



FIG. 71.—Construction for Determining Armature Demagnetizing Factor.

When the armature is supplying a current I_a, the electromotive force generated in it by its motion through the *resultant* magnetic field produced by the field current and armature current is the same as would be produced on open circuit by an *effective* field current $\overline{OG}$, where $\overline{GF}$ represents the demagnetizing action of the armature current.

To find the value of the *effective* field current $\overline{OG}$ corresponding to a given value $I_f = \overline{OF}$ of the *actual* field current, and given value I_a of the armature current, the procedure is as follows:

With the armature at rest measure the armature resistance R_a,

by sending a current through the armature from some external source and measuring the voltage drop from the positive to the negative brushes. The resistance R_a is then equal to this voltage drop divided by the current.

Next operate the machine as a separately excited generator at constant speed and determine the descending saturation curve *QCBM*. Then, with the field current held constant at the desired value I_f, connect a load (e.g., a rheostat) to the brushes and adjust this load to take the desired armature current I_a, and note the voltage V across the brushes. To this terminal voltage V add the armature-resistance drop R_aI_a. This gives the generated electromotive force E, viz.,

$$E = V + R_aI_a \tag{2}$$

Locate the point G on the axis of abscissas so that the corresponding ordinate $\overline{GC}$ of the magnetization curve is equal to this electromotive force E. The length of the line $\overline{GF}$, to the same scale as the field current, is then equal to AI_a, and therefore the armature demagnetizing factor, corresponding to the given field current I_f and armature current I_a, is

$$A = \frac{\overline{FG}}{I_a} \tag{3}$$

For example, the machine illustrated in Fig. 71 has an armature resistance of 0.07 ohm and, when separately excited at a field current of 3 amperes, gives a terminal voltage of 125 volts when the armature current is 100 amperes. The armature-resistance drop is then $0.07 \times 100 = 7.0$ volts. Therefore $\overline{GC} = 125 + 7 = 132$ volts, and since 132 volts on the magnetization curve corresponds to a field current of 2.4 amperes, $\overline{FG} = 0.6$ ampere. Hence, the armature demagnetizing factor is

$$\frac{0.6}{100} = 0.006$$

The variation of the armature-demagnetizing factor with field excitation and load may readily be determined by proceeding in the manner just described for successive values of I_f and I_a.

60. Voltage-regulation of Separate-excited Generator.—The relations developed in the preceding article lead to a simple graphical construction of the voltage-regulation curve, or external characteristic, of a generator. This construction, which will now be explained in detail for the various types of connections (sep-

arately excited, series, shunt and compound), is particularly useful in bringing out the effect, on the operating characteristics of the machine, of the various factors listed in Article 55.

Consider first a separately excited generator, and let the curve $QCBM$ in Fig. 71 represent the saturation curve of this machine corresponding to a given speed of the armature. Let R_a be the armature resistance, and let A be the armature demagnetizing factor determined as explained in the preceding article. For any given value I_f of the field current and any given value I of the line current (which for a separately excited generator is the same as the armature current I_a), calculate the effective field current

$$I_f' = I_f - AI \tag{4}$$

The ordinate of the saturation curve corresponding to this effective field current is then the generated electromotive force E, and the terminal voltage is, therefore,

$$V = E - R_a I \tag{5}$$

Consider, for example, the same machine as in the preceding Article, and let the field rheostat be set to give a field current of 4 amperes. For a line current of 250 amperes the effective field current is then $I_f = 4 - 0.006 \times 250 = 2.5$ amperes. From Fig. 71 the corresponding generated electromotive force is then 134 volts, and the corresponding terminal voltage is $134 - 0.07 \times 250 = 116$ volts.

The full-line curve in Fig. 72 is the voltage-regulation curve for this same generator for a constant field current of 3 amperes and for line currents from 0 to 400 amperes. The shape of this curve is typical of all separately excited generators, i.e., as the line current increases the terminal voltage decreases, the maximum line current occurring when the brushes are short-circuited (zero terminal voltage).

The method just described for deriving the voltage-regulation curve is approximate only, since it assumes (1) that the armature demagnetizing factor is constant and (2) that the resistance of the armature circuit, including the brush-contact resistance, is constant. Actually, the armature demagnetizing factor increases with increase in the degree of saturation of the magnetic circuit and with increase of load (as explained in Article 58), with the result that,

if this factor is determined at low saturation and light load, the terminal voltage falls off as the load increases more rapidly than indicated by this method, particularly if, under operating conditions, the magnetic circuit is well saturated. Also, under actual conditions, the brush-contact resistance is not constant, but decreases as the armature current increases (see Article 40).

However, although the graphical analysis given in this and the subsequent articles cannot in general be used for exact calculations,

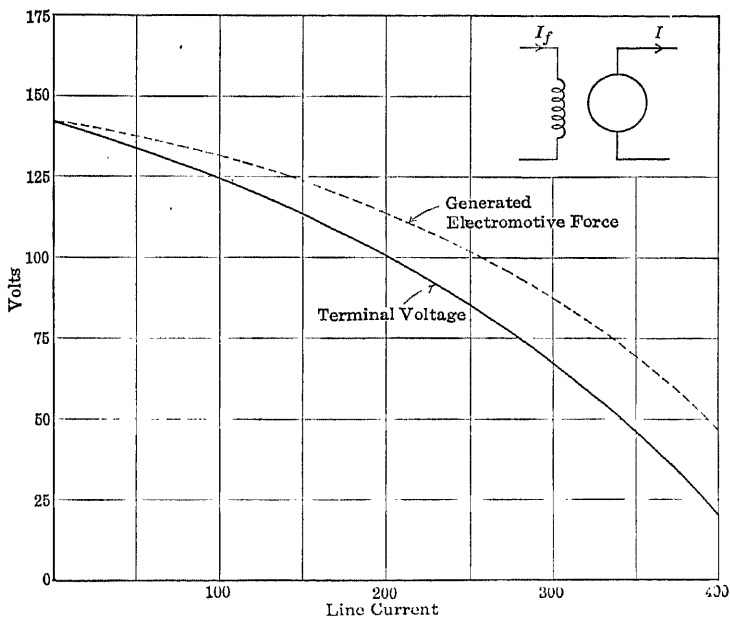



FIG. 72.—Voltage Regulation of Separately Excited Generator.

it does show very clearly the general effect of saturation and of armature reaction, and how the voltage regulation of a machine depends upon the resistances of the armature and field windings.

61. Armature Characteristic or Field-compounding Curve.—A characteristic curve which gives useful data relative to the design of the field rheostat and of the series winding (for a compound machine) is the so-called **armature characteristic** or **field-compounding curve.** This curve is one showing the relation between field current (ordinates) and armature current (abscissas), when the field current is adjusted to give rated terminal voltage for each

value of the armature current, the field being separately excited. The difference in the field current for any given load and that required at no load is proportional to the field ampere-turns required to overcome the armature-resistance drop and armature demagnetizing action.

The armature characteristic may be determined from the saturation curve as follows: Referring to Fig. 73 let $QDBM$ be the saturation curve. Let $\overline{HD}$ be the value at which the terminal

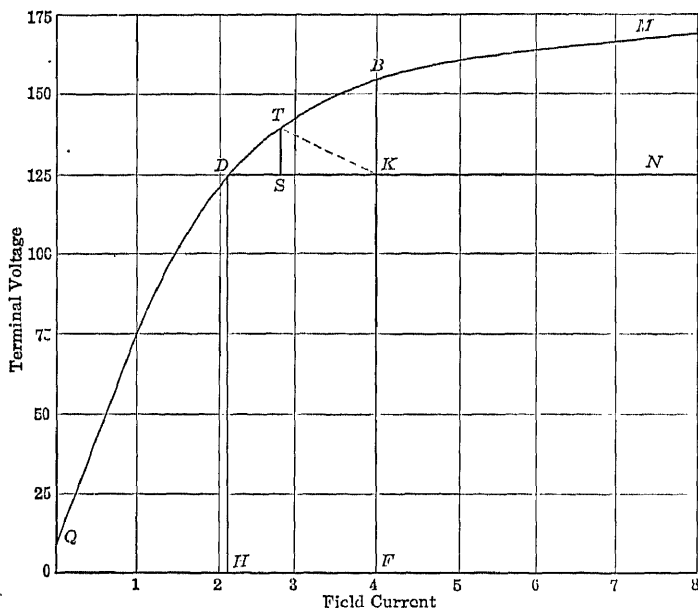



FIG. 73.—Construction for Armature Characteristic.

voltage is to be held constant, and draw the horizontal line $\overline{DN}$. Let I_a be any given value of the armature current. Locate the vertical line whose intercept $\overline{TS}$ between the magnetization curve and $\overline{DN}$ is equal to the armature-resistance drop R_aI_a. Lay off $\overline{SK}$ equal to the product AI_a, namely, the product of the armature-demagnetizing factor by the armature current. The corresponding abscissa $\overline{OF}$ is then the field current required to maintain the terminal voltage at the value $\overline{HD}$ when the armature current has the given value I_a.

For example, for the same machine as in Article 57, the field current required for a terminal voltage of 125 volts at no-load is 2.15 amperes. When the armature current is 200 amperes, the armature-resistance drop is $0.07 \times 200 = 14$ volts and the armature-demagnetizing effect is $0.006 \times 200 = 1.2$ amperes. Hence, laying off $\overline{TS} = 14$ and $\overline{SK} = 1.2$, it is seen that a field current of 4 amperes is required to maintain this same terminal voltage (125 volts) when the armature current is 200 amperes. The complete armature characteristic determined in this manner is shown in Fig. 74.

62. Voltage Regulation of Series Generator.—In a series generator the line current is also the field current and the armature

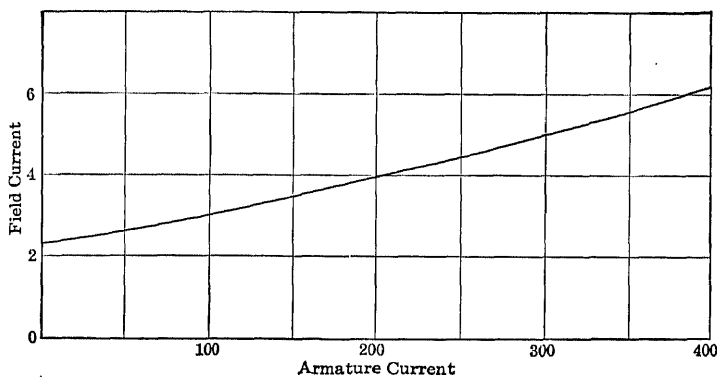



Fig. 74.—Armature Characteristic, or Field-compounding Curve.

current. Referring to Fig. 75, the upper curve represents the saturation curve of such a machine. For any given value $I = \overline{OF}$ of the line current the effective field current is $\overline{OG} = \overline{OF} - AI$, where A is the armature-demagnetizing factor. The armature-demagnetizing factor A may be determined in the manner explained in Article 59, operating the machine as a separately excited generator. The electromotive force generated in the armature when the line current is I is then $\overline{GC}$, where $\overline{GC}$ is the ordinate of the magnetization curve corresponding to the effective field current $\overline{OG}$. Draw the horizontal line $\overline{CD}$. Then $\overline{FD} = \overline{GC}$ is the generated electromotive force corresponding to the line current $\overline{OF}$ (which is also the actual field current).

Proceeding in this way for successive values of the line current, it is seen that the generated electromotive force will vary with

the line current as shown by the dotted curve in Fig. 75. Note that the horizontal distance between the saturation curve and this electromotive-force curve, for any point on the latter curve, is equal to the demagnetizing effect AI of the armature current which corresponds to this point.

The terminal voltage V corresponding to any value I of the line current is equal to the generated voltage E corresponding to this current, less the resistance drop (R_a+R_s) in the field and armature winding, viz.,

$$V=E-(R_a+R_s)I \tag{6}$$

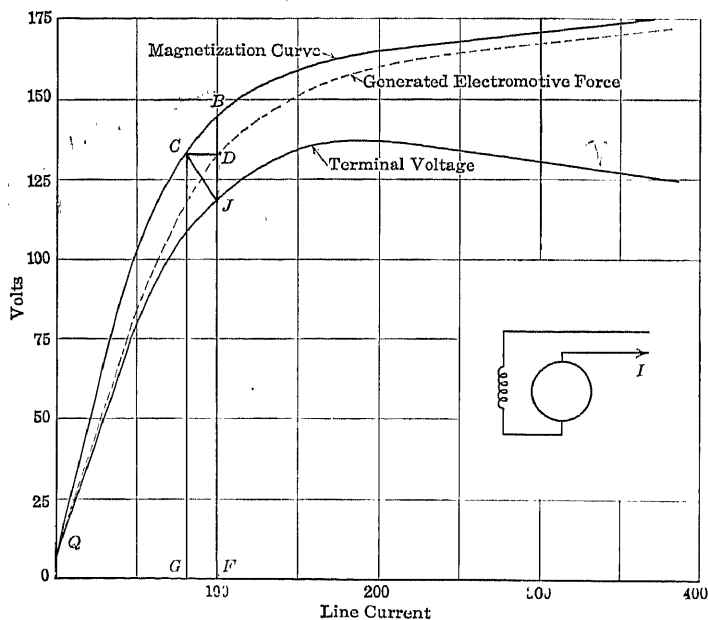



FIG. 75.—Voltage Regulation of Series Generator.

As just explained, corresponding to the line current $\overline{OF}$ in Fig. 75, the generated voltage is $\overline{FD}$. Hence, if $\overline{DJ}$ is laid off equal to the combined resistance drop $(R_a+R_s)I$ in the field and armature windings, then $\overline{FJ}$ is the corresponding terminal voltage.

Proceeding in this way, it is seen that the complete voltage-regulation curve will be as shown by the lower curve in Fig. 75, the vertical distance between this curve and the electromotive-force curve at any point being equal to the total internal resistance

drop $(R_a+R_s)I$ corresponding to the abscissa of this point. The particular voltage-regulation curve shown in Fig. 75 is for an armature resistance of 0.07 ohm and a series field resistance of 0.05 ohm, and an armature demagnetizing factor of 0.2.

From Fig. 75 it is evident that as the load on a series generator increases, the terminal voltage at first increases very nearly proportionally with the current, but as the knee of the magnetization curve is approached, the terminal voltage increases more and more slowly and ultimately reaches a maximum value. Beyond this point, any further increase in the current produces a decrease in the terminal voltage, this decrease being due primarily to the increase in the internal resistance drop. As in the case of a separately excited generator, the maximum current occurs when the terminals of the machine are short-circuited (zero terminal voltage).

Both the terminal voltage and the current which a series generator will supply depend upon the resistance and back electromotive force (if any) of the load connected to its terminals, i.e., upon the volt-ampere characteristic of the load. Since the terminals of the load are also the terminals of the generator, the terminal voltage of the generator must be the same as the terminal voltage of the load.

When the load is a resistance R with no back electromotive force, the terminal voltage of the load for any value I of the current is $V=RI$, where I is the line current. The volt-ampere characteristic of such a load is therefore a straight line through the origin (see Fig. 76), which makes with the horizontal axis an angle whose tangent, when current and voltage are plotted to the same scale,* is R. The ordinate of the point P at which this straight line intersects the voltage-regulation curve is then the terminal voltage at which the generator will operate when the resistance R is connected across its terminals, and the abscissa of this point is the current which will be supplied by the generator to this resistance.

For example, for the generator represented by Fig. 76, a resistance of 1 ohm connected across its terminals will take a current of 130 amperes, and the terminal voltage will be 130 volts.

* When the voltage and current are not to the same scale this line is the straight line through the origin and the point whose abscissa is 1 on the current scale and whose ordinate is R on the voltage scale.

63. Voltage Regulation of Shunt Generator.—In a shunt generator the line current I is less than the armature current I_a by an amount equal to the field current I_f. Referring to Fig. 77, let the curve QCM represent the saturation curve of the machine. When the shunt field is connected across the brushes for normal operation, and there is no current supplied to the line, the value to which the terminal voltage will build up is determined by the total resistance in the shunt-field circuit. This no-load terminal

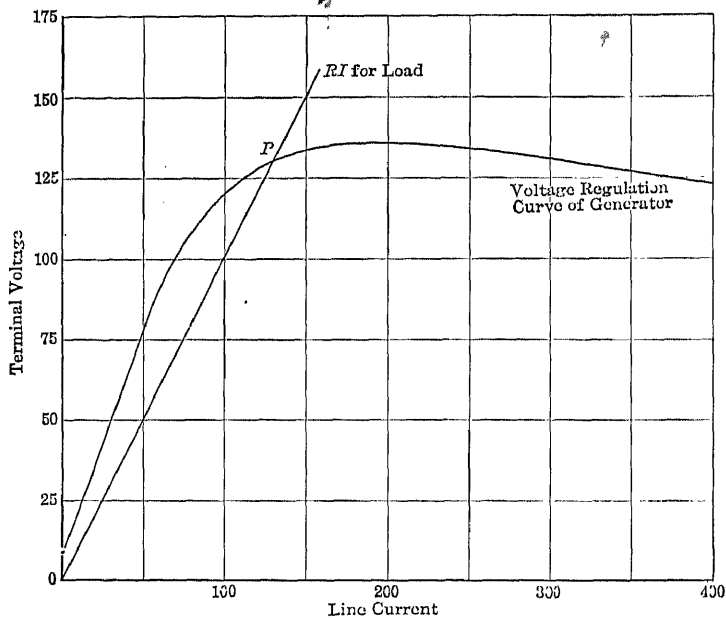



Fig. 76.—Building-up of Series Generator.

voltage and the corresponding shunt-field current are readily determined as follows:

Neglecting the armature-resistance drop and armature reaction due to the shunt-field current, which current is always small, the no-load terminal voltage for any value of the field current I_f must be equal to the corresponding ordinate of the saturation curve M, irrespective of the value of the shunt-field resistance. Calling R_f the total resistance of the shunt-field circuit, the terminal voltage must also be equal to R_fI_f, irrespective of the shape of the magnetization curve, for the terminals of the machine

are also the terminals of the shunt-field. Referring to Fig. 77, draw the straight line $\overline{ON}$ whose ordinate for any value I_f is equal to R_fI_f. The no-load terminal voltage for this value of R_f must then be equal to the ordinate which is common to the curve QCM and the line $\overline{ON}$, that is, to the ordinate of the point P_0 at which this line intersects the curve M. The corresponding abscissa $\overline{OF_0}$ is then the value of the shunt-field current at no load, corresponding to the given shunt-field resistance R_f.

The line $\overline{ON}$ whose ordinates are equal to the product of the shunt-field resistance by the shunt-field current is called the **shunt-field resistance line.** The slope of this line is proportional to the shunt-field resistance R_f; the greater this resistance R_f the steeper the line $\overline{ON}$. From Fig. 77, it is evident that if the shunt-field resistance is made so large that the resistance line $\overline{ON_1}$ cuts the magnetization curve at a point near the origin, such as P_1, the terminal voltage will build up to only the relative small value $\overline{F_1P_1}$. From Fig. 77, it is also evident that by varying the resistance of the shunt-field circuit the no-load terminal voltage may be adjusted to any desired value up to that corresponding to the resistance of the field winding only (field rheostat short-circuited).

Since the ordinates of the saturation curve are proportional to the speed (at least approximately), it is evident from Fig. 77 that a variation in speed will cause a variation in the shunt-field current. The higher the resistance of the shunt-field circuit, the greater will be the change in this current for a given change in speed, and therefore the greater will be the resultant change in the terminal voltage. Consequently, if the resistance of the shunt-field circuit is made too high, difficulty may be found in keeping the speed sufficiently near constancy to prevent relatively large variations in the terminal voltages; i.e., the terminal voltage may become decidedly unstable.

For a given setting of the shunt-field rheostat the terminal voltage will decrease as the load on the machine (i.e., the line current) increases, due (1) to the armature-resistance drop R_aI_a and (2) to armature reaction, and (3) to the decrease in the field current resulting from the decrease in the terminal voltage. For a given value I of the line current, the field current will therefore fall to some value $\overline{OF}$ (see Fig. 77) and the terminal voltage to a

corresponding value $\overline{FJ}$. Note that ordinate $\overline{FJ}$, which represents this new terminal voltage, must terminate at the resistance line $\overline{ON}$, since this terminal voltage is also equal to the voltage R_fI_f across the shunt field.

The armature current I_a corresponding to this reduced value $V=\overline{FJ}$ of the terminal voltage may be found as follows: Locate the point D vertically above J so that the ratio of length of the vertical line $\overline{DJ}$ to the length of the horizontal line $\overline{CD}$ (C being on the saturation curve) is equal to the ratio of the armature

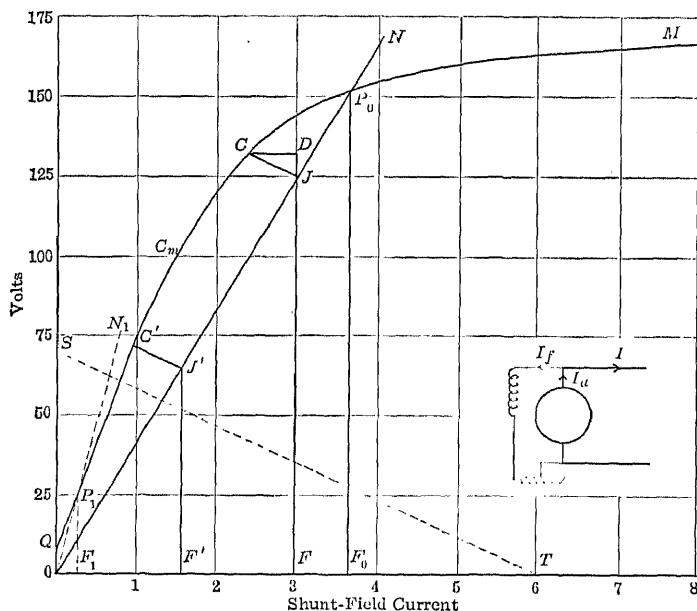



FIG. 77.—Construction for Voltage Regulation of Shunt Generator.

resistance R_a to the armature-demagnetizing factor A. Then the armature current I_a corresponding to the terminal voltage $V=\overline{FJ}$ is

$$I_a=\frac{\overline{CD}}{A} \tag{7}$$

or

$$I_a=\frac{\overline{DJ}}{R_a} \tag{7a}$$

That this relation is true follows from the fact that when I_a is determined in this manner, $\overline{DJ}$ is equal to the armature-resistance

drop R_aI_a, and therefore $\overline{FD}$ is equal to the generated electromotive force. This generated electromotive force in turn satisfies the condition that it is equal to the ordinate of the saturation curve corresponding to the effective field current, which current is equal to the actual field current $\overline{OF}$ less the demagnetizing effect $\overline{CD}=AI_a$ of the armature current.

In applying equation (6) or (6a), the length $\overline{CD}$ is to be expressed to the same scale as the field current (abscissas) and the length $\overline{DJ}$ to the same scale as the terminal voltage (ordinates).

On the assumption of constant armature resistance and constant armature-demagnetizing factor, both $\overline{DJ}$ and $\overline{CD}$ are proportional to the armature current. Hence, the slope of the line $\overline{CJ}$ is constant, irrespective of the value of the armature current, and its length also is proportional to the armature current. When the field current is plotted to a scale of i amperes per division of cross-sectioning, and the ordinate of the saturation curve to a scale of v volts per equal division of cross-sectioning, then the length of the line $\overline{CJ}$ is

$$\overline{CJ}=kI_a \qquad \text{divisions} \tag{8}$$

$$k=\sqrt{\left(\frac{R_a}{v}\right)^2+\left(\frac{A}{i}\right)^2} \tag{8a}$$

The terminal voltage corresponding to any value of the armature current may therefore be determined as follows: Lay off, along the Y-axis, to the same scale as the ordinate of the saturation curve, a distance $\overline{OS}$ equal to the armature resistance R_a times any arbitrarily chosen value of the armature current, and along the X-axis lay off, to the same scale as the field current, a distance $\overline{OT}$ equal to the armature-demagnetizing factor A times this same armature current. Draw the line $\overline{ST}$. Calculate the constant k from equation (8a).

Then for any value I_a of the armature current find the point J on the field-resistance line $\overline{ON}$ which is at the distance kI_a from the saturation curve, *measured parallel to the line* $\overline{ST}$. Then $\overline{FJ}$ is the terminal voltage corresponding to the armature current I_a, and $\overline{OF}$ is the corresponding field current. The line current corresponding to the armature current I_a is then

$$I=I_a-\overline{OF} \tag{9}$$

As a numerical example, consider the machine whose magnetization curve is shown in Fig. 77. This machine has an armature resistance of 0.07 ohm and an armature-demagnetizing factor of 0.006. The resistance of the shunt-field circuit (including the resistance of the shunt-field rheostat) is 41.7 ohms.

Locate the point which has an abcissa of, say, 4 amperes and an ordinate of $4 \times 41.7 = 167$ volts. (A field current of 4 amperes is here arbitrarily chosen, simply for convenience in plotting.) Through this point and the origin draw the straight line $\overline{ON}$. This is the field-resistance line, and its intersection with the saturation curve gives for the no-load terminal voltage 152 volts and for the no-load field current 3.65 amperes.

Lay off $\overline{OS} = 0.07 \times 1000 = 70$ volts and $\overline{OT} = 0.006 \times 1000 = 6$ amperes, and draw the line $\overline{ST}$. (A current of 1000 is here arbitrarily chosen, simply for convenience in plotting.) In Fig. 77 the field current is plotted to a scale of 1 ampere = 1 division of cross-sectioning, and the ordinates of the magnetization curve to a scale of 25 volts = 1 division of cross-sectioning. The factor k therefore has the value

$$k = \sqrt{\left(\frac{0.07}{25}\right)^2 + \left(\frac{0.006}{1}\right)^2} = 0.00662$$

To determine the terminal voltage corresponding to an armature current of, say, 100 amperes, calculate

$$\overline{JC} = 0.00662 \times 100 = 0.662$$

Make a straight-edged scale of a piece of the same kind of cross-section paper as that used for plotting the saturation curve. Keeping the edge of this scale parallel to the line $\overline{ST}$, find the position for which the distance, measured along the straight edge, from the magnetization curve to the field-resistance line, is 0.662. The ordinate corresponding to the intersection of this straight edge with the field-resistance line is the terminal voltage corresponding to the armature current of 100 amperes.

From Fig. 77 it is evident that there are two such positions of the straight-edge, namely, $\overline{CJ}$ or $\overline{C'J'}$, corresponding to two pair of values of the terminal voltage and field current, viz., 125 volts and 3 amperes, and 65 volts and 1.6 amperes.

The corresponding line currents are $100 - 3 = 97$ amperes and

$100-1.6=98.4$ amperes respectively. These two line currents correspond respectively to loads having equivalent resistances * of $\frac{125}{97}=1.29$ ohms and $\frac{65}{98.43}=0.92$ ohm.

Proceeding in this manner for successive values of the armature current, the complete voltage-regulation curve shown in Fig. 78 is obtained. This curve is typical, in its general shape, of the voltage-regulation curve of all shunt generators.

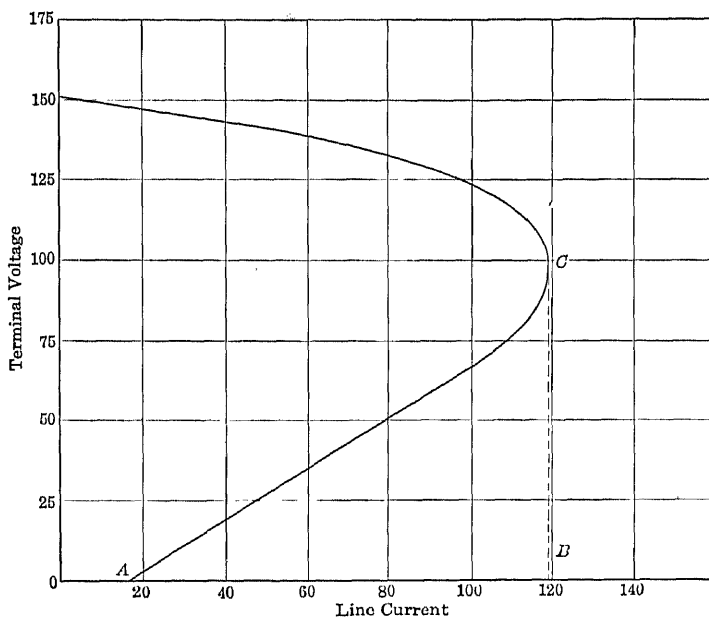



Fig. 78.—Voltage Regulation of Shunt Generator.

At no-load (i.e., no-line current) the terminal voltage is a maximum. No-load corresponds to infinite resistance connected across the terminals of the machine. As the resistance of the load * is decreased, the line current increases and the terminal voltage decreases. The terminal voltage at first falls off slowly, and then more and more rapidly as the line current increases.

When the load resistance is still further decreased, a critical

* By resistance of the load is here meant the equivalent resistance of the external circuit connected to the terminals of the machine, i.e., a resistance equal to the voltage impressed on this circuit divided by the total line current.

value is reached for which the line current has its maximum value, i.e., the value $\overline{OB}$ in Fig. 78. For any further decrease in the load resistance, the line current then *decreases*, and the voltage likewise decreases. When the terminals of the machine are short-circuited, the terminal voltage is of course zero, and therefore the field current is likewise zero, but there will still be some current, i.e., the current $\overline{OA}$ in Fig. 78, supplied by the armature to the load. This short-circuit current is due solely to the residual magnetism in the magnetic circuit of the machine.

From the manner in which the voltage regulation of the shunt generator is derived from its saturation curve it is evident that the maximum current $\overline{OB}$ in Fig. 78 occurs when the point C in Fig. 77 coincides with the point C_m, at which the tangent to the saturation curve is parallel to the resistance line $\overline{ON}$. The position of this point, and therefore the value of the terminal voltage at which maximum current occurs, depends upon the slope of the resistance line $\overline{ON}$, that is, upon the resistance of the shunt-field circuit.

The ratio of the maximum current which a shunt generator can supply to the rated current of the machine, depends not only upon the resistance in the shunt-field circuit, but also upon the shape of the saturation curve, the armature resistance and armature reaction. In the case of a large shunt generator with low armature reaction, it is usually impracticable to take from it, even under test conditions, the maximum current which it is theoretically capable of supplying, on account of the overheating of the armature which this current would produce.

The voltage-regulation curve shown in Fig. 78 corresponds to steady (unvarying) values of the field and armature currents. If a shunt generator operating at normal voltage and current is suddenly short-circuited, the current in the field circuit does not immediately fall to the low value $\overline{OA}$ in Fig. 78, but, on account of the relatively high self-induction of the field circuit, may take several seconds to fall to this value. During this interval the flux per pole, and therefore the generated voltage, will remain relatively high (i.e., the generator will operate more nearly like a self-excited machine), and therefore a relatively large current will flow in the armature during this interval. Consequently, in spite of the fact that the *steady* value of the short-circuit current of a shunt generator may be less than its full-load value, the sudden

short-circuit of such a machine may cause serious overheating and even burn out the insulation of the armature winding.

64. Voltage Regulation of Compound Generator.—Consider first a long-shunt compound generator. Under normal operation (shunt-field connected as indicated in Fig. 80) the effect of the series field is (1) to produce a magnetomotive force proportional to the armature current I_a and in the same direction as that due to the shunt-field current I_f and (2) to increase the internal resistance drop between brushes from R_aI_a to $(R_a+R_s)I_a$, where R_a and R_s are the resistance of the armature and series fields respectively.

The ampere-turns of the series field produce exactly the same effect as would be produced were there no series field but the shunt-field current increased from its actual value I_f to a value $I_f+A_sI_a$, where A_s, which may be called the *series field magnetizing factor*, is equal to the ratio of the number of turns in the series-field winding to the number of turns in the shunt-field winding. Hence, in a compound generator the combined effect of the series field and armature reaction is the same as would be produced were there no series field and no armature reaction, but the shunt-field current increased to an "effective" value

$$I_f'=I_f+(A_s-A)I_a \tag{10}$$

where A is the armature-demagnetizing factor. Or, putting $A'=A-A_s$, the effective shunt-field current, taking into account both the series field and the armature reaction, is

$$I_f'=I_f+A'I_a \tag{10a}$$

The factor A' may be called the **resultant magnetizing factor** of the series field and armature.

The resultant magnetizing factor A' of a compound generator may be determined in exactly the same way as the armature-demagnetizing factor of a shunt generator (see Article 59). Referring to Fig. 79, let $QBCM$ be the ascending saturation curve of the compound generator, determined by driving the armature at rated speed, without load, and with the shunt field excited from a separate source. With the shunt field *separately excited*, note the value I_f of the shunt-field current and measure the terminal voltage V (including the drop in the series field) for any value I_a of the armature current. To this terminal voltage V add the combined resistance drop $(R_a+R_s)I_a$ in the armature and series field. Let

$\overline{OF}$ be the value of the actual field current I_f, and locate the point G so that the corresponding ordinate GC of the magnetization curve is equal to the sum $V+(R_a+R_s)I_a$. The length of the line $\overline{GF}$, to the same scale as the field current, is then equal to $A'I_a$, and therefore

$$A'=\frac{\overline{GF}}{I_a} \tag{11}$$

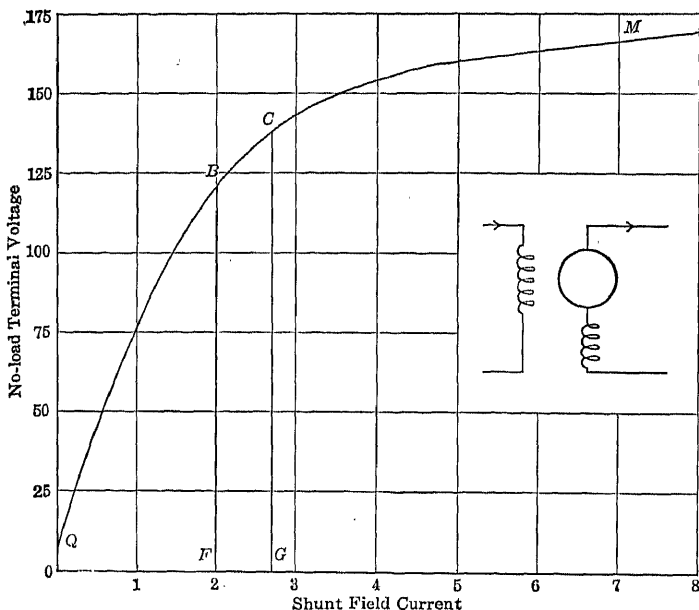



FIG. 79.—Construction for Determining Resultant Magnetizing Factor of Compound Generator.

For example, let the terminal voltage corresponding to an actual shunt-field current of 2 amperes and an armature current of 50 amperes be 132.5 volts. Let the armature resistance be 0.07 ohm and the series-field resistance be 0.03 ohm. Then $V+(R_a+R_s)I_a=132.5+3.5+1.5=137.5$ volts. The effective shunt-field current corresponding to 137.5 volts is 2.7 amperes. Hence, $\overline{FG}=2.7-2.0=0.7$, and, therefore, the resultant magnetizing factor is

$$A'=\frac{0.7}{50}=0.014$$

To find the terminal voltage of the machine when operating

normally as a compound generator (shunt field connected to the terminals of the machine itself), a method of procedure similar to that followed in the case of a shunt generator may be used. Referring to Fig. 80, lay off first the shunt-field resistance line $\overline{ON}$, corresponding to the combined resistance of the shunt field and the shunt-field rheostat. The ordinate of the intersection P_0 of this line with the saturation curve gives the no-load terminal voltage of the machine, and the corresponding abscissa $\overline{OF_0}$ gives the no-load shunt-field current.

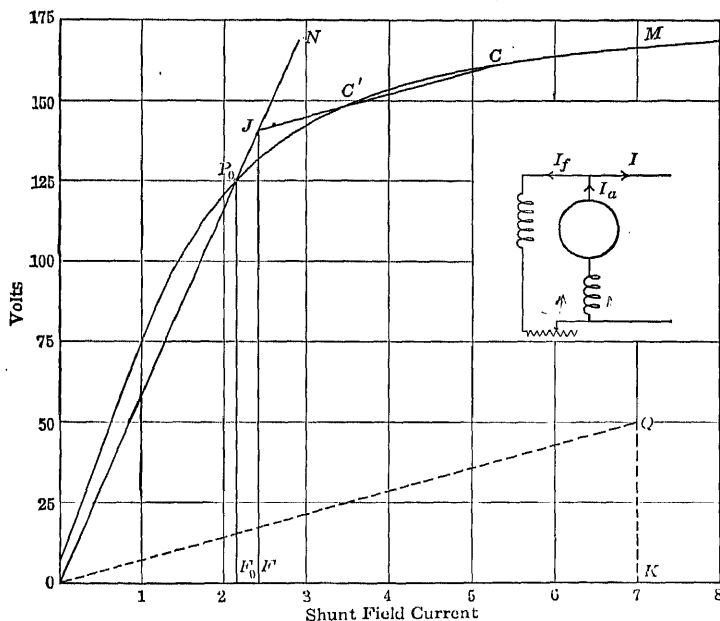



Fig. 80.—Construction for Voltage Regulation of Compound Generator.

Choose arbitrarily any value of the armature current and lay off $\overline{OK}$ equal to the resultant magnetizing factor A' times this current, and lay off $\overline{KQ}$ equal to the total armature and series-field resistance (R_a+R_s) times this same current. Draw the straight line $\overline{OQ}$. (This line $\overline{OQ}$ corresponds to the line $\overline{ST}$ in Fig. 77 for the shunt generator, but has a positive, instead of a negative, slope, since A' corresponds to a *magnetizing* action, whereas A for the shunt generator corresponds to a *demagnetizing* action.)

Next calculate the factor

$$k=\sqrt{\left(\frac{R_a+R_s}{v}\right)^2+\left(\frac{A'}{i}\right)^2} \qquad (11)$$

where v is the number of volts corresponding to one division on the scale of ordinates, and i is the number of amperes corresponding to an equal division on the scale of abscissas.

Then, for any given value I_a of the armature current, locate the point J on the shunt-field resistance line $\overline{ON}$ which is at the distance

$$\overline{JC}=kI_a \quad \text{divisions} \qquad (12)$$

from the saturation curve, *measured parallel to the line* $\overline{OQ}$. Then the ordinate $\overline{JF}$ is the terminal voltage corresponding to the armature current I_a, and the abscissa $\overline{OF}$ is the corresponding shunt-field current. The line current is then

$$I=I_a-\overline{OF} \qquad (13)$$

Note that, in general, for any given value $\overline{FJ}$ of the terminal voltage (Fig. 80) there are two possible values of the line current, one value corresponding to the point C and the other to the point C'. Compare with shunt generator (Article 61).

As a numerical example, consider a machine which has the saturation curve shown in Fig. 80, and which has an armature resistance of 0.07 ohm, series-field resistance of 0.03 ohm, shunt-field resistance (including shunt-field rheostat) of 58 ohms, and a resultant magnetizing factor of 0.014 (armature-demagnetizing factor of 0.006 and field-magnetizing factor of 0.02). The terminal voltage of this machine at no-load will be 125 volts, and the no-load shunt-field current 2.16 amperes.

Lay off $\overline{OK}=0.014\times500=7$, and $\overline{KQ}=(0.07+0.03)\times500=50$, and draw $\overline{OQ}$. (The armature current is here taken as 500 amperes simply for convenience in plotting.)

The factor k has the value

$$k=\sqrt{\left(\frac{0.07+0.03}{25}\right)^2+\left(\frac{0.014}{1}\right)^2}=0.0146$$

To determine the terminal voltage corresponding to an armature current of, say, 200 amperes, calculate

$$\overline{JC}=0.0146\times200=2.92$$

and proceed in the same r
61), noting that the line
sponds to the line $\overline{ST}$ i
voltage coresponding to
found to be 142.5 volts, and
is 2.45 amperes. The corresponding
amperes. The complete voltage-regulat
this manner is shown in Fig. 81.

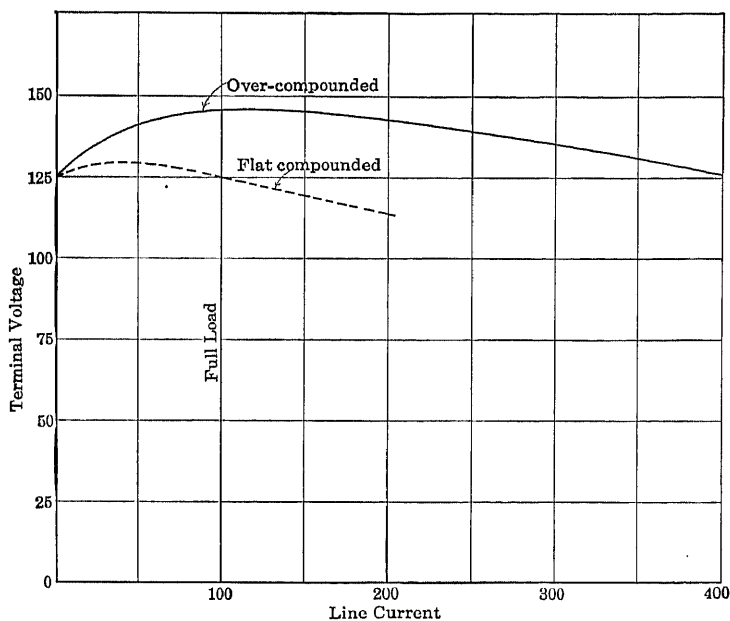



FIG. 81.—Voltage Regulation of Compound Generator.

From the construction above described it is evident that there is a definite maximum value of the terminal voltage of a compound generator for a given setting of the shunt-field rheostat, this maximum value being equal to the ordinate $\overline{FJ}$ when the line $\overline{JC}$ is tangent to the saturation curve. The particular load at which this maximum value of the terminal voltage occurs depends upon the relative number of ampere-turns of the series field and shunt field. As already noted, the ampere-turns of the series field may be adjusted by placing a shunt of suitable resistances across the terminals of this winding. The ampere-turns of the shunt field

can be adjusted by varying the setting of the shunt-field rheostat. By proper adjustment the point of maximum terminal voltage can be made to occur at any desired load.

In the particular case corresponding to Fig. 80 and illustrated by the upper curve in Fig. 81 the maximum terminal voltage occurs at a line current of 125 amperes, the terminal voltage rising from 125 volts at no-load to 145 volts at 125-amperes-line current. When the terminal voltage increases from no-load to full-load, the machine is said to be **over-compounded.** By properly adjusting the shunt on the series field (or the shunt-field rheostat), it is possible to make the terminal voltage at full-load equal to the terminal voltage at no-load. Under these conditions, the machine is said to be **flat-compounded.** Actually, however, the voltage curve is not flat, but between no-load and full-load rises to a maximum value slightly in excess of its no-load value, as indicated by the dotted curve in Fig. 81.

The construction shown in Fig. 80 is for the long-shunt connection. For a short-shunt compound generator the construction is slightly different. In the first place, the current in the series field of a short-shunt generator is the line current (see Fig. 17). Consequently, the magnetizing action of the series field is proportional to the *line* current and not to the *armature* current. However, since the shunt-field current is usually small, the resultant demagnetizing factor A' may, to a close approximation, be taken the same as for a long-shunt connection.

When this assumption is made, the construction for the long-shunt connection has to be modified only in the following particulars in order to give the voltage regulation of a short-shunt machine:

1. Instead of employing equation (11), calculate k by the formula

$$k=\sqrt{\left(\frac{R_a}{v}\right)^2+\left(\frac{A'}{i}\right)^2} \qquad (14)$$

2. In laying off the line $\overline{OQ}$ in Fig. 80, make $\overline{KQ}$ equal to the resistance drop in the *armature only,* exclusive of the series field.

3. The construction just described, without other modification, will then give the *armature* terminal voltage.

4. To find the terminal voltage of the *machine,* subtract from the armature terminal voltage the voltage drop in the series field, viz., R_sI, where I is the line current.

On account of the fact that the voltage impressed on the shunt field of a short-shunt generator is not affected by the voltage drop in the series field, the shunt-field current in a short-shunt generator remains more nearly constant than in a long-shunt machine. Consequently the compounding of a given machine is slightly more effective when the short-shunt connection is used. Most compound generators are therefore connected in this way.

65. Voltage Regulation of Commutating-pole Generators.—The voltage-regulation curve of a commutating-pole generator may be determined from its magnetization curve in exactly the same way as for non-interpole machines, the only difference being that:

1. When the brushes are exactly in the mechanical neutral, the armature-demagnetizing factor is very small. Were it not for the demagnetizing effect of the cross-magnetizing turns (see Article 56), the armature-demagnetizing factor of an interpole machine with brushes properly set, would be zero.

2. The internal-resistance drop between the terminals of the machine is the drop due to the combined resistance of the armature winding R_a, commutating pole R_c (and compensating winding if present), and the series-field winding R_s.

In the particular case of a commutating-pole shunt generator, operating at relatively low saturation, so that the demagnetizing action of the armature current may be neglected entirely, the horizontal line CD in Fig. 77 becomes zero, and the point D coincides with the point C (see Fig. 82). The armature current corresponding to any value $\overline{JF}$ of the terminal voltage is then simply

$$I_a = \frac{\overline{CJ}}{R_a + R_c} \tag{15}$$

As a numerical example, consider the same machine as in Article 61, and assume this machine to be equipped with commutating poles. Let the commutating-pole winding have a resistance of 0.01 ohm, and assume its armature demagnetizing factor to be zero. The armature resistance is 0.07 ohm and the shunt-field resistance (including shunt-field rheostat) is 41.7 ohms. For any value I_a of the armature current the length of the line $\overline{CJ}$ is then

$$\overline{CJ} = 0.08 I_a \quad \text{volts}$$

Hence, for an armature current of 100 amperes, $\overline{CJ}=8$ volts. From Fig. 82, the corresponding terminal voltage is then either 129 volts or 13 volts, accordingly as the equivalent load resistance is high or low. The corresponding field currents are 3.1 amperes and 0.3 ampere, and the corresponding line currents 196.9 amperes and 199.7 amperes respectively.

Proceeding in this manner for successive values of the armature current the complete voltage-regulation curve shown in Fig. 83 is obtained.

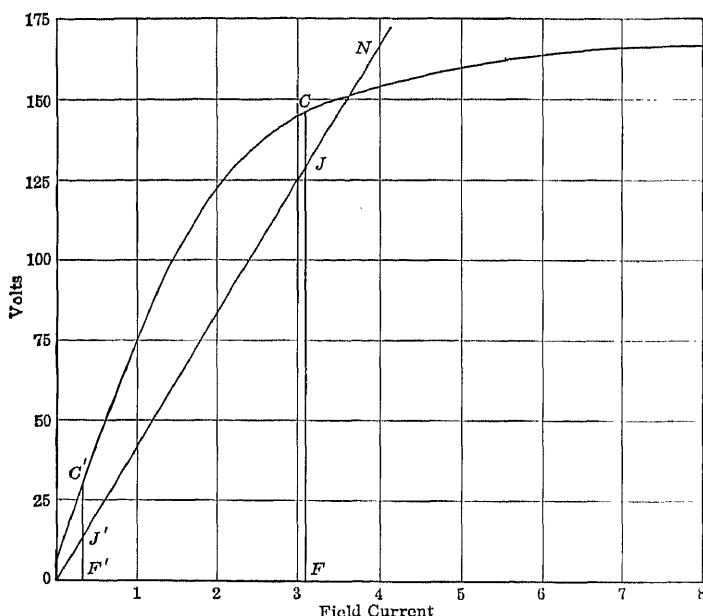



Fig. 82.—Construction for Regulation of Commutating-pole Shunt Generator.

A comparison of Fig. 83 with the voltage-regulation curve of the non-interpole generator shown in Fig. 78 (noting the difference in scale shows clearly the effect of commutating-poles in improving the voltage regulation of a generator. For example, for a load of 80 amperes the percent voltage regulation of the non-interpole machine is $\frac{151-132}{132}=14.4$ percent, and for the commutating-pole machine is $\frac{151-143}{143}=5.6$ percent.

The reader should keep in mind, however, that this improvement in voltage regulation is not due to the elimination of all armature reaction, but is due to the elimination of the direct demagnetizing effect of the armature current (no demagnetizing turns). The field-distorting effect of the armature current is still present, and can be eliminated only by the use of a compensating winding.

It is also of interest to note (though of no particular practical significance) that the use of commutating poles greatly increases

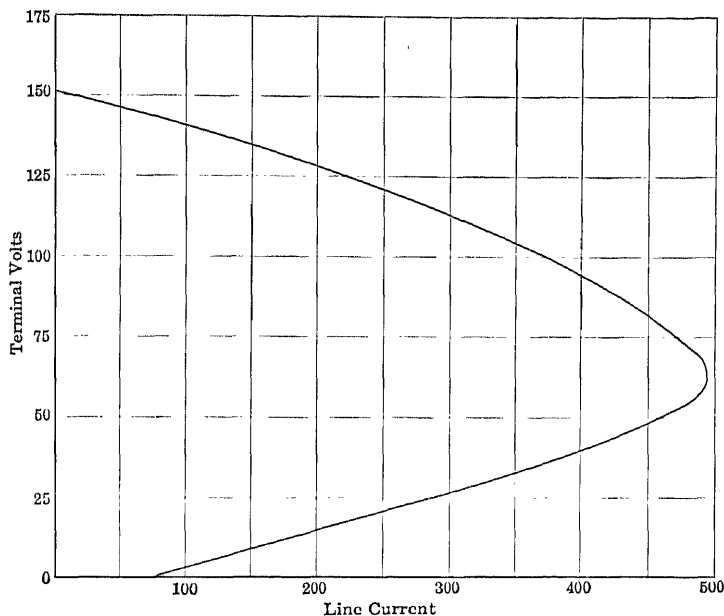



FIG. 83.—Voltage Regulation of Commutating-pole Shunt Generator.

the *maximum* current which it is possible to obtain from a given shunt generator. For example, for the particular machine under consideration, this maximum current is 490 amperes when commutating poles are used (Fig. 83), as against 119 amperes when the machine is not so equipped (Fig. 78).

66. Fluctuation in Voltage due to Sudden Change in Load.—On account of the self-inductance of the armature circuit of a generator, a sudden increase in the load connected to its terminals will always produce a momentary decrease in the terminal voltage

of the machine (see Fig. 84). Similarly, a sudden decrease in the load will produce a momentary rise in voltage. These momentary variations in voltage, though they may be of extremely short duration (one-hundredth of a second or less), may cause an appreciable flicker, or "winking," of the lamps connected to the generator. This effect is often pronounced when the light and power supply is from a single generator, as is sometimes the case in an office building.

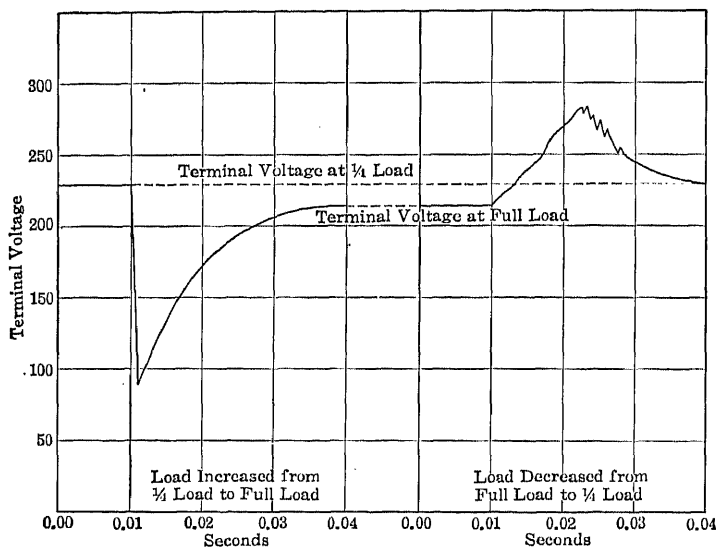



Fig. 84.—Effect on Terminal Voltage of Sudden Change in Load.

The effect of the self-inductance of the armature circuit is readily understood when it is remembered that a change in the current in a circuit always sets up an electromotive force in this circuit, due to the change in the magnetic field which this varying current itself produces (see Article 19). While the current is decreasing, this self-induced electromotive force is always in the direction of the current; while the current is increasing the self-induced electromotive force is always in the direction opposite to that of the current.

Consequently, when the load on a generator is suddenly increased (for example, by the starting of an elevator in an office building), the terminal voltage of the machine *while the current is increasing*

is not equal to the generated electromotive force E less the resistance drop RI_a, but is still further reduced by an amount equal to the self-induced electromotive force. This self-induced electromotive force is equal to the self-inductance L of the armature circuit multiplied by the rate of increase of the armature current, namely, is equal to $L\frac{dI_a}{dt}$, where dI_a is the increase in the armature current in time dt. The terminal voltage of the machine during an increase in the armature current is therefore

$$V=E-RI_a-L\frac{dI_a}{dt} \tag{16}$$

In this expression R is the total resistance of the armature circuit, including the resistance of the armature, the resistance of the series field (if any), the resistance of commutating-pole winding (if any), and the resistance of the compensating winding (if any). Similarly, L is the resultant self-inductance of the armature winding, series-field winding, commutating-pole winding, and compensating winding.

From equation (16) it is apparent that the *greater* the change in the current and the more *suddenly* this change takes place, the greater will the momentary change in voltage be. Also, the greater the self-inductance of the armature circuit, the greater will this change be.

The reader should note that this momentary dip in the voltage is quite distinct from the additional resistance drop resulting from the increased armature current required to supply the additional load connected to the machine. Just as soon as the current reaches the steady value corresponding to the increased load, the self-induced electromotive force disappears (see Fig. 84). *The self-induced electromotive force exists only while the current is changing.*

In a machine with a compensating winding the resultant self-inductance is *less* than that of the armature alone, since the resultant magnetic field due to the current in the compensating winding neutralizes, to a large extent, the magnetic field which the current in the armature tends to produce (see Article 52). Consequently, when a compensating winding is used, the change in voltage caused by a sudden change in load is much less than it would be in a similar machine with no compensating winding.

When the load on a generator is suddenly decreased (for example,

by the opening of a circuit breaker on one or more feeders), the self-induced electromotive force, being in the same direction as the generated electromotive force, increases the terminal voltage. Equation (16) is also applicable to this case, provided dI_a is still interpreted as an *increase.* When so interpreted dI_a for an actual decrease becomes a *negative* quantity, and therefore $-L\frac{dI_a}{dt}$ becomes positive.

Fig. 84 shows the results of an actual test (by means of an oscillograph) of a 250-kilowatt generator. When the load on the machine was suddenly increased from $\frac{1}{4}$-load to full-load, there was a momentary drop in the terminal voltage from 230 volts to 90 volts. The terminal voltage then rapidly rose, in less than 0.03 second, to a steady value of 215 volts. The load was then suddenly decreased from full-load to $\frac{1}{4}$-load, by opening the circuit breaker in the feeder supplying the load which had been added. The terminal voltage rose momentarily to 290 volts, but at the end of about 0.03 second came back a steady value of 230 volts. The irregularities in the voltage curve when the load was suddenly switched off are due to arcing at the circuit-breaker terminals.

67. Voltage Regulators.—The voltage at the terminals of a generator, either shunt or compound, may always be kept constant, irrespective of the load, by properly adjusting the shunt-field rheostat. This method of maintaining constant voltage may be satisfactory when the fluctuations in the load are gradual and infrequent, but when the load fluctuations are frequent and rapid, and the service requires a voltage even more nearly constant than can be obtained from a compound generator, an automatic voltage regulator is usually employed.

One of the most satisfactory forms of automatic voltage regulator is that known as the **Tirrill regulator.** This device not only prevents variations in voltage due to armature resistance and armature reaction, but also voltage variations due to changes in speed of the prime mover driving the generator. It operates on the principle of intermittently short-circuiting the field rheostat, thus maintaining a definite *average* value of the resistance in the field circuit for each value of the load on the machine.

Referring to Fig. 85, a shunt circuit of negligible resistance containing a pair of contacts (marked "relay contacts" in the

figure) is placed around the shunt-field rheostat. These contacts are controlled by a spring and an electromagnet (relay magnet) whose winding, in series with an external resistance, is connected directly across the terminals of the machine. The external resistance is simply for the purpose of adjusting the current in the relay magnet to the desired value. A condenser is placed around the relay contacts to prevent sparking.

Neglecting for a moment those parts of the diagram to the left of the relay magnet, it is evident that as the terminal voltage of the generator rises, the current through the relay magnet

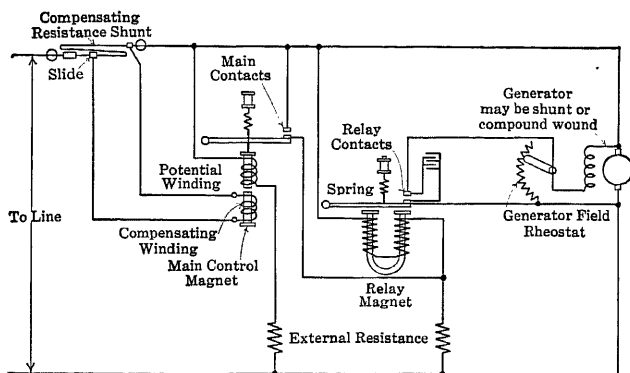



FIG. 85.—Elementary Connections of Type T-D. Regulator.

will increase, and, at a predetermined voltage, dependent upon the setting of the spring attached to the relay contacts, the pull of the relay magnet will open the contacts, thereby inserting full resistance into the field circuit. This will decrease the field current, which in turn will cause the terminal voltage to fall. The spring will then again close the relay contacts, again short-circuiting the field rheostat, and the terminal voltage will rise, and this cycle of changes will be repeated over and over.

The voltage at which the relay contacts close will always be higher than the voltage at which these contacts open, so that there will always be a slight fluctuation in the terminal voltage of the machine. This fluctuation, however, in voltage, can be reduced to a practically inappreciable amount by making the armature (or moving arm) of the electromagnet light, and laminating the core of the magnet.

This simple arrangement, however, has one serious defect, namely, that the *opening* of the relay contact is sluggish, and, due to the high self-inductance of the field winding, serious sparking will occur at the contacts, even when a condenser is used. To prevent this, a second pair of contacts (marked "main contacts" in the figure) are shunted around the winding of the relay magnet. These main contacts are likewise controlled by a spring and an electromagnet. The winding of the latter, in series with a current-limiting resistance, is also connected directly across the terminals of the generator.

When the terminal voltage is below the value for which the spring on the main contacts is set, these contacts remain closed, the relay magnet is short-circuited, and the spring on the relay contacts holds these contacts closed. When the terminal voltage rises to such a value that the pull of the main control magnet exceeds the pull of its spring, the contacts open, and full-line voltage is thrown on the relay magnet. The sudden pull of this magnet, due to the sudden application of voltage, causes a quick opening of the relay contacts, and no sparking at the contacts will occur, provided the condenser shunted around these contacts has the proper capacity. With this arrangement the relay magnet and contacts may be made rugged and the adjustment of the spring controlling the relay contacts need not be particularly close. On the other hand, the moving parts of the control magnet are made light, and the desired terminal voltage is secured by careful adjustment of the spring controlling its contacts.

By using a so-called compensating winding on the main control magnet, connected to the terminals of a low resistance in series with the generator and load, as shown in Fig. 85, the regulator can be made to produce a terminal voltage which increases with the load by an amount equal to the resistance drop in the line. In this way a constant voltage can be maintained at the load. The compensating winding is so connected that the current in it tends to demagnetize the core of the main control magnet, and therefore the greater this current the higher must the terminal voltage rise before the main contacts open.

The appearance of one form of Tirrill regulator, made by the General Electric Co., is shown in Fig. 86. There are other forms of regulators of this same general type which may be used for

controlling the voltage of a single generator or of any number of generators operating in parallel.

68. Parallel Operation of Shunt Generators.—Two or more shunt generators of the same, or approximately the same, rated voltage may always be operated in parallel, the connections being as indicated in Fig. 87. The machines need not have the same

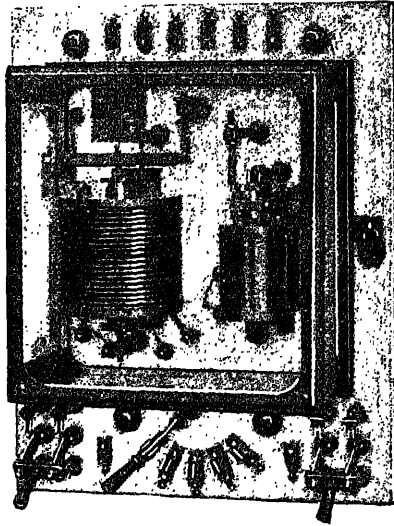



FIG. 86.—Tirrill Regulator (Type T-D.).

kilowatt rating, one may be small and the other large. The proportion of the total load which each machine will carry depends solely on the voltage-regulation curves of the two machines, which in turn depend not only upon the design of the machines, but also upon the speeds at which they are driven and their field excitations.

Referring to Fig. 88, let the curves V_1 and V_2 represent the voltage-regulation curves of two shunt generators when operating separately at given speeds and given settings of their field rheostats. When these two machines are connected in parallel to supply a given total line current I, the terminal voltages of the two machines must have the same value. That is, the current I_1 supplied by No. 1 must correspond to the same terminal voltage

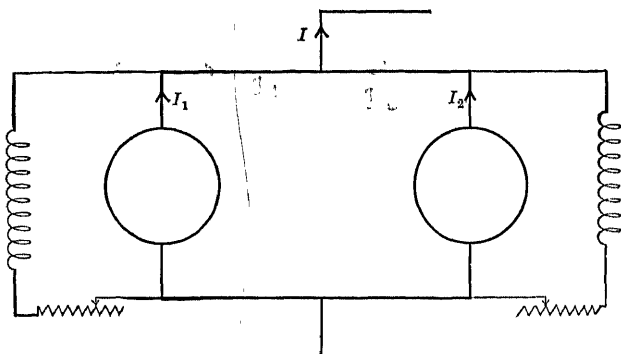



FIG. 87.—Two Shunt Generators in Parallel.

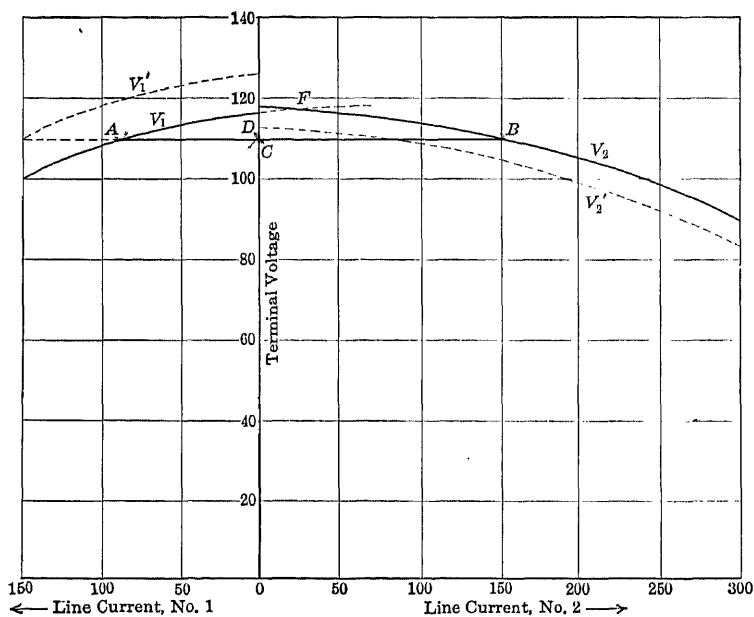



FIG. 88.—Division of Load between Two Shunt Generators in Parallel.

as the current I_2 supplied by No. 2, and I_1+I_2 must equal the total current I.

Locate the horizontal line $\overline{AB}$ whose intercept between the two regulation curves is equal to I. Then the portion $\overline{AC}$ of this intercept to the left of the Y-axis will give the current I_1 supplied by No. 1, and the portion $\overline{CB}$ of this intercept to the right of the Y-axis will give the current I_2 supplied by No. 2. The distance $\overline{OC}$ of this intercept above the X-axis gives the terminal voltage of each machine.

For example, when the total current supplied by the two machines in Fig. 88 is 240 amperes, the terminal voltage will be 110 volts, and machine No. 1 will supply 90 amperes and machine No. 2 will supply 150 amperes.

As the total load on the bus bars (i.e., the total line current) varies, the proportion of the total load carried by each machine may, or may not, remain constant. If the voltage-regulation curve for each machine plotted against *percent* of full-load current for that machine is the same for the two machines, then the total load will divide between the two machines in proportion to their ratings. This fact is usually expressed by the statement that if the voltage-regulation curves of the two machines are of the same *shape* the machines will divide the load in proportion to their ratings.

The division of the load between the two machines may always be controlled either by changing the resistance of the shunt-field rheostats or by changing the relative speeds of the two machines. In practice, the adjustment of the load on each machine is always accomplished by means of the field rheostats. For example, referring to Fig. 88, if it is desired to make machine No. 1 carry 150 amperes of the total of 240 amperes, and keep the terminal voltage constant, the field resistance of No. 1 is cut down, thereby increasing the field current, and raising the voltage-regulation curve of this machine, as indicated by the dotted line V_1'. At the same time the field resistance of No. 2 is increased, thereby decreasing the field current of No. 2 and lowering its voltage regulation curve, as indicated by the dotted curve V_2'.

When there is no external load, i.e., no line current, the current supplied by each machine will be zero (neglecting the field current), only if the field rheostat of each is so adjusted that when operating

separately each machine has the same no-load terminal voltage. When this is not the case, one machine will operate as a generator supplying a current to the other, which will operate as a motor. For example, referring to Fig. 88, if the field rheostats are set to give the voltage-regulation curves V_1 and V_2, then for no external load, machine No. 1 will act as a generator supplying a current of

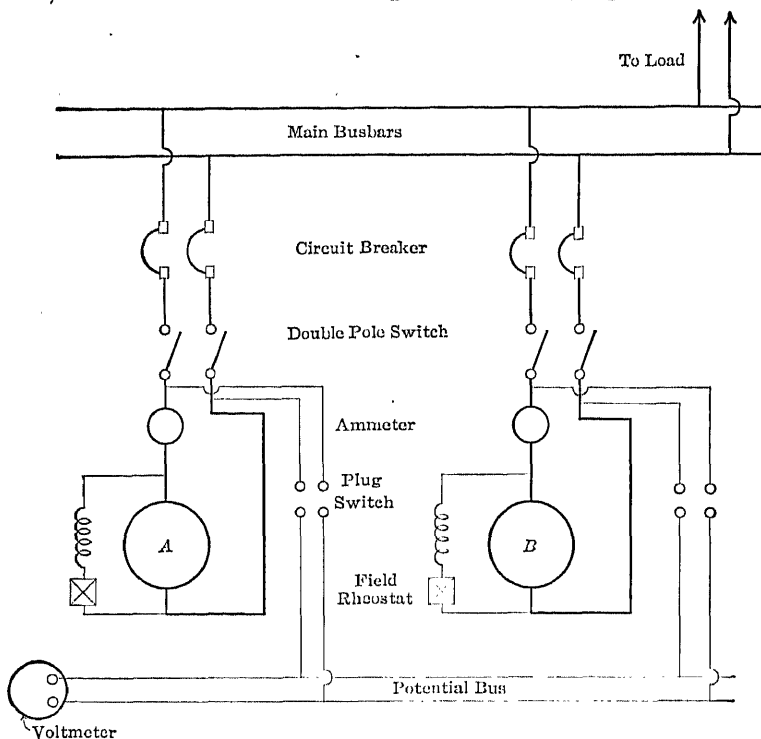



FIG. 89.—Detail Connections for Two Shunt Generators in Parallel.

33 amperes to No. 2, which will operate as a motor, actually driving, instead of being driven by, its prime mover.

In Fig. 89 are shown the instruments, switches and circuit-breakers and the connections usually employed when shunt generators are to be operated in parallel. Instead of having a voltmeter for each generator, a single voltmeter is usually employed, with plug switches arranged as shown, so that the voltmeter may be connected at will to any one of the two or more generators.

To connect a generator A in parallel with one or more generators

B which are already connected to the busbars, the procedure is as follows:

1. With the switch and circuit-breaker of *A* open, start the prime mover of *A* and bring it up to speed.

2. Adjust the field rheostat of *A* until the terminal voltage of *A* is equal to the busbar voltage (which is the same as the terminal voltage of *B*). This is determined by connecting the voltmeter first to *B* and then to *A*.

3. Close the circuit breaker of *A*.

4. Close the switch of *A*.

When the switch of *A* is closed, no appreciable current will be delivered by *A* to the busbars, for its generated voltage (no-load terminal voltage) was made equal to the busbar voltage, and therefore there can be no resistance drop in the armature of *A*, and consequently no current. To make *A* take its share of the load it is merely necessary to increase its field current by manipulating its field rheostat, thereby increasing its generated voltage. To keep the busbar voltage constant, it will also be necessary, as a rule, to decrease the field current of *B*.

To disconnect a generator from the busbars, its load is first shifted to the other machine with which it is operating in parallel, by manipulating its shunt-field rheostat, its circuit breaker is then tripped, its switch opened and its prime mover shut down.

69. Parallel Operation of Compound Generators.—Two or more compound generators may be operated in parallel in the same way; as shunt generators, **provided their series fields,** as well as the generators as a whole, **are connected in parallel,** as shown in Fig. 90 for the particular case of two generators in parallel. The additional connections between the series fields is called an "equalizer bus," or "equalizer." When an equalizer is employed, the currents in the several series fields divide inversely as their resistances, and therefore the series-field current of each machine, and consequently the compounding of each machine, is directly proportional to the *total* load current, and not to the current which the armature of this particular machine may be supplying.

When an equalizer is not used, the division of load between the several generators will be unstable, except in the special (and infrequent) case of under-compounded machines. An inspection of Fig. 91, which shows the voltage-regulation curves

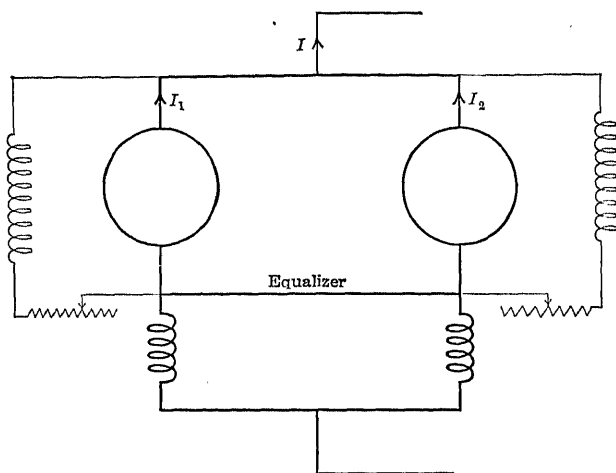



FIG. 90.—Two Compound Generators in Parallel.

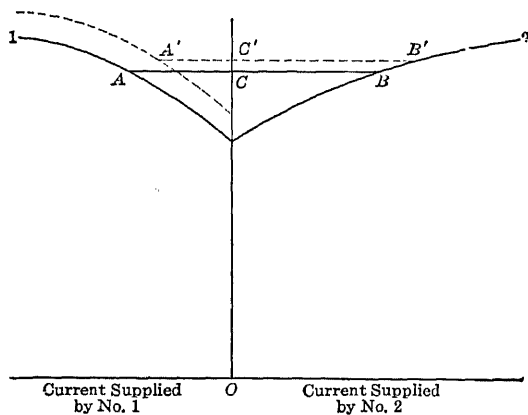



FIG. 91.—Division of Load between Two Compound Generators WITHOUT EQUALIZER.

of two over-compounded generators when operating separately, will make this clear. Let these two machines be connected in parallel, without an equalizer, to supply a total load current $\overline{AB}$. If the speeds of the two machines remain absolutely constant at the values corresponding to the voltage-regulation curves shown in the figure, generator No. 1 will supply a current $\overline{AC}$, and No. 2 a current $\overline{CB}$. If, however, the speed of No. 1 should increase slightly, due to any cause, thereby giving for No. 1 a higher regulation curve, as shown by the dotted curve in Fig. 91, the current output of No. 1 will decrease from $\overline{AC}$ to $\overline{A'C'}$. The load on the prime mover of No. 1 will therefore be reduced, which will cause a further increase in its speed, and therefore a further reduction in its load. This cumulative action may continue until the load originally on No. 1 may all be shifted to No. 2. In fact, under certain conditions, No. 1 may actually operate as a motor, requiring No. 2 to supply not only the entire line current, but also the current to drive No. 1 as a motor.

When an equalizer is used, the series-field current of each machine, for a given total line current, remains constant, irrespective of the speed, since the series-field windings are all in parallel. *For a given total line current* the series-field winding of each generator is therefore equivalent to a separately excited field winding carrying a constant current, and its effect is therefore independent of the division of the load between the two (or more) machines.

When an equalizer is used, the terminal voltage of each generator, *for a given total line current*, is therefore equal to a constant (determined by the series-field winding) plus the terminal voltage which the shunt field alone would produce. The voltage regulation curve of each machine, *for a given total line current*, is therefore a drooping curve, as shown in Fig. 92.

Note, however, that these curves apply only for one value of the total line current. Since the series-field current in each generator is proportional to the total line current, the height of each of these curves increases as the total line current increases, and therefore the group of generators as a whole will have a rising-voltage characteristic, i.e., the terminal voltage will increase as the total line current increases.

Referring to Fig. 92, it is evident that, for a given total line

current $\overline{AB}$, should the speed of No. 1 increase slightly, due to any cause, thereby giving a voltage-regulation curve represented by the dotted curve, the load on No. 1 will increase from $\overline{AC}$ to $\overline{A'C'}$, and this increase of load will tend to prevent any further increase of speed. There will therefore be no cumulative action, such as takes place when an equalizer is not used, and the operation of the two machines will be stable.

From the foregoing discussion it is therefore evident that the condition for stable operation of two or more direct-current generators in parallel is that *for a given total line current* the individual machines have drooping regulation curves. This condi-

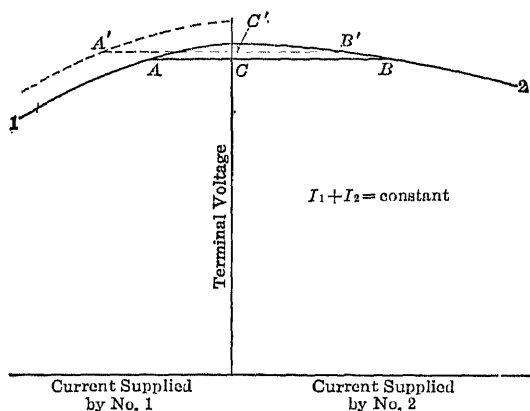



FIG. 92.—Division of Load between Two Compound Generators WITH EQUALIZER.

tion is inherent in shunt and separately excited machines. To secure this condition with compound generators, or with series generators, it is necessary that the series-fields be connected in parallel, i.e., that an equalizer connection be used.

The division of the load among two or more compound generators in parallel is controlled by manipulating the shunt-field rheostats, just as in the case of shunt generators. It is usually desirable, when the field rheostats have been set to make the load divide among the several machines in proportion to their ratings, that the machines continue to divide the load in this ratio, without change of the field rheostats, irrespective of the value of the total line current. This will happen only when the regulation curves of the several machines

are of the same shape (i.e., when the curves showing the relation between terminal voltage and *percent* of full load are identical).

As noted in Article 64, the regulation curve of a single compound generator may be given any shape, within limits, by placing a shunt of suitable resistance across the terminals of the series field. The less the resistance of this shunt the less will be the

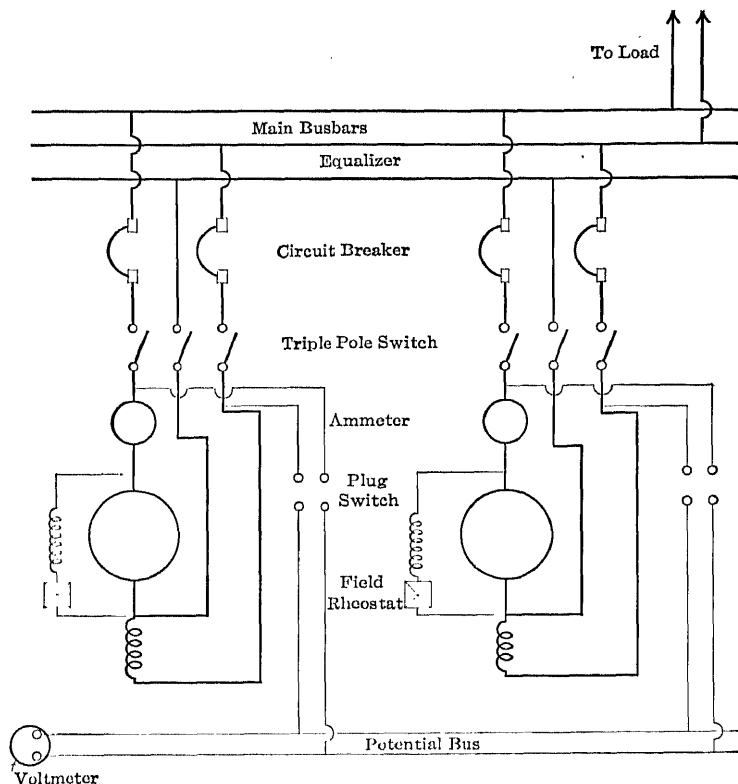



Fig. 93.—Detail Connections for Two Compound Generators in Parallel.

proportion of the line current which flows through the series-field winding, and therefore the less the degree of compounding.

When two or more generators are operated in parallel, a shunt of this kind placed around any one of the series fields also acts as a shunt around each of the other series fields, for all the fields are connected in parallel by the equalizer. Consequently such a shunt will affect the degree of compounding of *all* the machines,

but will have practically no effect on the division of the load among them. The only way in which the division of load can be affected, for given settings of the shunt-field rheostats, is to insert resistance *in series* with one or more of the series fields.

In Fig. 93 are shown the instruments, switches and circuit breakers and the connections usually employed when compound generators are to be operated in parallel. These connections are similar to those used for shunt generators in parallel (Fig. 89), with the addition of the equalizer and connections thereto. The main switch is provided with three sets of poles, one of which is in the lead from the series field to the equalizer. It should be particularly noted that the ammeter for each machine should be *in the lead from the armature* to the main bus and not in the lead from the series field. If it is placed in the latter lead it will read the series-field current, which, as noted above, may be quite different from the current supplied by the generator to the load connected to the busbars.

Compound generators are put in parallel and the load shifted from one to the other in the same manner as shunt generators (see Article 68).

PROBLEMS

1. A 250-volt generator supplies three loads, connected in parallel across the supply mains at distances of 200 feet, 700 feet and 1400 feet respectively, from the generator. The supply mains are of No. 0000 A. W. G. stranded copper wire. The currents taken by the three loads are respectively 100 amperes, 50 amperes and 75 amperes. What is the voltage across each load?

2. A series circuit supplies 50 incandescent lamps, each of which is rated at 10 amperes and 20 volts. The wire connecting the lamps is No. 6 A. W. G. solid copper, and, has a total length of 1.5 miles.

(*a*) What voltage must be impressed on the terminals of the loop formed by this circuit in order to produce rated current through the lamps.

(*b*) If 25 of the lamps in this circuit are short-circuited, to what value would this voltage have to be reduced in order to maintain this same current?

3. If the generator in Problem 1 is flat compounded (and its full load current is 225 amperes), what will be the per cent voltage regulation at each of the three loads?

4. The saturation curve and the voltage regulation curve of a certain shunt generator were both determined experimentally, at a speed of 1440 r.p.m., with the results given at the top of p. 195.

The armature resistance (including brushes and brush contacts) was found to be 0.4 ohm.

(*a*) Plot these curves and determine the value of the armature demagnetizing

factor corresponding to line currents of 20 amperes, and also its value corresponding to a line current of 40 amperes. Compare these two values.

(*b*) If the field rheostat is set to give a terminal voltage of 168 volts at no load, by how much must the resistance of this rheostat be decreased in order to give this same terminal voltage when the line current is 40 amperes?

Saturation Curve.		Voltage Regulation Curve.	
Field Amperes.	Armature Voltage.	Line Amperes.	Terminal Voltages.
0.0	6	0	168
0.6	83	10	159
0.8	103	20	150
1.0	122	30	140
1.2	136	40	126
1.5	153		
2.0	169		

(*c*) What will be the corresponding values of the no-load and full-load field currents?

5. The following data were obtained from test of a 1000-kw., 275-volt, 720 r.p.m. shunt generator:

Armature resistance, including brushes and brush contacts, 0.0021 ohm.

When the armature was driven at rated speed without load, and the field winding separately excited, the following readings were obtained, the brushes being set in their normal operating position:

Field amperes,	0	5	8	10	15	20	30	40	50
Arm. Ter. Volts,	5	145	208	238	292	324	356	372	381

Full-load current was then taken from the armature, the field winding being separately excited as before. To obtain rated terminal voltage under these conditions it was found necessary to use a field current of 15.2 amperes.

(*a*) What must be the resistance of the shunt field circuit, when the machine is operating as a shunt generator, in order to obtain rated voltage at full load?

(*b*) What will be the field current when, for this setting of the field rheostat, the load is disconnected?

(*c*) What will be the no-load terminal voltage?

(*d*) What is the armature demagnetizing factor of this machine?

6. (*a*) Calculate and plot the voltage regulation curve of the generator described in Problem 5, when this machine is operated as a separately excited generator with a constant field current of 15.2 amperes. Make calculations for at least six points, between zero load and 150 per cent overload.

(*b*) What is the theoretical maximum current which this machine could supply when operated as a separately excited generator at a constant field current of 15.2 amperes? What would be the corresponding terminal voltage?

(*c*) Why is it impossible to obtain practically this theoretical maximum current?

7. (*a*) Calculate and plot the armature characteristic of the generator described in Problem 5 for a constant terminal voltage of 275 volts. Calculate at least six points between no-load 150 per cent of rated armature current.

(*b*) If there are N shunt-field turns per pole, how many series-field turns per pole would be required in order to convert this machine into a long-shunt, flat-compound generator?

8. Calculate and plot the voltage regulation curve of the generator described in Problem 5, when this machine is operated as a shunt generator and its field adjusted to give rated voltage at rated load. Make calculations for at least six points between zero load and 150 per cent load. Compare this curve with that obtained in Problem 6.

9. The following are the data for the saturation curve of a certain series generator when driven at a constant speed of 1200 r.p.m.:

Field amperes,	0	50	100	150	200	300
Terminal volts,	3	59	107	135	150	160

The armature demagnetizing factor of this machine is 0.08, its armature resistance (including brushes and brush contacts) is 0.05 ohm; and the resistance of its field winding 0.01 ohm.

Calculate and plot the voltage regulation curve of this machine. Make calculations for at least six points between zero line current and a line current of 300 amperes.

10. The generator described in Problem 5 is now equipped with a series-field winding, connected long-shunt. The resistance of this series winding is 0.00025 ohm. The number of turns in this winding is so chosen, and the shunt-field current so adjusted, that the terminal voltage of the machine will increase from 250 volts at no-load to 275 volts at rated load.

(*a*) What will be the resistance of, and current in, the shunt-field circuit?

(*b*) What will be the value of the series-field magnetizing factor?

(*c*) If the number of shunt-field turns is N, how many series-field turns will be required?

(*d*) Calculate and plot the voltage regulation curve of this compound generator, and determine the value of the line current for which the terminal voltage will be a maximum (for the given speed and setting of the shunt-field rheostat).

11. (*a*) Plot the voltage regulation curve which would result were the shunt generator described in Problem 5 equipped with commutating poles, the total resistance of the winding of which is 0.0002 ohm. The field rheostat is adjusted to give rated terminal voltage at rated load. In calculating this curve neglect the demagnetizing effect of the cross-magnetizing turns.

(*b*) Why, how and to what extent will the actual voltage regulation curve differ from this calculated curve?

12. The electromotive force generated in the armature of a certain generator is 230 volts. The armature resistance is 0.004 ohm and the total self-inductance of the armature is 0.8 millihenry. Part of the load supplied by this generator is suddenly disconnected, resulting in a decrease in the armature current from

1000 to 750 amperes. At the instant immediately after the switch is opened, the armature current is decreasing at the rate of 50 amperes in 0.001 second. What is the terminal voltage of the armature at this instant?

13. Two 125-volt shunt generators A and B are operating in parallel at rated load and rated voltage. Generator A is a 100-kilowatt machine and generator B is a 250-kilowatt machine. The voltage regulation of A at rated load 8 per cent, and the voltage regulation of B at rated load is 5 per cent.

Assuming the voltage regulation curves of the two machines to be straight lines, and the field rheostats not to be altered, what will be the bus-bar voltage and the current output of each (*a*) when the total load on the bus-bar is 125 kilowatts and (*b*) when there is no load on the bus-bars?

14. Each of two over-compound generators A and B are connected short-shunt. Generator A is 1000-kilowatt, 250-volt machine and generator B a 500-kilowatt, 250-volt machine. The resistance of the series field of A is 0.0015 ohm, and the resistance of the series field of B is 0.002 ohm. When operated separately each of these machines has a voltage regulation of 5 per cent (i.e., the terminal voltage rises from 237.5 volts at no load to 250 volts at full load).

These two machines are first operated separately, each at its rated load and rated voltage. The two machines are then connected in parallel, with equalizer as shown in Fig. 90. Neither the total load nor the shunt-field rheostats is changed.

(*a*) What will be the series-field current of each machine?

(*b*) Will the current supplied by the armature of machine A increase or decrease? Explain fully.

(*c*) Will the terminal voltage of the two machines increase or decrease? Explain fully.

(*d*) How can the armature current of each machine be brought back to its rated value?

(*e*) Why is the equalizer used?

CHAPTER VII

OPERATING CHARACTERISTICS OF MOTORS

70. Service Requirements.—Electric motors are used for driving practically all kinds of machines. The particular type of motor best suited for a given service depends primarily upon the relation between speed and torque which this service demands. The service may require:

1. Constant, or substantially constant, speed at all loads. Examples: line shafting in a factory or shop, fans, pumps, etc. This type of service is usually referred to as **constant-speed service.**

2. Speed which can be varied at will, but when once adjusted remains substantially constant. Example: machine tools, such as lathes, milling machines, etc. This type of service is usually referred to as **multispeed,** or **variable-speed service.** The former term is used when the service requires several distinct speeds with no graduation between, and the latter for a service which requires a gradual variation of speed over a considerable range.

3. Speed which decreases as the torque increases, thereby making the power required increase less rapidly than the torque. Examples: railway service, hoisting, rolling mills, etc. This type of service is usually referred to as **varying-speed service.**

The reader should note particularly the distinction made between the terms "variable-speed" and "varying-speed." A variable-speed service is one which, for different conditions requires different speeds, but for a given set of conditions, requires that the speed, whatever its value may be, remain substantially constant irrespective of the load. A varying-speed service, on the other hand, is one in which the speed may vary with the load, usually decreasing as the load increases.

As will be shown in the subsequent articles of this chapter, the speed of a shunt motor is approximately constant, falling off

only slightly with the load. It is therefore well adapted to constant speed, or approximately constant-speed service. The shunt motor, particularly when equipped with commutating poles, is also well suited for variable speed or multispeed service, since its speed is readily controlled by varying its field current by means of the field rheostat.

The series motor, on the other hand, is inherently a varying-speed machine, its speed falling off rapidly as the load (opposing torque) increases. Series motors are therefore almost invariably used for traction work.

One peculiarity of a series motor, however, is that should the mechanical load which it is driving be thrown off (i.e., should the opposing torque become zero), its armature may attain such a high speed as to wreck itself by centrifugal action. With traction motors there is no danger of this ever happening, for the motor is mechanically connected to its load (the car or locomotive), which always offers an appreciable opposing force, namely, the inertia of the car and the friction between the wheels and track.

By using a compound motor with series field and shunt field acting in the same direction (cumulative compound), the difficulty of high no-load speed may be avoided, and at the same time high torque at heavy loads (e.g., at starting) may be obtained. The cumulative compound motor is therefore well suited for driving elevators, hoists, rolling mills, etc.

71. Torque of a Motor.—The total torque T developed in a motor, due to the mutual reaction between the field and armature currents, is proportional to the product of the current I_a in its armature and the resultant useful flux per pole ϕ_r, viz.,

$$T = K_1 \phi_r I_a \tag{1}$$

where K_1 is a constant whose value depends upon (1) the number of poles, (2) the number of armature conductors, (3) the type of winding, and (4) the units in which the various quantities are expressed, *but which is independent of the speed at which the armature rotates.*

By the resultant useful flux per pole ϕ_r is meant the change in the resultant number of lines of force which thread an armature coil as the commutator segment to which either end is connected moves around the air-gap from one brush to the next. Under

esultant flux is due to the combined effect of the field d armature current, i.e., the flux ϕ, is the resultant o the field excitation and armature reaction. See Article 32.

† Equation (1) can be deduced directly from the fundamental principle, stated in Article 20, that the resultant torque exerted by a magnetic field on a coil and core, due to a current in this coil, is equal to the product of this current by the change in the flux linkages of this coil per unit angular displacement of it. Another method is the following, which brings out certain other useful relations.

Let V = the armature terminal voltage,
I_a = the armature current,
R_a = the armature resistance, including the brush-contact resistance.

Then, of the total electric power input (V_aI_a) to the armature, an amount $R_aI_a^2$ is dissipated in the armature winding as heat. The remainder, namely,

$$P_m = V_aI_a - R_aI_a^2 = (V_a - R_aI_a)I_a \qquad \text{watts} \qquad (2)$$

is converted into mechanical power, as a consequence of the mutual forces between the magnetic field and the currents in the armature conductors.

But ($V_a - R_aI_a$) is the back electromotive force generated in the armature winding due to its motion in the resultant magnetic field of the machine. Let E be the value of this back electromotive force, i.e., put

$$E = V_a - R_aI_a \qquad (3)$$

As pointed out in Article 32, the back electromotive force of a motor bears exactly the same relation to the number of poles p, the number of armature conductors Z, the number of parallel paths a through the armature, the resultant useful flux per pole ϕ_r and the speed n, as does the electromotive force of a generator, namely,

$$E = \left(\frac{10^{-8}pZ}{60a}\right)n\phi_r \qquad (4)$$

Put

$$K = \frac{10^{-8}pZ}{60a} \qquad (5)$$

For a given machine this factor K is a constant. The back electromotive force of the motor, equation (4), may then be written

$$E = Kn\phi_r$$

This value of E substituted for $(V_a - R_a I_a)$ in equation (2) gives for the mechanical power developed by the motor the expression

$$P_m = Kn\phi_r I_a \qquad \text{watts} \qquad (7)$$

Let T be the total torque or turning moment, exerted on the armature by the magnetic field of the machine, in pound-feet. Then, for a speed of n revolutions per minute, the mechanical power developed by the motor is also

$$P_m = \frac{2\pi nT}{33{,}000} \times 746 = \frac{nT}{7.04} \qquad \text{watts} \qquad (8)$$

Equating equations (7) and (8) gives for the torque developed by the motor the expression

$$T = 7.04\, K\phi_r I_a \qquad \text{pound-feet} \qquad (9)$$

This relation is identical with that stated by equation (1), when $7.04\,K$ is put equal to K_1.

It should be carefully noted that the torque given by equation (1) or (9) is the *total* torque, or turning moment, exerted on the armature as a consequence of the mutual action of the field current and the armature current. This torque is less than that exerted by the armature on the external mechanical load connected to its shaft, since it includes (1) the torque required to overcome the friction of the motor bearings, (2) the torque required to overcome air friction and windage, and (3) the torque required to overcome the opposing torque due to eddy currents and hysteresis in the armature core and pole faces.

Let T_l be the total internal opposing torque due to the combined effect of windage and core loss. Then the net *output torque*, or *useful torque*, is

$$T_0 = T - T_i \qquad (10)$$

where T is the total *generated* torque, given by equation (1) or (9). The internal opposing torque T_i is usually but a small percentage (5 percent or less) of the full-load torque.

When there is no mechanical load connected to the motor, the output torque T_0 is zero, but the generated torque T is not zero, but is equal to the internal opposing torque T_i. The armature current at no load is therefore not zero, but must have a value sufficient to produce the torque necessary to overcome the internal opposing torque T_i.

It is particularly important to note that *for a given armature current the torque developed by a motor is independent of its speed.* Conversely, to develop a given torque, a definite value of the armature current is required, this current being independent of the speed of the motor. The value of the armature current required to produce a given torque is of course a function of the shunt-field current, since the resultant flux per pole ϕ_r depends upon this latter current.

72. Speed of a Motor.—From equation (6) the speed of a motor, in revolutions per minute, is

$$n=\frac{E}{K\phi_r} \qquad \text{r.p.m.} \qquad (11)$$

where E is the back electromotive force, and ϕ_r the resultant flux per pole. Substituting for E its value from equation (3), the speed may also be written

$$n=\frac{V_a-R_aI_a}{K\phi_r} \qquad \text{r.p.m.} \qquad (11a)$$

Except for very heavy loads, the resistance drop R_aI_a is small in comparison with the impressed voltage V_a. Hence, to a first degree of approximation, the speed of a motor is directly proportional to the voltage impressed across the terminals of its armature, and inversely proportional to the resultant flux per pole.

In a series motor, the resultant flux per pole is determined solely by the value of the armature current. As pointed out in the preceding article, the torque is proportional to the product of the resultant flux per pole and the armature current. Hence, for a given opposing torque both ϕ_r and I_a in equation (11a) are fixed in value, irrespective of the value of the impressed voltage. Consequently, to a close approximation (provided the armature resistance drop R_aI_a and the field resistance drop R_sI_a are small in comparison with the impressed voltage, which is usually the

case), *the speed of a series motor for a given opposing torque is directly proportional to the voltage impressed across its terminals.*

In a shunt motor, the flux per pole ϕ_r increases as the impressed voltage increases, since the shunt-field current is directly proportional to the impressed voltage Were the magnetization curve a straight line, and were there no armature resistance and no armature reaction, the ratio of V_a to ϕ_r would therefore be constant, and the speed, see equation (11*a*), would be independent of the value of the impressed voltage. Actually, however, due primarily to the increase in the saturation of the magnetic circuit as the shunt-field current increases, the speed of a shunt motor does increase with increase of impressed voltage, but much less rapidly than in the case of a series motor.

The speed of a series motor is usually controlled by varying the voltage impressed on its terminals, by means of a rheostat connected in series with the motor and the supply mains, or "line." The greater the resistance in series with the motor the lower will its speed be.

This method may also be used to control the speed of a shunt motor, but a much more efficient arrangement is to connect the armature directly to the mains, and to vary the shunt-field current *only* (by means of the shunt-field rheostat). *A reduction in the resistance of the shunt-field circuit decreases the speed.* Conversely, to increase the speed, resistance must be inserted in the shunt-field circuit.

73. Variation of Speed with Torque.—As noted in Article 70, the characteristic of a motor which is of prime importance in determining its adaptability to a given service, is the manner in which its speed varies with the opposing torque which the motor has to overcome.

The speed regulation of a motor, for a given impressed voltage, depends upon the saturation curve of the machine, the armature reaction, and the resistances of its various windings, just as the voltage regulation of a generator for a constant speed depends upon these several factors.

The speed regulation of a shunt motor is frequently expressed as a percentage, viz., by its **percentage speed regulation** at a given impressed voltage and given load is meant the change in speed which occurs when this load is thrown off, expressed

as a percentage of the speed at *no load.* Compare with percent voltage regulation of a generator.

It will now be shown how the speed n and total torque T of a given motor, for any given value V of the impressed voltage and any given value I of the current taken by it, may be determined when its saturation curve, its armature demagnetizing factor A, and the resistance R_a, R_f and R_s of its armature, shunt field, and series field windings are known.

The saturation curve of the motor may be determined in exactly the same way as the saturation curve of a generator, by driving the machine at any arbitrarily chosen constant speed n_0 and separately exciting the field (see Article 35). The armature demagnetizing factor A may also be obtained by driving the machine as a generator at a constant speed and proceeding in the manner described in Article 59.

74. Speed and Torque Characteristics of the Series Motor.— The curve QCM in Fig. 94 is the saturation curve of a series motor, determined by driving the armature at a constant speed (by a second motor, for example) and separately exciting the series field.* As pointed out in Article 35, the ordinate of the saturation curve, for any given value of the field current, is directly proportional to the speed, assuming the reaction of the current in the coils undergoing short-circuit by the brushes to be negligible.

Let n_0 be the speed at which the magnetization curve is determined, and let n be the actual speed at which the armature rotates when the motor is taking a current of I amperes. The actual field current is then also I amperes. The effective field current (see Article 58) is

$$\overline{OG} = (1-A)I \qquad (12)$$

where A is the armature demagnetizing factor. Note that this factor A is independent of the speed; see Article 59.

Referring to Fig. 94, find the ordinate $\overline{GC}$ of the saturation curve corresponding to the effective field current $\overline{OG}$. This ordinate $\overline{GC}$ would then be the back electromotive force of the motor were the speed equal to that at which the saturation curve is taken, namely n_0.

* In making this test the brushes should be set in their normal position for motor operation.

The actual speed, however, depends upon the voltage impressed across the motor terminals. Let this voltage be V. Corresponding to any value I of the line current the actual back electromotive force will then be

$$E = V - (R_a + R_s)I \tag{13}$$

where R_a is the resistance of the armature and R_s the resistance of the field winding. The actual speed n must then be in the same

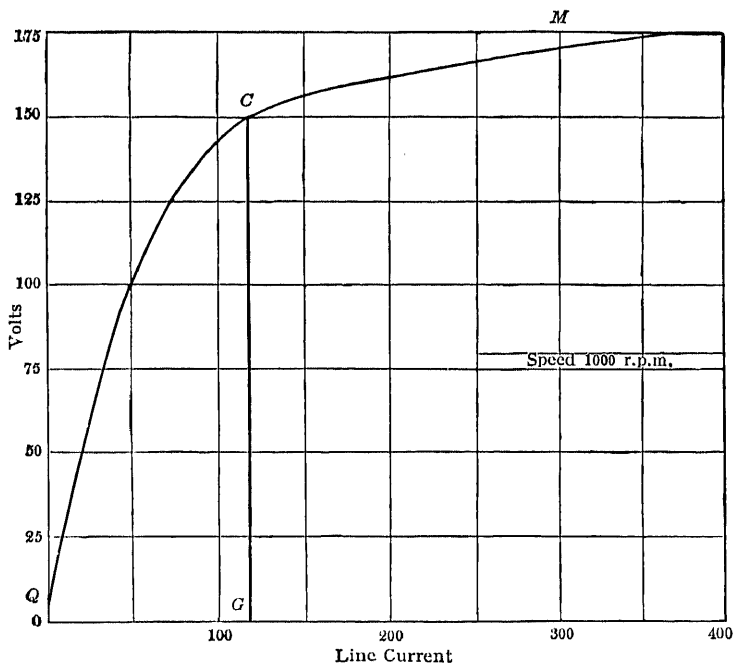



FIG. 94.—Saturation Curve of Series Motor.

ratio to the speed n_0 as the actual electromotive force E. is to the ordinate $\overline{GC}$. That is, the actual speed is

$$n = n_0 \cdot \frac{E}{\overline{GC}} = n_0 \cdot \frac{V - (R_a + R_s)I}{\overline{GC}} \tag{14}$$

Example.—Let the saturation curve shown in Fig. 94 correspond to a constant speed of 1000 revolutions per minute. Let the armature resistance be 0.07 ohm, the series-field resistance,

0.05 ohm, and the armature demagnetizing factor 0.02. Let the voltage impressed on terminals of the machine, when operated as a motor, be 125 volts. Then, when this motor is taking a current of 150 amperes, the effective field current is

$$\overline{OG} = (1-0.2)\times 150 = 120 \text{ amperes}$$

From Fig. 94, the back electromotive force corresponding to a speed of 1000 revolutions per minute is 151 volts. The actual back electromotive force, however, is $125-(0.07+0.05)\times 150=107$ volts. Hence the actual speed is

$$n = 1000\,\frac{107}{151} = 702 \text{ r.p.m.}$$

From the construction just described it is evident that the smaller the current taken by the motor, the greater will be the speed. For example, when the current taken by the particular motor under consideration is 10 amperes, the effective field current is 8 amperes, and the corresponding ordinate of the magnetization curve is 25 volts. The actual back electromotive force, however, is 123.8 volts. Hence the speed corresponding to a current of 10 amperes would be 4950 revolutions per minute.

A small current corresponds to a small power input, which in turn means a small opposing torque. It is therefore evident that if the torque opposing the rotation of a series motor is small, say only that due to the friction and windage of the motor itself, the armature may reach such a high speed that it may destroy itself by centrifugal action.

In Fig. 95 is shown the speed-characteristic of the particular motor under consideration, for all values of the line current up to 400 amperes. The shape of this curve is typical of all series motors.

From equation (9), the total torque exerted by the magnetic field on the armature of a motor, when the armature current is I_a and the resultant flux per pole is ϕ_r, is

$$T = 7.04\,K\phi_r I_a \qquad (15)$$

From equation (6), the back electromotive force corresponding to the resultant flux ϕ_r and the speed n_0 is $K\phi_r n_0$. But this electromotive is also equal to the ordinate $\overline{GC}$ of the saturation

curve (Fig. 94) corresponding to the effective field current $I_a = OF$. Hence

$$\overline{GC} = K\phi_r n_0 \qquad (16)$$

Substitute for $K\phi_r$ in equation (15) its value from equation (16); there results for the total torque T the expression

$$T = \frac{7.04}{n_0}\overline{GC} \times I_a \qquad (17)$$

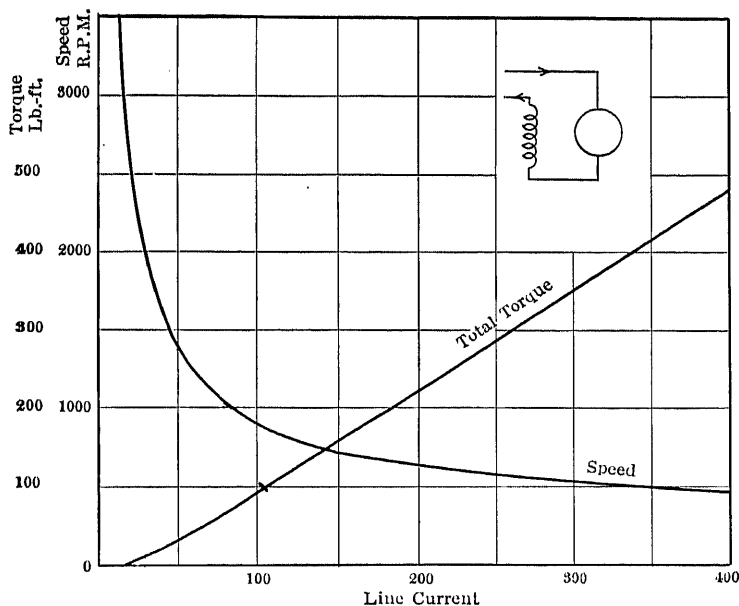



FIG. 95.—Speed and Torque Characteristics of Series Motor.

Hence, to find the total torque developed by a series motor when it is taking a current of I amperes, first find from the saturation curve, determined at a speed of n_0 revolutions per minute, the ordinate $\overline{GC}$ corresponding to the effective field current $\overline{OG} = (1 - A)I$. The total torque in pound-feet is then equal to the product of $\overline{GC}$ by the line current I, multiplied by the factor $\frac{7.04}{n_0}$.

The reader should bear in mind that this total torque is greater than the useful torque available at the motor shaft, by an amount

(usually small) equal to the torque required to overcome the friction and windage of the armature and the losses due to hysteresis and eddy currents. The torque required to overcome friction and windage increases with the speed, and the torque required to overcome the core-loss increases both with speed and with the degree of saturation of the magnetic circuit (see Articles 16 to 18).

In Fig. 95 is shown the torque-current characteristic, calculated as just described, for the particular motor to which the magnetization curve in Fig. 94 applies. For example, for a line current of 300 amperes, the effective field current is $(1-0.2)\times 300=240$ amperes, and the corresponding value of $\overline{GC}$ is 166 volts. Hence, the torque corresponding to 300 amperes is

$$\frac{7.04\times 166\times 300}{1000}=350 \text{ pound-feet}$$

The useful, or output, torque is a curve of the same general shape as that for the total torque, but will lie slightly below the latter.

An inspection of Fig. 95 will show that the torque-current characteristic of a series motor is approximately a straight line, except for small values of the torque. For these low values of torque, corresponding to small values of the current, the flux is practically proportional to the current (see Fig. 94), with the result that, for small currents, the torque varies as the square of the current; see equation (9).

75. Speed and Torque Characteristics of the Shunt Motor.—The same general method of procedure as just described may also be used to determine the speed and torque characteristics of a shunt motor. The saturation curve of a shunt motor is shown in Fig. 96. This curve is determined by driving the armature at constant speed (by a second motor, for example), and separately exciting the shunt field.

Let n_0 be the speed at which the saturation curve is determined, and let n be the actual speed at which the armature rotates when the total current supplied to the motor is I amperes. Let V be the impressed voltage and R_f the resistance of the shunt-field circuit (including the shunt-field rheostat). The actual field current is then

$$I_f=\frac{V}{R_f} \tag{18}$$

and the armature current is

$$I_a = I - I_f$$

Let A be the armature demagnetizing factor, determined in the same manner as for a shunt generator (Article 59). Then the effective field current is

$$\overline{OG} = I_f - AI_a \tag{20}$$

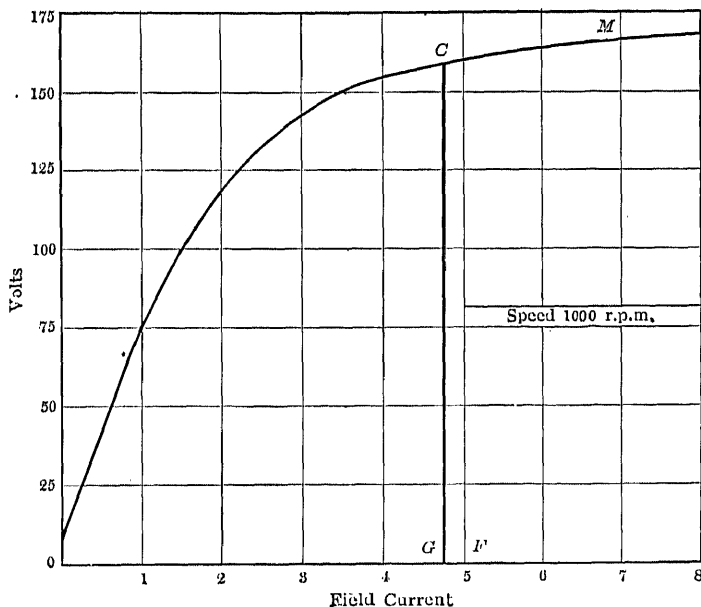



FIG. 96.—Saturation Curve of Shunt Motor.

The ordinate $\overline{GC}$, Fig. 96, would then be the back electromotive force of the motor were the speed equal to that at which the saturation curve is taken, namely n_0.

However, just as in the case of the series motor, the actual back electromotive force is

$$E = V - R_a I_a \tag{21}$$

where R_a is the armature resistance. Hence, the actual speed n must be to the speed n_0, as the actual back electromotive force E, equation (21), is to the ordinate $\overline{GC}$, viz.,

$$n = n_0 \cdot \frac{V - R_a I_a}{\overline{GC}} \tag{22}$$

Example.—The saturation curve shown in Fig. 96 corresponds to a constant speed of 1000 revolutions per minute. The armature resistance is 0.1 ohm, the armature demagnetizing factor 0.003. Let the voltage impressed across the terminals of the machine, when operated as a motor, be 125 volts, and let the field rheostat be set to give a field current of 5 amperes (resistance of shunt-field circuit 25 ohm). For a constant impressed voltage the field current will be constant, independent of the line current.

For a line current of 100 amperes the armature current is then $100-5=95$ amperes. The effective field current $\overline{OG}$ is

$$5.0-0.003\times95=4.71$$

and the ordinate $\overline{GC}$ is 159 volts. The actual back electromotive force is $125-0.1\times95=115.5$ volts. The actual speed of the motor is then

$$n=1000\,\frac{115.5}{159}=726 \text{ r.p.m.}$$

For a line current of 10 amperes, the armature current is 7 amperes, and the effective field current $5.0-0.003\times5=4.98$ amperes. The ordinate $\overline{GC}$ is then 161 volts. The actual back electromotive force is $125-0.1\times5=124.5$ and therefore the speed is

$$n=1000\,\frac{124.5}{161}=774 \text{ r.p.m.}$$

The speed curve in Fig. 97 gives the speed of this particular shunt motor, for an impressed voltage of 125 volts and field current of 5 amperes, for all values of the line current up to 400 amperes. As shown by this curve, the speed of the motor is a maximum at no load, but falls off as the load increases. However, compared with a series motor, the decrease of speed with increase of load is relatively slight. In other words, a shunt motor is an approximately constant-speed machine.

The particular motor to which the curves in Fig. 97 apply has a relatively high armature resistance. The less the armature resistance the more nearly constant will the speed be; see equation (11*a*). However, the speed-current characteristic shown in Fig. 97 is typical, as far as its general shape is concerned, of that of the usual shunt motor.

From equation (11a) it is evident that should the armature reaction be sufficiently large to cause the resultant flux ϕ to decrease faster than the back electromotive force $(V-R_aI_a)$, the speed of the motor would *increase* with increase in load. This condition is sometimes encountered in non-interpole shunt motors when the brushes have a large backward lead.

The torque corresponding to any value I_a of the armature current of a shunt motor is determined from the saturation curve

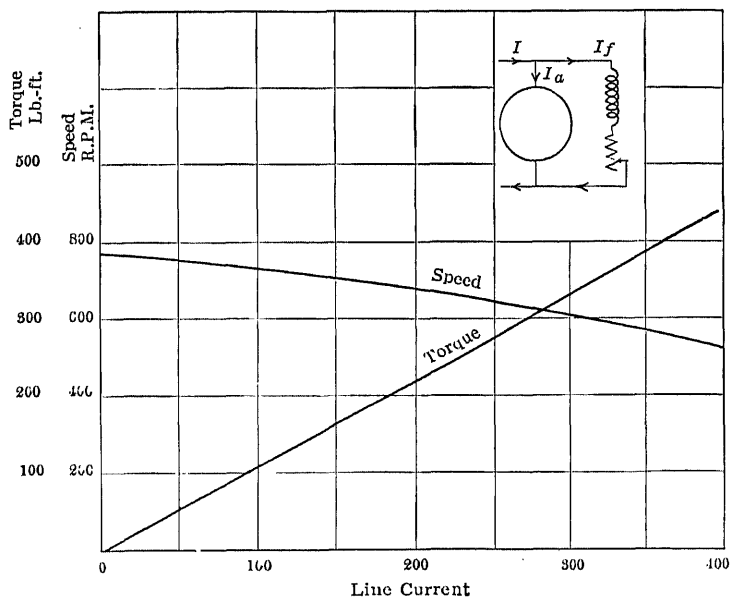



FIG. 97.—Speed and Torque Characteristics of Shunt Motor.

in exactly the same way as in the case of a series motor. That is, referring to Fig. 96, the total torque in pound-feet corresponding to the current I_a is

$$T=\frac{7.04}{n_0}\overline{GC}\times I_a \tag{23}$$

where $\overline{GC}$ is the ordinate corresponding to the effective field current $\overline{OG}=I_f-AI_a$, and n_0 is the speed at which the saturation curve is determined.

For example, for the particular shunt motor to which the saturation curve in Fig. 96 applies, the ordinate $\overline{CG}$ for a line

current of 100 amperes is 159 volts. Hence the total torque corresponding to this line current is 106 pound-feet. The complete torque-current curve determined in this manner is shown in Fig. 97. The torque-current characteristic is slightly concave downward, but the curvature is so slight that the curve is very close to a straight line. This is typical of the torque-current characteristic of all ordinary shunt motors.

The reader should not lose sight of the fact that the above construction gives the total torque, which is slightly in excess of the useful torque, on account of friction, windage and core loss; see Article 71.

76. Speed and Torque Characteristics of a Compound Motor.—The speed and torque characteristics of a compound motor may be derived from the saturation curve of the machine in a manner similar to that employed for a shunt motor. Consider, for example, a long-shunt **cumulative-compound motor** (Fig. 98). For a constant impressed voltage V the shunt-field current I_f will be constant, just as in the case of a shunt motor. Let I be the line current, A the armature demagnetizing factor, and A_s the series-field magnetizing factor. (A_s is the ratio of the number of turns in the series-field winding to the number of turns in the shunt-field winding.) The armature current will then be:

$$I_a = I - I_f \tag{24}$$

and the effective field current

$$\overline{OG} = I_f + (A_s - A)I_a \tag{25}$$

Referring to Fig. 96, let the saturation curve there shown be that of the compound motor under consideration, determined at a speed of n_0 revolutions per minute, with brushes in normal position for motor operation, and with the shunt field separately excited. Then the back electromotive force corresponding to an effective shunt field-current $\overline{OG}$, were the machine operating as a compound motor at a speed of n revolutions per minute, would be the ordinate $\overline{GC}$. The back electromotive force corresponding to the actual speed n, however, is

$$E = V - (R_a + R_s)I_a \tag{26}$$

The actual speed of the motor is then

$$n = n_0 \cdot \frac{\overline{V} - (R_a + R_s) I_a}{\overline{GC}} \qquad (27)$$

and the total torque developed is

$$T = \frac{7.04}{n_0} \overline{GC} \times I_a \qquad (28)$$

In Fig. 98 are shown the speed and torque characteristics, calculated as just described, for the same motor as considered in

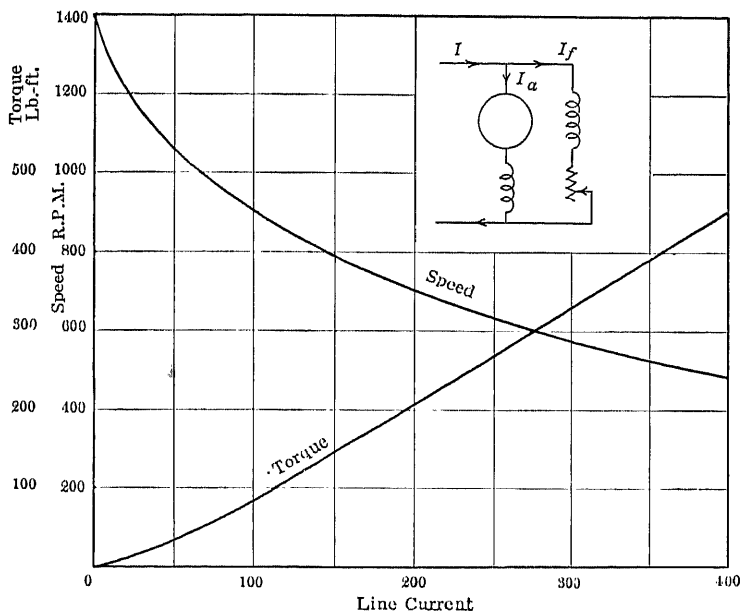



FIG. 98.—Speed and Torque Characteristics of Cumulative-compound Motor.

the last article, assuming the machine to be equipped with a series-field winding, connected long-shunt to aid the shunt field, and the resistance of the shunt-field circuit to be increased from 25 to 100 ohms. The resistance of the series-field winding is taken as 0.2 ohm and the ratio of the series turns to the shunt turns is taken as 0.013. The armature resistance is 0.1 ohm and the armature demagnetizing factor 0.003. The line voltage is 125 volts. For these conditions the actual shunt-field current

is 1.25 amperes, the armature current for any value I of the line current is $I_a = I - 1.25$, and the effective field current is

$$\overline{OG} = 1.25 + 0.01\ I_a$$

Comparing Fig. 95 (series motor) with Fig. 98 (cumulative compound motor), it is seen that the speed-torque characteristic of the cumulative compound motor is similar to that of a series motor, with the important difference that the no-load speed of the cumulative compound motor is relatively much lower. As noted in Article 70, the cumulative compound motor is therefore particularly suited to a varying-speed service where the mechanical load may at times be thrown off.

Differential-compound motors are seldom used in practice. However, should the series winding of a cumulative-compound motor be accidentally connected in the wrong way, or should the brushes of an commutating-pole shunt motor accidentally be shifted backward from the neutral position (thereby causing the interpole winding to produce a demagnetizing effect), the motor will operate as a differential-compound motor.

One peculiarity of a differential-compound motor, which is sometimes encountered when the brushes of an commutating-pole motor are accidentally given a backward lead, is that, when the motor is connected to the supply mains, it may start to rotate in the wrong direction, then stop, start up again in the right direction, stop again, reverse, and so on indefinitely.

This peculiar action is due to the low self-inductance of the series winding (or commutating-pole winding) compared with that of the shunt winding. When the voltage is first impressed on the terminals of the motor, the current in the series winding builds up faster than that of the shunt-field current, so that at the start the series ampere-turns completely overbalance the shunt ampere-turns thus producing a resultant flux in the wrong direction, and therefore a torque and rotation in the wrong direction. However, in a very short time, the shunt-field current will build up to its normal value. The resultant flux, and therefore the driving torque, will then reverse, and the armature will come to rest and start to rotate in the opposite direction.

As the armature slows down and reverses, the back electromotive force becomes zero, the armature current may therefore

rise to a very high value. This large armature current, which, on account of the self-inductance of the series winding, may not reach its maximum value until after the direction of rotation has reversed, may again cause the series ampere-turns to overcome the effect of the shunt-field ampere-turns, with the result that a second reversal of rotation may occur, and so on.

77. Starting of Motors.—When the armature of a motor is at rest, there is of course no back electromotive force of rotation. Consequently, should a motor, whose armature is at rest, be connected directly to the supply mains, the initial rush of current through its armature will be limited only by the resistance and self-inductance of the armature and series-field winding (if any). This current may reach a value many times greater than the normal full-load current of the machine, and serious sparking or flashing at the commutator may result.

For example, a 220-volt shunt motor designed for a normal full-load current of 100 amperes will have an armature resistance of the order of 0.1 ohm. Neglecting the effect of the self-inductance of the armature, the initial rush of current into the armature, should this motor be suddenly connected to 220-volt mains, would be 2200 amperes, or 22 times the normal full load current. Of course, just as soon as the armature begins to turn over, the back electromotive force developed will immediately tend to reduce this current, but serious damage would probably occur before the current is reduced to a safe value.

This initial rush of current may be readily prevented by inserting in series with the armature a variable resistance, as indicated in Fig. 99. This resistance is gradually cut out of circuit as the armature speeds up (by moving the contact C in Fig. 99 to the left). The starting rheostat should as a rule have a maximum resistance (all coils in circuit) not less than twice the line voltage divided by the rated full-load armature current.

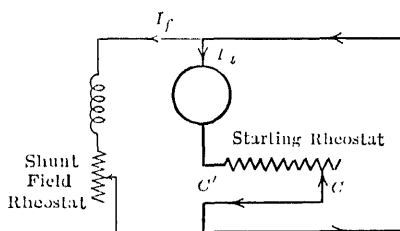



FIG. 99.—Correct Connection for Starting Rheostat.

When it is desired to start a motor which is permanently connected to its mechanical load, it is necessary that the motor develop a relatively high starting torque. In the case of a shunt motor, therefore, it is *important that the starting rheostat be connected in series with the armature only*, as shown in Fig. 99, and not in series with the line, as shown in Fig. 100. Should the starting rheostat be connected as shown in Fig. 100, it is evident that the voltage across the shunt-field circuit will be only a fraction of the line voltage (on account of the voltage drop in the starting rheostat). Consequently, the shunt-field current, and therefore the main flux, will be only a fraction of their normal values. The torque developed, being proportional to the flux, may therefore be insufficient to start the armature against the opposing torque of the load to which it is mechanically connected.

FIG. 100.—Incorrect Connection for Starting Rheostat.

78. Starting Boxes for Shunt Motors.—Should the supply circuit to a motor be interrupted while the motor is running, by the opening of the circuit-breaker at the generator, for example, the motor will stop. Under running conditions all the starting resistance is normally out of circuit, i.e., the contact C in Fig. 99 is at C'. Hence, should voltage again be impressed on the line, by closing the circuit-breaker at the generator, there will be a rush of current into the armature, which may again open the circuit-breaker or damage the machine.

To prevent this action, it is usual practice to employ a **no-voltage release** in connection with the starting rheostat. The starting resistance and no-voltage release are usually mounted together, forming a so-called "starting box," a simple form of which is shown in Fig. 101. The coils or grids which form the resistance are enclosed in a metal box with a slate top, on which are mounted contacts connected to the several sections of the resistance, a suitable spring-controlled arm D, a small electromagnet M, called the **hold-up magnet,** and the necessary terminals A, F and L.

When the arm D is moved to the right, the first connection is to put the shunt-field circuit, including the hold-up magnet M, directly across the line. When the arm is moved over to the running position (all the way to the right), it is held there by the attraction between the hold-up magnet and the soft iron plate P on the arm D. Should the voltage go off the line, or should the shunt-field circuit be accidentally interrupted, the hold-up magnet becomes de-energized, and the arm D is carried back to the off position by the spring S.

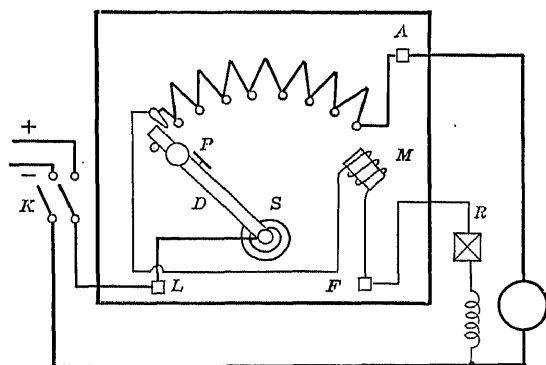



FIG. 101.—Three-point Starting Box.

The starting-box terminals are usually marked L, F and A as indicated in Fig. 101.

To connect the motor and starting box to the line, proceed as follows:

(1) Connect the common terminal of the armature and shunt field directly to one of the free terminals of the switch K.

(2) Connect the other free terminal of this switch to the starting-box terminal L.

(3) Connect the other terminal of the shunt field to one terminal of the shunt-field rheostat R and the other terminal of R to the starting-box terminal F.

(4) Connect the other terminal of the armature to the starting-box terminal A.

In addition to a no-voltage release, some starting boxes are also provided with an **over-load release.** The over-load release is merely an electromagnet whose winding is in series with the line,

and so adjusted that when the line current exceeds a predetermined value, the plunger of this magnet releases the rheostat arm and allows it to be pulled back (by the spring) to the starting position.

In Fig. 102 is shown one type of starting box equipped with both no-voltage release and over-load release. In this particular type of box, the no-voltage release is connected directly across the line, in series with a high resistance r, instead of being in series with the shunt field. The plunger of the over-load release T, should the load on the motor become excessive, is drawn up and

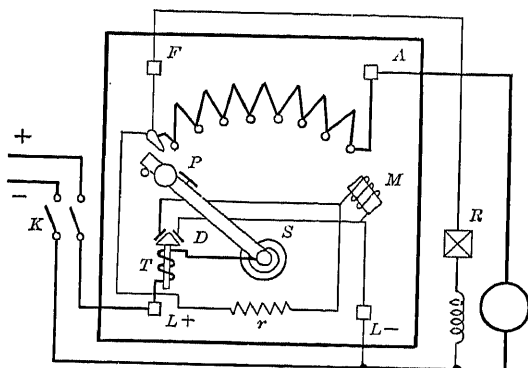



Fig. 102.—Four-point Starting Box.

short-circuits the winding of the hold-up magnet M, thereby releasing the arm D which is pulled back to the off position by the spring S.

This type of starting box, with or without the over-load release, is always used with variable speed motors having field control (see next Article). The three-point type of box shown in Fig. 101 is not suitable when the field current is varied over a wide range, for should the field current be reduced to a relatively low value (to give high speed), the pull of the hold-up magnet might not be sufficient to hold the arm in the running (right-hand) position against the pull of the spring S.

The wire or grids which form the resistance of a starting box are usually designed to carry the starting current *only for a relatively short period of time.* The arm of a starting box should therefore never be held permanently in any position between the

on and off positions, as otherwise a "burn-out" may occur, i.e., the heat developed by the current may melt the conductor which forms its path.

79. Starting Boxes and Controllers for Series Motors.—For stationary series motors starting boxes similar to those used for shunt motors are employed. The hold-up magnet constituting the no-voltage release, if provided, is connected directly in series with a high resistance directly across the line.

Should a series motor be employed under conditions where there is a possibility of the load dropping to such a low value that the speed may become excessive, a starting box with **no-load release** is used. This no-load release is simply a hold-up magnet, similar in construction to that shown in Fig. 101, except that it has a winding of a relatively few heavy turns connected in *series* with the motor. Should the motor current fall below the value corresponding to a safe speed, the hold-up magnet becomes partially de-energized and releases the rheostat arm, which is pulled back by spring tension to the off-position.

Fig. 103.—Platform Controller for Railway Motor.

The starting rheostat for the motors on a trolley car consists of a set of resistance grids, or resistors, placed under the body of the car, with a controlling device, called a controller, mounted on the car platform. The handle of the controller (see Fig. 103) turns a drum on which are mounted a number of copper segments. As the handle is turned, these segments make the proper connections, through a set of stationary fingers, to cut the resistors in or out of circuit. A typical grid resistor is shown in Fig. 104.

The resistance units, or resistors, employed in connection with

railway motors are used not only as starting resistances, but also to maintain the speed of the car at whatever value may be desired. These units are therefore relatively much larger than in an ordinary starting box, in order that they may have sufficient radiating capacity to prevent overheating.

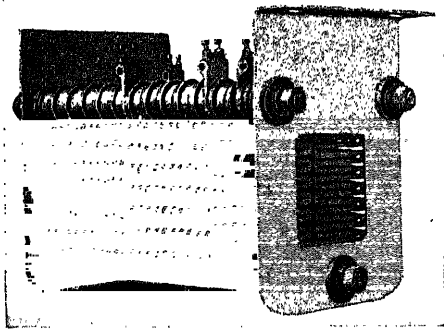



FIG. 104.—Typical Grid Resistor.

When a car is equipped with two motors, as is commonly the case, the controller is designed to perform the following sequence of operations:

(*a*) Connect the two motors in series with each other and with an external resistance R, as indicated at A in Fig. 105.

(*b*) Cut the resistance out, step by step, until the two motors are in series and directly across the line, B, Fig. 105.

(*c*) Connect the two motors in parallel with the resistance in series between the two motors and the line as indicated at C in Fig. 105.

(*d*) Cut the resistance out, step by step, until the two motors in parallel are directly across the line, D, Fig. 105.

At the instant of starting, the current taken by the two motors is limited only by the external resistance R, and the internal resistance of the two motors. As the car speeds up, a back electromotive force is developed in each armature, causing the current to decrease. The controller handle is then moved to the next notch, which cuts out some of the resistance R. The current then increases, producing an increase of torque, which causes the speed to increase. The back electromotive force of each motor therefore increases still more, the currents falls, and the controller handle is advanced another notch, and so on, until all the resistance R is cut out.

When all the resistance R is cut out, the voltage impressed on each motor is half the line voltage. If the controller handle is held on this notch, the car will run at half-normal speed, and

since there is no loss of power in the external resistance thi[s] be an efficient running position.

To obtain full speed, it is necessary to connect each motor directly across the line (full-line voltage across the terminals of each motor). If this were done in a single step, there would be a heavy rush of current, equal (per motor) to roughly half the line voltage divided by the internal resistance of one motor. Hence the external resistance *R* must again be connected in circuit, as shown at *C*, Fig. 105, and cut out step by step in the same manner as when passing from zero speed to half speed.

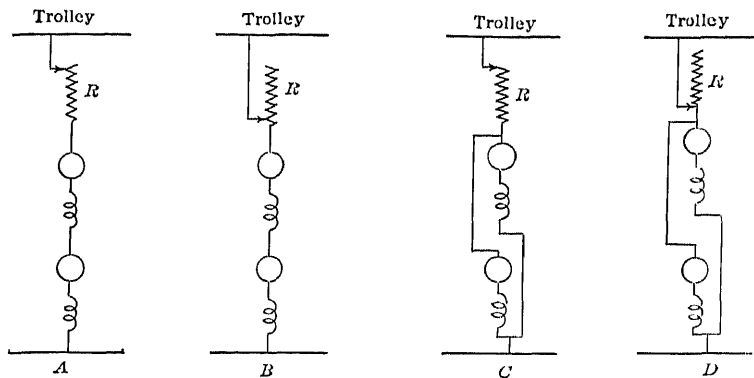



Fig. 105.—Sequence of Operations in Series-Parallel Control.

This method of starting and controlling the speed of two series motors is known as the **series-parallel method of control.** The loss of power in the external resistance *R* is much less when this method is used than it would be were the two motors permanently connected in parallel at the start. In the latter case, at half speed, half of the power supplied to the car would be wasted in the external resistance.

The current taken by a heavy interurban or suburban car is usually so large that a platform controller such as shown in Fig. 103 becomes too bulky to be practicable. Under such conditions, the connections between the resistance units, motors and trolley are usually made by solenoid-operated switches, called **contactors,** mounted under the car body alongside the resistors. To close any one of these contactor switches, a relatively small current,

about 2.5 amperes, is sent through the solenoid winding, which causes the plunger to rise and press the switch contacts together.

The sequence of closing and opening the contactors is controlled by a controller of the same general appearance as shown in Fig. 103, but very much smaller and of much simpler construction, since the maximum current it has to handle is only about 2.5 amperes.

When two or more cars are coupled together to form a train, the corresponding contactors on the several cars are connected in parallel, and are operated simultaneously from a single master controller on the head car. This is known as the **multiple-unit system** of control. The connections between the contactors on the several cars are made by means of a single cable formed of the necessary number of small wires (usually six), insulated from one another, and made continuous at the car couplings by plug and socket connectors.

80. Speed Control of Shunt Motors.—As shown in Article 72, the speed n of a motor is directly proportional to the back electromotive E, and inversely proportional to the resultant per pole ϕ_r. The back electromotive force E is always equal to the voltage V_a impressed on the armature terminals less the armature-resistance drop R_aI_a. These relations are conveniently expressed by the formula:

$$n = \frac{V_a - R_aI_a}{K\phi_r} \qquad (29)$$

where K is a constant whose value depends on the design of the motor.

As pointed out in Article 75, the armature resistance drop R_aI_a, and to a less extent the flux per pole ϕ_r, of a shunt motor vary with the load which the motor is supplying. Consequently a slight speed variation is inherent in a shunt motor. This inherent speed variation is usually referred to as the **speed regulation** of the motor, and should not be confused with the term **speed control**, which refers to a variation of speed produced by some external change in the conditions of operation.

From equation (29), it is evident that, to produce a change in the speed of the motor when it is supplying a given load, it is necessary to alter either (1) the voltage V_a impressed on the armature, or (2) the flux per pole ϕ_r.

The several methods of speed control may therefore be classified as follows:

Methods for varying armature terminal voltage:
(*a*) Armature-resistance control.
(*b*) Multivoltage system.
(*c*) Ward-Leonard system.

Methods for varying flux per pole:
(*d*) Field rheostat control.
(*e*) Field-reluctance control.

(*a*) **Armature Resistance Control.**—One method of varying the voltage across the armature terminals of a shunt motor is to place a variable resistance R_a' in series with the armature, as shown in Fig. 106. For a given opposing torque, the armature current will then have a fixed value I_a, see Article 71, and the corresponding armature terminal voltage will then be $V_a = V - R_a' I_a$, where V is the line voltage. By varying the resistance R_a', the motor may be made to run at any desired speed for this particular value of the opposing torque.

Fig. 106.—Armature-resistance Control.

There are, however, two serious objections to this method of speed control, namely, (1) the power $R_a'I_a^2$ dissipated as heat in the resistance R_a' is wasted, and (2) the speed regulation of the machine becomes worse the greater the value of R_a'.

For example, to reduce the speed at full load to one-half its normal value, the resistance R_a' must have such a value that $R_a'I_a$ is approximately equal to one-half the line voltage. Under these conditions, as much power would be dissipated in the resistance R_a' as is taken to drive the motor.

As to speed regulation, should the load on the motor be suddenly thrown off, the resistance drop due to R' becomes practically negligible (armature current practically zero), and the speed will therefore rise to approximately twice the value it would have at full load with the resistance R_a' in circuit. The speed-torque

characteristic under these conditions is practically a straight line, the speed falling off approximately 50 percent from no load to full load.

This method of speed control by armature resistance is therefore seldom used in industrial service, although it is sometimes the simplest method of securing the desired range of speed for testing purposes.

(*b*) **Multivoltage Control.**—A direct method of providing several different voltages which may be impressed on the armature of a motor, is to employ a multi-wire distributing system with different voltages between the various pairs of mains, as indicated in Fig. 107. These several voltages may be obtained from a

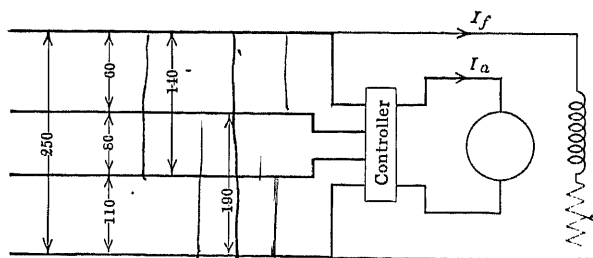



Fig. 107.—Multivoltage Control.

"balancer set" (see Article 83) located in the power house and operated in connection with a single generator.

From the four-wire distributing system shown in Fig. 107 six different voltages may be obtained, namely 60, 80, 110, 140, 190 and 250. By means of a suitable controller any one of these voltages may be connected across the armature terminals. The shunt field is permanently connected across one pair of mains. For any one of the six voltages impressed on the armature, the speed of the motor will remain approximately constant. There will then be six different speeds, each approximately constant, these six speeds being proportional to the six voltages.

The disadvantage of this system is its complication and high initial cost. With modern interpole motors, an equal, or even greater, range of speed can be obtained by the much simpler and cheaper method of field-rheostat control described in paragraph (*e*).

(*c*) **Ward-Leonard System of Control.**—In this system of control, the shunt field of the motor is connected across the mains, but the armature is supplied from a motor-generator set $M'G'$, connected as shown in Fig. 108. The motor of the motor-generator set is a shunt motor, connected to the supply circuit in the usual manner. The generator of this set is separately excited from the supply mains, the field circuit containing a reversing switch *S* and rheostat, for reversing and adjusting the voltage supplied to the armature of the main motor.

With this arrangement close adjustment of the speed of the main motor may be secured over a wide range. This system was formerly used for the operation of the turrets of battleships and

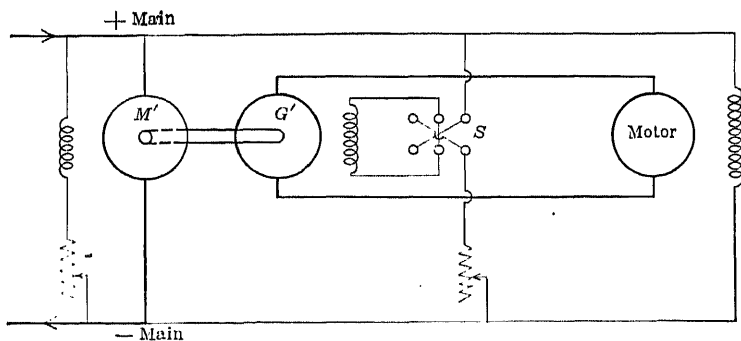



FIG. 108.—Ward-Leonard System.

for the operation of electricially controlled rudders. It is still used to some extent for the control of large hoist motors, and in other special cases requiring close adjustment of speed over a very wide range. However, on account of the necessity of providing three full-sized machines instead of one, it is a relatively expensive system of speed control, and is used only where the required range and closeness of speed adjustment cannot be secured by a less costly method.

(*d*) **Field Rheostat Control.**—The simplest and cheapest way of controlling the speed of a shunt motor is to vary the flux per pole by means of the field rheostat. To increase the speed, resistance is inserted in the field circuit, which reduces the field current and therefore the flux; see equation (29).

With a non-interpole motor, the maximum range in speed which

it is practical to obtain by this method is about 3 to 1. The range is limited by commutation difficulties at high speed, due to the reduction in the flux from the trailing pole tip.

As pointed out in Article 50, the commutation electromotive force in a motor is proportional to the resultant flux density under the trailing pole tip, which flux density is due to the differential action of the field ampere-turns and the armature ampere-turns. When the field current is reduced to a relatively low value (to give high speed), the commutating electromotive force may therefore become too small to produce the proper rate of change of the current in the coils undergoing commutation, or may even reverse in direction, and sparking will result.

With commutating-pole motors, however, it is entirely practicable to obtain a range of speed of 5, or even 6, to 1 by varying the resistance of the field rheostat. In such motors the commutating electromotive force is independent of the field current, being produced by a flux set up by the armature current itself.

For industrial applications, a variable speed motor is usually provided with a combination starting box and field rheostat, so that the same handle is used both for starting and for controlling the speed. This combination starting box and field rheostat, usually referred to as a controller, may also be designed to make the proper connections for operating the motor in either direction.

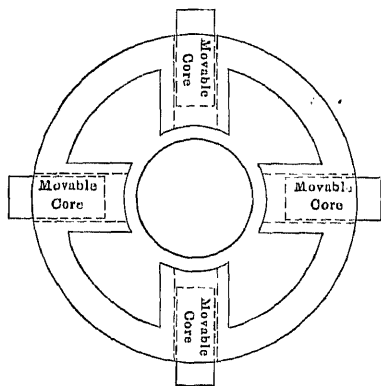



Fig. 109.—Diagram of Stow Variable-speed Motor.

(*c*) **Field-Reluctance Control.**—The flux per pole of a dynamo, neglecting armature reaction, is equal to the magnetomotive force due to the field current, divided by the reluctance of the magnetic circuit of the machine. Hence the flux per pole can be varied not only by varying the field current, but also by varying the reluctance of the path of the flux.

In the Stow motor, each pole is made up of two parts, a station-

ary outer shell fastened rigidly to the field yoke, and a movable cylindrical core which fits into this shell, as indicated diagrammatically in Fig. 109. The cores are geared together at their outer ends (by means of bevel gears and cross rods), so that they may all be moved simultaneously in or out by means of a handwheel. The further away from the armature the cores are moved, the higher is the speed. As the cores are moved outward, the flux density in the air-gap is reduced only under the central portions of each pole face. The flux density at the pole tips actually increases, due to the tendency of the lines of force to crowd into the stationary iron

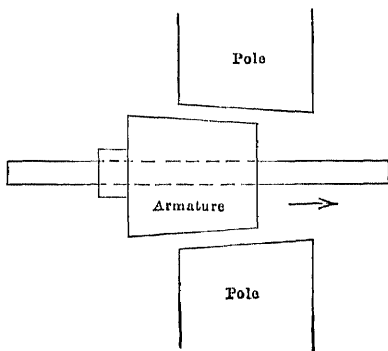



Fig. 110.—Diagram of Reliance Variable-speed Motor.

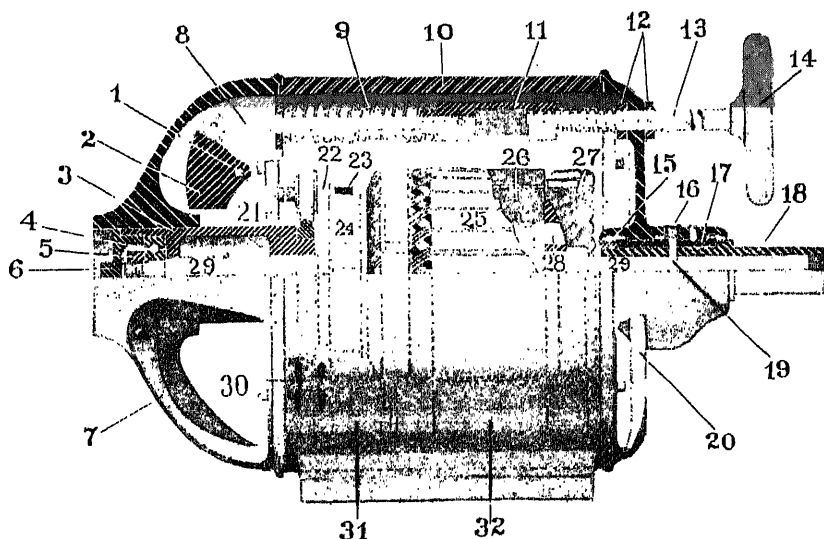



Fig. 111.—Reliance (Lincoln) Variable-speed Motor.

shell. Commutation is therefore perfectly satisfactory over a wide range of speed.

In the Reliance adjustable-speed motor (formerly known as the Lincoln motor), change in the reluctance of the magnetic circuit is obtained by moving the armature axially, as indicated diagrammatically in Fig. 110. The details of the mechanical construction employed are shown in Fig. 111. Referring to Fig. 110, when the center of the armature is directly under the center of the poles, the effective cross-section of the air-gap is a maximum and the length of the air-gap a minimum; the flux is therefore a maximum and the speed has its lowest value. When the armature is moved to the left, the effective cross-section of the air-gap decreases and its length increases (due to the taper on the armature and poles); the flux therefore decreases, and the speed increases.

This type of motor is built for various speed ranges, the lowest being about 5 to 1 and the highest 10 to 1. Commutation difficulties are avoided by the use of commutating-poles.

PROBLEMS

1. A certain 50-H.P., 550-volt railway motor has an efficiency of 90 per cent at rated load. The speed at rated load is 750 r.p.m. The armature has a 6-pole simplex wave winding of 240 conductors. The armature resistance (including brushes and brush contacts) is 0.32 and the resistance of the series field is 0.13 ohm.

Calculate, at rated load:

(*a*) The net output torque, in pound-feet.

(*b*) The combined core-loss and friction in watts, (i.e., the power loss due to hysteresis, eddy-currents, friction and windage).

(*c*) The opposing torque in pound-feet, due to the combined core-loss and friction.

(*d*) The resultant useful flux per pole.

(*e*) Assuming the combined core-loss and friction to be directly proportional to the speed (which is only a rough approximation), what would be the net output torque, in pound-feet, for rated current input at a speed of 1000 r.p.m.?

(*f*) What voltage would have to be impressed on the motor in order to make it run at a speed of 1000 r.p.m. and develop the same total torque as at rated speed and rated voltage.

(*g*) On the assumption that, for a given flux per pole, the combined core-loss and friction is proportional to the speed, how much more heat will be developed in this motor if operated at 660 volts instead of at rated voltage, the opposing torque being the same in each case?

2. A 220-volt shunt motor has an armature resistance (including brushes and brush contacts) of 0.1 ohm. This motor is connected to 220-volt mains,

the field rheostat has a fixed setting, opposing torque is constant at such a value that the armature current is 100 amperes. An external resistance of 1 ohm is now connected in series with the armature.

(*a*) What will be the percentage change in the total current taken by this motor from the supply mains.

(*b*) What will be the percentage change in the speed of the motor, and will the speed increase or decrease?

(*c*) Neglecting armature reaction and the armature resistance drop due to the no-load current, what will be the percentage change in speed when the mechanical load is thrown off, first, when there is no external resistance in the armature circuit, and second, when the external resistance is in this circuit?

3. (*a*) Calculate and plot the speed-current and torque-current characteristics of the series generator described in Problem 9 of Chapter VI, when this machine is operated as a motor on a 125-volt circuit. (The brushes are to be assumed set with a backward lead equal to this forward lead for generator operation.) Make calculations for at least six points between no load and a current input of 300 amperes.

(*b*) On this same sheet plot, against line current input as abscissas, the gross mechanical horse-power developed by the mutual action between the magnetic field of the machine and the armature current.

(*c*) Why is the horse-power thus calculated greater than the useful horse-power output of the motor?

4. (*a*) Calculate and plot the speed-current and torque-current characteristics of the shunt generator described in Problem 5 of Chapter VI when this machine is operated as a motor on a 250-volt circuit, with the resistance of the field circuit adjusted to give rated speed (720 r.p.m.) at no load. Make calculations for at least six points between zero load and a power input of 1500 kilowatts.

(*b*) On this same sheet plot, against line current as abscissas, the gross horse-power developed by the mutual action between the magnetic field of the machine and the armature current.

5. (*a*) Calculate and plot the speed-current and torque-current charactersities of the compound generator described in Problem 10 of Chapter VI when this machine is operated as a cumulative-compound motor on a 250-volt circuit, with the resistance of the shunt-field circuit adjusted to give rated speed (720 r.p.m.) at no load.

(*b*) What changes in connections, if any, will have to be made in the field windings of this machine in order to make it operate as a cumulative-compound motor?

(*c*) On the same sheet as the speed and torque curves plot, against line current as abscissas, the gross horse-power developed by the mutual action between the magnetic field of the machine and the armature current.

6. The series motor whose speed-torque characteristics (impressed voltage 125) are shown in Fig. 95 is permanently connected to a mechanical load which develops a constant opposing torque of 100 pound-feet. The combined armature and series field resistance of this motor is 0.12 ohm.

(*a*) What would be the initial current taken by this motor were it con-

nected directly across the 125-volt mains, neglecting the self-inductance of the armature and field windings?

(*b*) When the armature has been properly brought up to speed, and all external resistance cut out of circuit, at what speed will it run, and what will be the value of the current taken by the motor? This current will be referred to as the full-load current.

(*c*) How much external resistance would have to be inserted in series with this motor in order to limit the initial current (armature at rest) to a value equal to twice this full-load current?

(*d*) With this external resistance in series with the motor, what would be the voltage across the motor terminals?

(*e*) Up to what speed would the armature accelerate with this external resistance in series, the opposing torque being held constant at 100 pound-feet?

(*f*) How much of this external resistance may now be cut out of circuit, without causing the current to exceed twice its full load value?

(*g*) Up to what speed will the armature accelerate with this new value of the external resistance, the opposing torque being 100 pound-feet?

(*h*) Proceeding in the manner here indicated, determine the number of steps and the resistance of each step of the starting rheostat required for this motor, in order to bring the armature up to speed under load without having the current exceed at any time a value equal to twice its full-load value.

7. (*a*) The efficiency of a d.c. railway motor (including gears) is usually about 85 per cent at rated lead. A trolley car is equipped with two motors and series-parallel control. During the starting of this car, the average current *per motor* is equal to that corresponding to the motor rating. Determine the energy efficiency during the starting period, i.e., the ratio of the mechanical work developed at the car wheels to the electric energy input during this period.

(*b*) Determine the energy efficiency of this same car, during the starting period, assuming the motors to be permanently connected in parallel, and the starting-current *per motor* to be kept at the same average value as in (*a*), by straight resistance control. Compare these two efficiencies.

8. (*a*) How much external resistance must be inserted in series with the armature of the motor considered in the numerical example in Article 75, in order to reduce the speed to one-third its normal value, the opposing torque being constant at 100 pound-feet? The field rheostat is kept unaltered.

(*b*) What will be the current taken by the motor under these conditions.

(*c*) Plot the speed-current and torque-current curves when this external resistance is in series with the armature, and compare with Fig. 97.

(*d*) What is the per cent speed regulation corresponding to a torque of 100 pound-feet, both with and without this external resistance in the armature circuit.

9. (*a*) How much resistance must be inserted in the field circuit of the motor considered in the numerical example in Article 75, in order to increase the speed to three times its normal value, the opposing torque being constant at 100 pound-feet?

(*b*) What will be the value of the field current under these conditions?

(*c*) What will be the percentage change in the resultant useful flux per pole?

CHAPTER VIII

MISCELLANEOUS DIRECT-CURRENT MACHINES

81. Motor-Generators and Dynamotors.—A motor-generator, as the name implies, consists of two machines, a motor and a generator with their armatures mechanically coupled together. The motor may be either a direct-current or an alternating current motor, and the generator may likewise be either a direct-current or an alternating-current generator. Motor-generator sets have a wide variety of uses, such as transforming from direct to alternating current, or conversely; transforming from high-voltage direct current to low-voltage direct current, or conversely; transforming from alternating current at one frequency to alternating current at another frequency, etc.

A motor-generator set is usually the most satisfactory means of obtaining high-voltage direct-current energy from a low-voltage direct-current supply, or conversely. Since the magnetic circuits of the two machines are independent, there is no fixed relation between the voltage impressed on the motor and the terminal voltage of the generator. Irrespective of the value of the supply voltage, the terminal voltage of the generator element may be adjusted, by means of its field rheostat, to any desired value within the range of this rheostat.

The overall efficiency of a motor-generator set is relatively low, being equal to the product of the efficiencies of the two machines (when each is expressed as a fraction).

Another method of transforming direct-current from one voltage to another is to provide two armature windings on a common armature core which is arranged to rotate in a single field structure; see Fig. 112. This type of machine is called a **dynamotor.** The two armature windings are "sandwiched" between each other, and each is provided with a separate commutator and a separate set of brushes. One commutator is at one end of the armature and one at the other.

When the field winding and the brushes which make contact with commutator No. 1 are connected to a source of supply, the machine will operate as a motor, as far as the armature winding connected to this commutator is concerned. At the same time, due to the rotation of the second armature winding in the magnetic field of the machine, there will be an electromotive force generated in this winding. Consequently with respect to a load connected to the second set of brushes, the machine will operate as a generator, supplying current to this load.

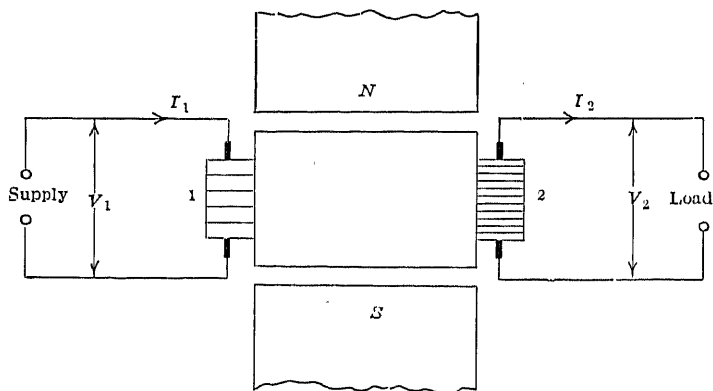



FIG. 112.—Diagram of Dynamotor.

Since both armature windings rotate in the same magnetic field at the same speed, the electromotive force developed *per conductor* in each winding will be the same. In the armature winding connected to the source of supply, this electromotive is a back electromotive force (opposing the current supplied to this winding), whereas in the second winding, this electromotive force is in the same direction as the current supplied by it to the load. The first winding may be referred to as the motor-winding and the second winding as the generator-winding.

Neglecting the losses in the machine, the power input V_1I_1 to the motor-winding must be equal to the power output V_2I_2 of the generator-winding, that is

$$V_1I_1 = V_2I_2 \tag{1}$$

Neglecting the resistance drop in the motor-winding, the

voltage V_1 impressed on this winding is equal to the total back electromotive force E_1 developed in it, which in turn is equal to the electromotive force e developed *per conductor* times the number of conductors Z_1 in this winding, viz., $V_1 = E_1 = Z_1e$. Similarly, neglecting the resistance drop in the generator-winding, the terminal voltage V_2 of this winding is equal to the electromotive force E_2 generated in it, which in turn is equal to the electromotive force e developed *per conductor* times the number of conductors Z_2 in this winding, viz., $V_2 = E_2 = Z_2e$.

Whence

$$\frac{V_2}{V_1} = \frac{Z_2}{Z_1} \tag{2}$$

Combining equations (1) and (2) there results

$$\frac{I_2}{I_1} = \frac{Z_1}{Z_2} \tag{3}$$

Consequently, neglecting the losses in the machine, the output voltage of a dynamotor is to the input voltage *directly* as the number of conductors in the two armature windings, and the output current is to the input current *inversely* as the number of conductors in the two armature windings. Since this relation is independent of the field excitation, it is evident that varying the field current of a dynamotor will produce no effect on the output voltage, but merely causes the speed of the armature to change.

The only way in which the output voltage can be altered is by varying the impressed voltage on the input side. It is on this account that a motor-generator set is usually preferable to a dynamotor, unless the amount of power to be converted is small (as in radio work), in which case the impressed voltage may be conveniently controlled by a rheostat in series with the source of supply.

82. Boosters.—A booster is a generator whose armature is connected in series with the source of supply, to increase or decrease the line voltage. Such a generator is usually driven by an electric motor supplied from the same source, in which case it is called a motor-booster. A motor-booster is therefore one form of motor-generator.

A booster may be either a series generator, a shunt generator or a compound generator. When both series and shunt fields are

provided they are usually connected so that they oppose each other, in which case the booster is called a **differential booster.**

A **series booster** is frequently used to compensate for the *RI* drop in a long heavily loaded feeder, in which case it is connected as shown in Fig. 113. The magnetic circuit of the machine is so designed that the flux density is relatively low (well below saturation), in which case the working portion of the saturation curve is practically a straight line. Under these conditions, the electromotive force generated by the booster is practically proportional to the current through its field winding, i.e., to the feeder current.

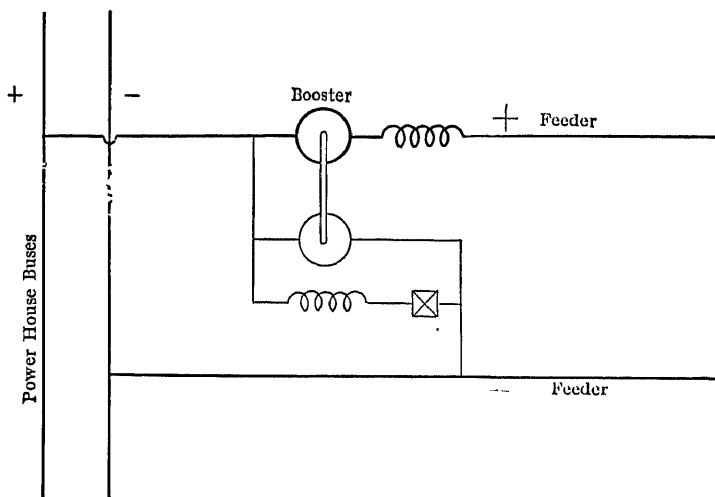



Fig. 113.—Series Booster.

It is therefore obvious that, if the series field is given the proper number of turns to produce an electromotive force equal to the *RI* drop in the feeder for any one value of the feeder current *I*, the voltage at the load end of the feeder will remain equal to the bus-bar voltage at the power-house, irrespective of the load supplied over this feeder.

When the fluctuations in the load on the feeder are very rapid, the entire magnetic circuit of the booster is sometimes laminated, in order to reduce to a minimum the eddy currents which the corresponding rapid variations of flux produce in the iron.

Boosters are also frequently used to reduce the voltage drop

in the track-return circuit of electric railways, and thereby to reduce the electrolytic action produced by the leakage current from the track rails. When used for this purpose, the booster is connected in series with a copper feeder which is run from the negative bus in the substation to some point of the track at a considerable distance from the substation, as shown in Fig. 114.

A booster used in this manner is called a **negative booster.**

The action of a negative booster may be seen by considering the simple case, in which all the cars on the track are at a distance from the substation greater than that of the point P

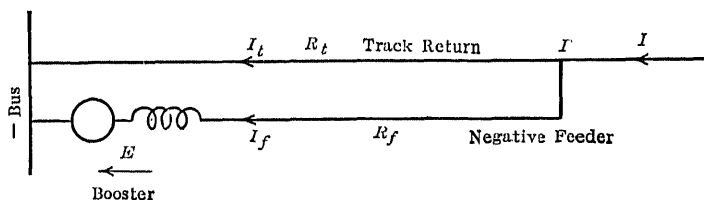



FIG. 114.—Negative Booster.

at which the negative feeder is connected to the track. Let the total return current at P be I, let R_t and R_f be respectively the resistance of the track and the resistance of the feeder (including the resistance of the booster), and let E be the electromotive force of the booster. Applying Kirchhoff's Laws to the loop formed by the track and feeder, it is readily seen that the current in the track between P and the substation is

$$I_t = \frac{R_f I - E}{R_f + R_t} \quad (4)$$

Consequently, if the electromotive force of the booster is made equal to $R_f I$, there will be no current in the track, and therefore no drop of voltage between the point P and the substation. Under these conditions, all the current returns through the feeder.

In general, the cars on the track would not all be on one side of the point at which the feeder is connected, and the calculations would not be as simple as for the special case just considered. See the article on *Trolley Systems, Overhead* in Pender's *Handbook for Electrical Engineers.*

Boosters are also used **for the control** of the charge and discharge **of storage batteries.** For example, in an isolated plant

supplying light and power for an office building or hotel, it is often economical to use a storage-battery in parallel with the generator, letting the battery supply part of the load during the hours of heavy load, and letting the generator charge the battery during the periods of light load. Under these conditions, there is a reduction in the size of the generator required. The generator also operates constantly at or near full load, resulting in a higher efficiency. Another advantage of this arrangement is that, in case of damage to the generator, the battery can supply a part, at least, of the load.

The terminal voltage of a lead storage cell, when fully charged and floating on the line, i.e., neither supplying nor receiving current from the line, is about 2.2 volts. When the cell is discharging, its terminal voltage falls off, due to its internal resistance and to the chemical changes which take place within the cell at its plates or "poles." Both the resistance and polarization of the cell increases as the discharge goes on. When the voltage falls to 1.8 volts, at normal discharge rate, the cell is, from a practical standpoint, completely discharged. On the other hand, when a lead cell is being charged, its terminal voltage increases, reaching a value of about 2.6 volts when fully charged and when taking current at the normal charging rate.

The variation in the terminal voltage of a lead storage battery during a complete cycle of charge and discharge is therefore

$$\frac{2.6-1.8}{1.8}=0.45$$

or 45 percent of the lowest value.

Consequently, in order to operate a lead storage battery in parallel with a constant-voltage generator, it is necessary to provide some means whereby an additional electromotive force may be inserted in series with the battery, which, during discharge adds to the electromotive force of the battery, helping it to discharge into the supply mains, and which, during charge, opposes the battery electromotive force and aids the supply voltage in charging the battery.

In Fig. 115 is shown one method by which this result is accomplished. The booster armature is driven by a motor (not shown) connected across the supply mains. The field winding of the

booster is connected in series with the armature of a small motor-driven generator, or "exciter," E. The field winding of this exciter is connected across a low-resistance shunt R in series with one line wire.

The adjustments are so made that, for a predetermined value of the load current, the electromotive force of the exciter E is exactly equal and opposite to the potential difference between the supply mains. Under these conditions, there is no current through the field winding of the booster, and the terminal voltage of the battery is equal to the line voltage, and the battery neither charges nor discharges. When the load current increases, the electromotive force of the exciter over-balances the line voltage,

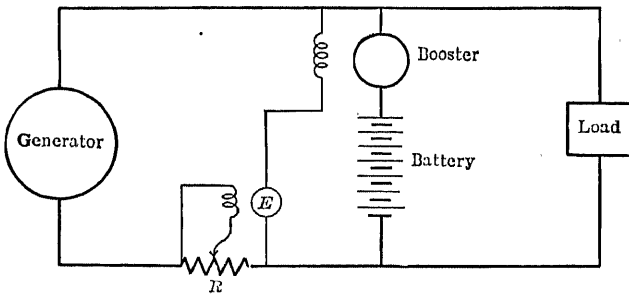



Fig. 115.—Booster Control of Storage Battery.

a current is established in the field winding of the booster in the proper direction to produce in the booster armature an electromotive force in the same direction as the battery electromotive force, and the excess load is supplied by the battery. Conversely, when the load current decreases, the line voltage over-balances the exciter electromotive force, a current is established in the field winding of the booster in the reverse direction, and current is forced into the battery from the line.

For other methods of controlling the electromotive force of a battery booster see Langsdorf, *Principles of Direct Current Machines.*

83. Balancers.—The weight and therefore the cost, of the conductors required to transmit a given amount of power at a given efficiency (or at a given drop in voltage) is inversely proportional to the square of the line voltage. On the other hand,

experience has shown that the most satisfactory voltage for incandescent lamps for interior use is from 100 to 120 volts. To secure the advantage of a higher transmission voltage, and at the same time keep the required lower voltage on the lamps, the three-wire system of distribution, shown schematically in its simplest form in Fig. 116, is commonly used.

At the generator end, a voltage equal to the required lamp voltage plus the line drop is maintained between each of the outside wires and the middle, or neutral, wire. Various methods for doing this are in use, the simplest (though seldom used) method is to connect two like generators in series, and to connect

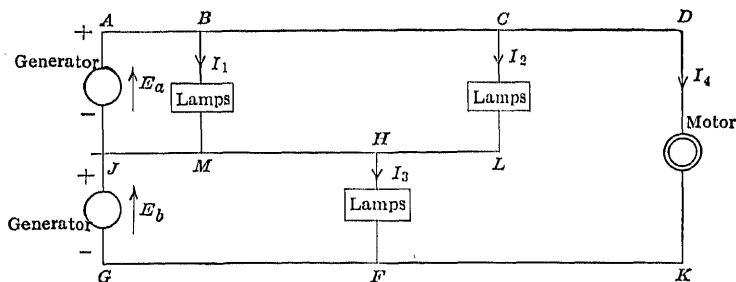



FIG. 116.—Three-wire System with Two Main Generators.

the neutral wire to the junction between the two generators. Except for the voltage drop in the neutral, the voltage impressed across the loads on one side of the system is then independent of the loads on the other side of the system. The two outer wires may also be used to supply 220-volt motors, by connecting the motors, as shown in Fig. 116, between the two outer wires, thereby making possible the use of more economical motors.

When the neutral wire is made the same size as each of the outer wires, as is usual practice, the amount of copper required, for a given total load and a given maximum variation in voltage at the lamps, is only three-eighths, or 37.5 percent, of that required for a two-wire system.

Of course, the cost of two generators, each having a capacity to supply half the total load, is greater than the cost of a single generator of the same total capacity. In order to reduce the cost of the generating equipment, the arrangement shown in Fig. 117 is commonly used, in which the loads on the two sides of the

system are balanced by a small motor-generator set, called a **balancer.** The two units of a balancer are exact duplicates.

As a rule, the several loads (groups of lamps or motors) can be so connected that the total load on one side will not differ by more than 15 percent from the load on the other side, i.e., so that the current in the neutral wire will not be more than 15 percent of the full load current of the main generator. The current through each unit of the balancer is always less than this neutral current. (Were there no losses, the balancer current would be one-half the neutral current). Hence, since the voltage across each of these units is one-half (approximately) of the voltage of

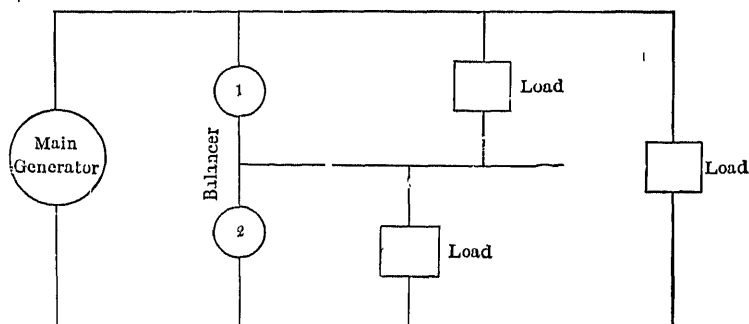



Fig. 117.—Three-wire System with Balancer.

the main generator, the balancer units are relatively very small machines, seldom having a rating greater than 5 percent of that of the main generator.

The action of a balancer set will be readily understood by reference to Fig. 118, in which the shunt-field windings of the two units are shown connected in series across the outer wires. When the load on the system is balanced, i.e., when there is no current in the neutral, the two units of the balancer will act simply as two shunt motors in series, without load. The speed of the set and the current through each armature will then be the same as would be produced by applying half line voltage to one unit by itself. Neglecting the armature resistance drop due to this no-load current, the electromotive force generated in each armature is then equal to one-half the voltage V between the two outer wires.

When the loads on the two sides of the system are equal, the voltage across each load (and therefore the terminal voltage of each unit of the set) is also equal half the voltage V between the outer wires. Should the load 1, say, be greater than the load 2 (equivalent resistance of 1 less than the equivalent resistance of 2), the voltage V_1 across 1 will decrease and the voltage V_2 across 2 will increase. The electromotive force of the balancer unit A, connected across the load 1, will then be in excess of the terminal voltage of A, and the electromotive force of the other unit B will be less than the terminal voltage of B. Consequently, A will act as a generator, supplying current to the heavily loaded

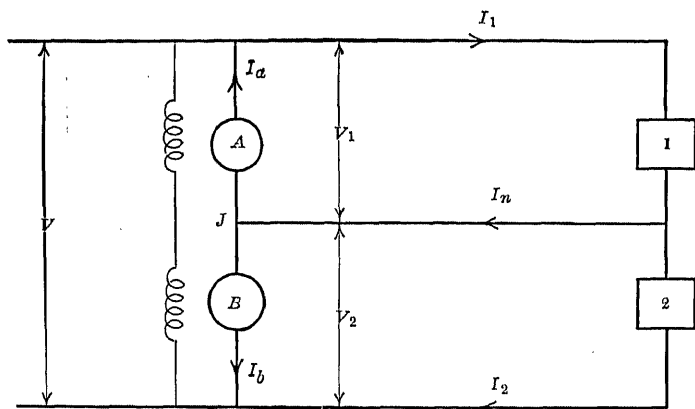



FIG. 118.—Shunt Balancer with Fields in Series.

side of the system, and B will act as a motor, and therefore put more load on the lightly loaded side of the system.

Since the two armatures are mechanically coupled and since, for the connection shown in Fig. 118, each has the same field excitation, the electromotive force generated in each armature would have the same value E, provided there were no armature reaction. Also, were the friction, windage and core-loss of each machine negligible, the mechanical output EI_b of the motor armature would be equal to the mechanical input EI_a to the generator armature, for the mechanical output of the set as a whole is zero. Consequently, under these assumptions, the currents I_a and I_b would be equal to each other, i.e., the current I_n in the neutral

would divide equally at the junction J, half going through the generator and half through the motor.

Let r be the resistance of the armature of each unit, including the brush contact resistance. Then, under the assumptions just stated, the voltage across the heavier load will be

$$V_1 = E - r\frac{I_n}{2} \qquad (5)$$

and the voltage across the lighter load would be

$$V_2 = E + r\frac{I_n}{2} \qquad (6)$$

Let V be the voltage between the two outer wires. Then

$$V_1 + V_2 = V$$

Therefore, adding (5) and (6), it is evident that under the assumption of no armature reaction and no friction, windage or core-loss, the generated voltage E of each unit would be equal to half the voltage V between the two outer wires, irrespective of the degree of unbalancing of the loads on the system, viz.,

$$E = \frac{V}{2} \qquad (7)$$

and therefore

$$V_1 = \frac{V - rI_n}{2} \qquad (8)$$

$$V_2 = \frac{V + rI_n}{2} \qquad (9)$$

Under actual conditions, due to the friction, windage and core-loss in the two units of the balancer, the generator current is not equal to the motor current, but is always less. In sets of average size, the generator current is about 42 percent and the motor current 58 percent of the neutral current. In very large sets the generator current may be as high as 47 percent, whereas in very small sets it may be only 38 percent, of the current in the neutral. For a detailed analysis of the performance of the balancers, taking into account armature reaction, friction, windage and core-loss, the reader is referred to an article by A. C. Lanier in the *Electrical Journal*, Vol. 9, p. 1036 (1912).

From equations (8) and (9) it is evident that the voltages across the two sides of the system, for the connections shown in Fig. 118, are never exactly equal when the loads are unbalanced. The voltage across the more heavily loaded side is always less than that across the other side.

By connecting the shunt-field windings of the two balancer units in the manner shown in Fig. 119, a closer voltage balance is obtained. This connection puts the field winding of the generator across the higher voltage side of the system, thereby increasing the field current, flux per pole and generated voltage, which results in raising the terminal voltage of the generator armature.

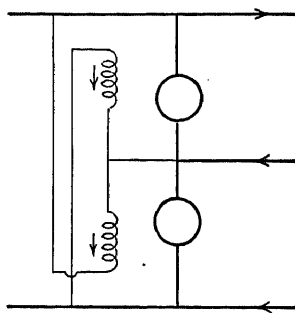
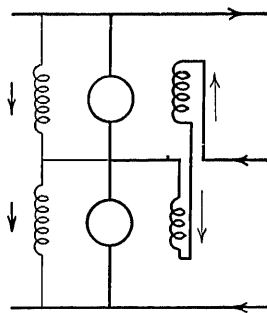



Fig. 119. Fig. 120.

Similarly, the field winding of the motor is connected across the lower voltage side, which lowers the terminal voltage of the motor armature.

Practically equal voltages can be maintained on the two sides of an unbalanced three-wire system by using a balancer made up of two compound-wound machines, illustrated in principle in Fig. 120. An inspection of this figure will show that the neutral current, flowing through the series winding of the generator, aids the shunt-field current, thereby increasing the terminal voltage of the generator armature. On the other hand, this current in the series winding of the motor opposes the shunt-field current, thereby decreasing the terminal voltage of the motor armature. By proper adjustment of the series-field turns, the unbalancing action of the armature resistances, see equations (8) and (9), may therefore be completely counteracted. Since the compounding of the motor element of the balancer is differential, the speed of the

set is higher the greater the difference in the loads connected to the two sides of the system. Armature reaction produces the same effect, but to a less degree, in shunt-wound sets.

84. Three-wire Generators.—An objection to the use of a balancer set in connection with a three-wire system of distribution is the extra rotating machinery required. This objection is overcome by the arrangement shown diagrammatically in Fig. 121, first suggested in principle by Dobrowolsky. To an ordinary generator designed to give a terminal voltage equal to that between the two outer wires, two slip-rings A_1 and A_2 are added, which slip rings are connected to armature conductors which are 180 electrical degrees apart.* Brushes bearing on these slip rings

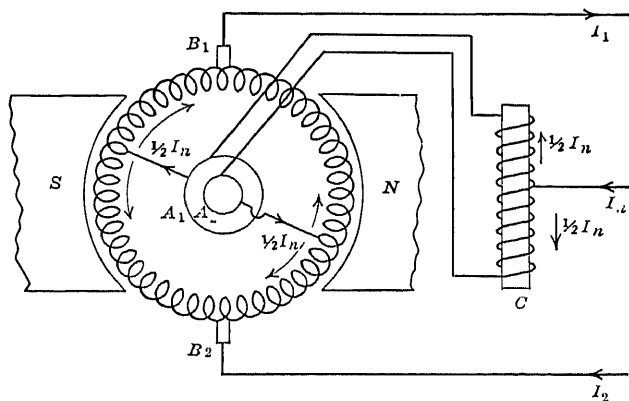



FIG. 121.—Three-wire Generator with External Balance Coil.

are connected to the two ends of a coil C, wound on an iron core, and the middle point of this coil is conected to the neutral wire. The two outer wires are connected to the positive and negative brushes of the generator.

The coil C, usually referred to as the balance coil, or **static balancer,** is designed so that it has a low resistance, but a high self-inductance. High inductance is secured making the coil of a relatively large number of turns and winding it on an iron core.

As the armature rotates, an alternating electromotive force is set up between the two slip-rings A_1 and A_2, which electromotive

* 360 electrical degrees correspond to the distance between the center of a given pole to the center of the next pole of the *same polarity.*

force has a maximum value equal to the electromotive force between the main brushes, and a period equal to the time required for an armature conductor to move past a pair of poles. This electromotive force sets up in the coil C an alternating current of the same period. Due to the rapidity with which this current varies with time, it has to have only a very small value in order to set up a back electromotive force of self-induction $\left(L\frac{di}{dt}\right)$ sufficient to balance the electromotive force generated in the armature (see Article 19). By proper design of the coil C this alternating current can be made negligibly small.

However, due to the flow of this alternating current in the coil C, the middle point of its winding is maintained constantly at a potential half way between that of the two slippings, which, from the symmetry of their connection to the armature winding, is also half way between that of the two main brushes B_1 and B_2. Consequently, the voltages between the two outer wires and the neutral are maintained equal.

When the currents I_1 and I_2 taken by the loads on the two sides of the system are unequal, the current in the neutral wire will be the difference between these two currents, namely,

$$I_n = I_1 - I_2$$

This neutral current, when it enters the balancing coil C, will divide in half, one half entering the armature through the slip-ring A_1 and the other half through the slip-ring A_2.

Since this current from the neutral is a direct current, the self-inductance of the coil C will offer no opposition to its flow. The resistance of the coil C, and also the resistance of the armature, will, however, cause a small drop of potential from the neutral point toward each of the outer wires. This will decrease the potential difference between the more heavily loaded outer and the neutral, and will increase the potential difference between the other outer and the neutral. Consequently, unbalanced currents will produce a slight unbalance of the voltages on the two sides of the system. By making the coil C of sufficiently low resistance, this difference can readily be kept within practical limits. A well-designed three-wire generator should give not over 2 per cent unbalance in voltage for 10 per cent unbalance in current.

The balance coil C is sometimes mounted on the armature spider of the generator, in which case only one slip-ring, connected to the middle point of the coil, is required.

Instead of one balance coil, two such coils are sometimes used, connected as shown diagrammatically in Fig. 122. This reduces the effect of armature resistance on unbalancing the voltages. When two external balance coils are used, four slip-rings are required.

FIG. 122.— Armature of Three-wire Generator with Two Internal Balance Coils.

For a detailed analysis of the action of balance coils, or static balancers, the reader is referred to a paper by C. C. Hawkins, in the *Journal of the Institution of Electrical Engineers* (British), Vol. 45, p. 704 (1910).

When a three-wire generator is provided with both shunt and series-field windings, the series winding is made in two equal parts, half of the turns being connected in series with one outer wire, and the other half in series with the second outer wire. In this way, the compounding is made independent of the amount of unbalancing. The commutating-pole windings are also split. When two such generators are to be operated in parallel, two equalizer buses are necessary, one for each half of the series field.

85. Homopolar, or Unipolar, Generators.—As noted in Article 11, when a conductor moves across the lines of force of a magnetic field an electromotive force is always induced in it. This electromotive force is proportional to the length of the conductor, the flux density of the magnetic field at the conductor, and the speed at which the conductor moves. The direction of this electromotive force is that in which the middle finger of the right hand points when the thumb, forefinger and middle finger of this hand are held mutually perpendicular, and the forefinger is pointed in the direction of the lines of force, and the thumb in the direction of the motion.

Referring to Fig. 123, A and A_1 represent two rings joined by

a conductor C, and NS represents a magnet with *one* of its poles only between the two rings. The light lines with arrows on them represent the lines of force due to the magnetic pole between the two rings (here assumed as a north pole).

From the fundamental relation just stated, it follows that if the conductor C is rotated about the axis of the magnet, in the direction indicated by the arrows on the rings, an electromotive force will be induced in it, in the direction from right to left, and this electromotive force will be in this same direction as long as the conductor is kept moving in a given direction about the axis of the magnet. The electromotive force between the brushes B and B_1 will therefore be a unidirectional electromotive force, and for a constant speed of rotation of the conductor will have a constant value. A load connected to the brushes B and B_1 will therefore have a constant voltage impressed across its terminals.

Fig. 123.—Elementary Homopolar Dynamo.

The reader should note that the electromotive force is induced *in the conductor* C, not in the rings. The rings merely serve to keep the conductor connected to the brushes B and B_1. As the conductor and rings rotate there is no change in the number of lines of force which link the rings, and therefore there is no electromotive force induced in them. In fact, the conductor C may be spread out to form a cylinder concentric with the magnet, and the brushes B_1 and B_2, placed in contact with this cylinder, thus doing away with the rings entirely.

It is perfectly possible to build a direct-current generator upon the principle illustrated in Fig. 123. Such a generator, called a "homopolar," or "unipolar" generator, possesses one great advantage over the ordinary direct-current generator, namely,

no commutator is required. However, there is one serious limitation in the design of such a machine, viz., its electromotive force for any practicable length of conductor, speed of rotation, and flux density is relatively low; see equation (26), Article 11. For example, in a 300-kilowatt machine of this type, built by the General Electric Co., the electromotive force in the rotating conductor under normal operating conditions, is only 41.5 volts,

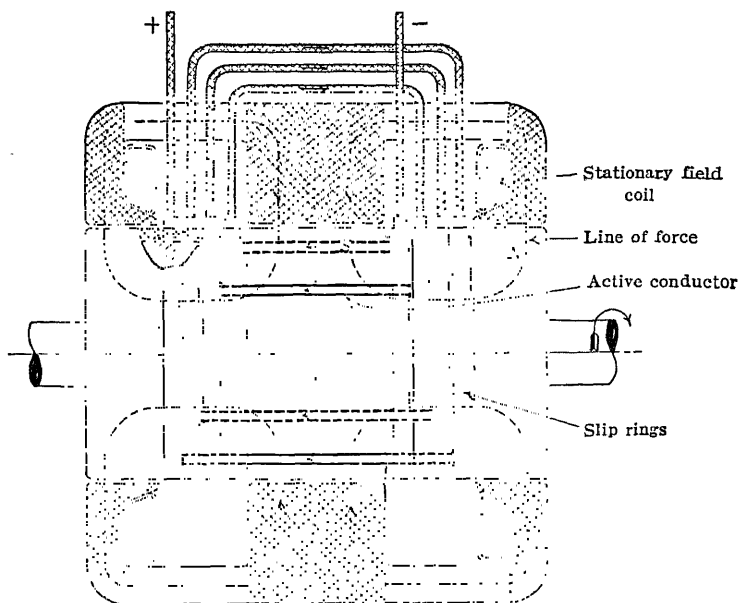



FIG. 124.—Four-stage Homopolar Generator.

and in a similar machine built by the Westinghouse Co., 32.5 volts.

By using several conductors, each provided with a pair of slip-rings, as indicated in Fig. 124 it is possible to obtain a higher terminal voltage by connecting the several conductors in series. However, for each slip-ring there must be provided a set of brushes sufficient to carry the entire current output of the machine. The higher the terminal voltage required, the greater the number of slip-rings and brushes, and the limit is soon reached where the homopolar machine with its multiplicity of slip-rings and brushes

becomes more complicated than the ordinary generator with a commutator.

The 300-kilowatt General Electric machine above referred to was designed for a terminal voltage of 500 volts, and required 12 conductors (24 slip-rings). The 2000-kilowatt Westinghouse machine was designed for a terminal voltage of 260 volts and required 8 conductors (16 slip-rings).

Homopolar generators have never been extensively used, for in spite of their apparent simplicity, this type of machine is actually more expensive to build than the ordinary commutator type, except possibly for very low voltages and high currents. Modern practice even for low-voltage machines, is definitely in favor of the commutator type.

A point of interest in regard to the theory of homopolar machines is that it is immaterial whether the magnet NS (Fig. 123) remains stationary as the conductor rotates, or whether the conductor C is mounted on the magnet and rotates with it. On the other hand, should the conductor be kept stationary, and only the magnet rotated, no electromotive force will be induced in the conductor. These facts are explicable on the assumption that the lines of magnetic force are fixed with respect to the space occupied by the magnet, but are not attached to the *substance* of the magnet. Another explanation is that when the magnet rotates, the lines of force rotate with it and cut the conductors C' which form the external circuit between the brushes.

86. Rosenberg Generator.—For the lighting of railroad cars, it is common practice to use a generator belted to the car axle. A storage battery supplies current to the lamps when the car is at rest, or running at a low speed. This storage battery is charged by the generator when the output of the latter is in excess of the requirements of the lamps.

Since the electromotive force of an ordinary shunt generator increases with the speed, some auxiliary means must be provided in order to maintain a constant terminal voltage, as the speed of the car increases. Various types of automatic regulators are used for this purpose. These regulators consist essentially of one or more solenoids whose plungers vary the pressure on a carbon-pile rheostat which is connected in series with the shunt-

field, increasing this resistance when the speed increases, and decreasing it when the speed decreases.

In connection with the regulator, there is provided a "main switch," which automatically connects the generator tc the battery when the voltage of the former exceeds that of the latter, and which automatically disconnects the battery when the voltage of the generator falls below that of the battery. In addition to the main switch, there is also a double switch, or "pole-changer," which automatically reverses the connection between the generator and battery when the direction of travel of the car is reversed.

By making use of the principle of armature reaction, Rosenberg, in 1905, developed a special form of generator which is particularly applicable to train lighting, in that no pole changer or external regulator is required. The special features of this generator are: (1) *it develops an electromotive force the direction of which is independent of the direction of rotation of the armature, and* (2) *its current output, when the speed exceeds a predetermined value, is substantially independent of the speed at which the armature is driven.*

The principle of the Rosenberg generator is shown in Fig. 125. The field is separately excited from the battery, and tends to produce a flux ϕ_f through the armature in the direction indicated. The brushes bb', placed in the mechanical neutral, are *short-circuited* on each other as shown. A relative small value of the flux ϕ_f will then tend to cause a relatively large current I_1 to flow between the brushes b and b'. The direction of this current in the armature conductors is indicated by the outside circle of + and − signs.

A current I_1 entering the armature through the brush b' and leaving through b will establish a flux ϕ_1 at right angles to the flux ϕ_f, as shown. (This flux ϕ_1 is the same as the ordinary armature reaction flux.) The armature conductors, cutting this flux ϕ_1, have electromotive forces generated in them in the directions indicated by the inside circle of + and − signs. The resultant of these electromotive forces in the conductors which form either of the two parallel paths between the brushes bb' is zero, since, with respect to these brushes, the electromotive force in half of each path is in the opposite direction to that in the other half. However, between the brushes B and B', which are placed

90 electrical degrees from the brushes bb', the electromotive forces produced by the armature reaction flux ϕ_1 are all *additive,* and give rise to a resultant electromotive force E proportional to the speed, number of armature conductors and to the flux ϕ_1.

The current I supplied to the load through the brushes BB' has the same direction as that of the electromotive force which gives rise to it, namely, the direction indicated by the inner circle of + and − signs. This currrent will therefore tend to set up

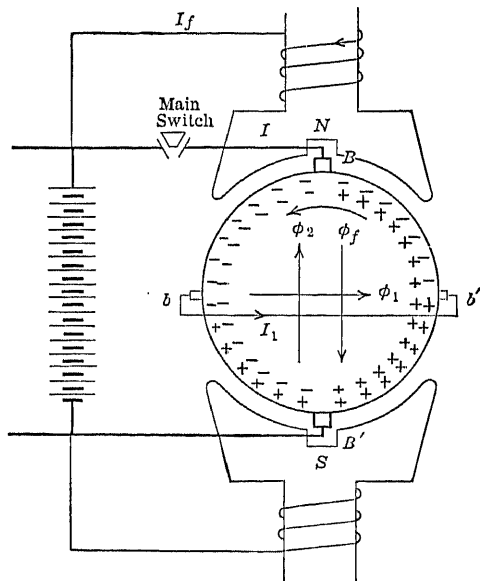



FIG. 125.—Rosenberg Generator.

a flux ϕ_2 along the same axis as the flux ϕ_f, but in the *opposite* direction. Hence the tendency of the field current I_f to produce the current I_1 in the short-circuited path bb' is always, in part at least, counteracted by the armature reaction of the load current I. That is, the current I_1 through the brushes bb' is not proportional to ϕ_f, but to the difference $(\phi_f - \phi_2)$. Therefore ϕ_1, and the generated voltage E between the main brushes BB', are likewise proportional to this difference.

At low speeds the electromotive force generated between the brushes bb' (due to the flux ϕ_f) will be relatively low, and the cur-

rent I_1 will therefore be relatively small. The load current I and the flux ϕ_2 will therefore likewise be small. As the speed increases, I, I_1 and ϕ_2 all tend to increase, but this tendency to increase is checked by the decrease in the resultant flux $(\phi_f - \phi_2)$, which is the flux which produces the current I_1.

Although $(\phi_f - \phi_2)$ decreases as the speed increases, this resultant flux cannot become zero. Consequently, as the speed increases, the load current I approaches a definite maximum value I_m, which maximum value is equal to the ampere-turns per pole $(N_f I_f)$ of the field winding divided by the effective number of turns per pole in the armature winding. Hence, no matter how great the speed may become, the load current can never exceed a definite limiting value, which limiting value is readily controlled by varying the field current I_f.

An inspection of Fig. 125 will make evident that the reversal of the direction of rotation will not reverse the direction of the current supplied by the generator. Were the armature rotated in the direction opposite to that indicated in the figure, the current I_1 would reverse in direction, which in turn would reverse the direction of the flux ϕ_1. Hence, since both the direction of rotation and the direction of the flux which produces the electromotive force E are reversed, the direction of this electromotive force is not changed.

In order to secure a weak field at the conductors undergoing commutation by the main brushes BB', a groove is cut in the center of each pole, as indicated in Fig. 125.

87. Third-brush Generator.—In Fig. 126 is shown another type of generator in which armature reaction is utilized to prevent changes in speed, above a predetermined value, from increasing the current output of the machine. In this generator an auxiliary brush b, usually referred to as the third brush, makes contact with the commutator between the two main brushes B and B'. Between this third brush b and the main brush B, which is behind it with reference to the direction of rotation, is connected the field winding.

Neglecting the reaction, in the armature, of the current supplied to the shunt field, the flux distribution in the air-gap of such a machine is as shown in Fig. 127. In this figure curve F represents that part of the flux due to the shunt-field current only, curve A

that part of the flux due to the armature current only, and curve R the resultant flux due to the combined action of the field and armature currents (see Article 49).

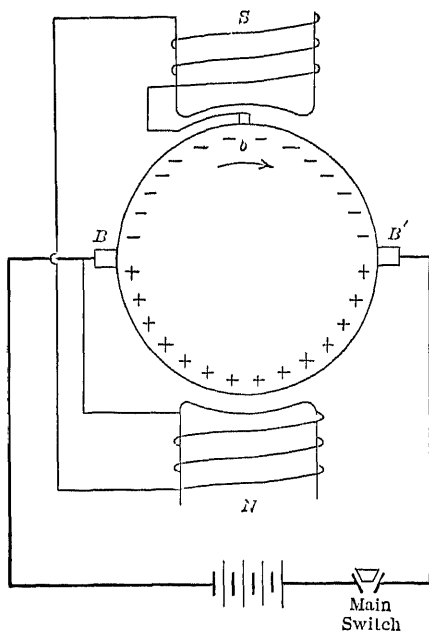



FIG. 126.—Third-brush Generator.

Since the ordinate of the resultant flux distribution curve at any point in the air-gap is proportional to the electromotive force generated in the armature conductor which is at this point, it follows that the total electromotive force generated between the brush B and the auxiliary brush b is proportional to the difference between the areas M and N in Fig. 127: viz.,

$$E_b = K(M-N)$$

where K is a constant.

As the speed increases, thereby increasing the electromotive force between the two main brushes BB', the armature current will increase, thereby increasing the flux represented by the area N in Fig. 127. This will result in a decrease in the electromotive force between the brushes B and b, which in turn will cause the field-current to decrease. As a result of this decrease in the field-current, the flux represented by the area M will decrease, resulting in a still further reduction in the electromotive force between B and b and in the field-current. A condition of equilibrium will be reached for some value of the armature current less than that corresponding to equality between the two areas M and N.

The area N can never be greater than the area M, for this would result in a reversal of the field current, which would cause reversal of the electromotive force between the main brushes.

A complete analysis of the action of a third-brush generator

used in connection with a storage battery, taking into account the reaction, in the armature, of the shunt-field current, will show that a certain (relatively low) critical speed must first be reached be-

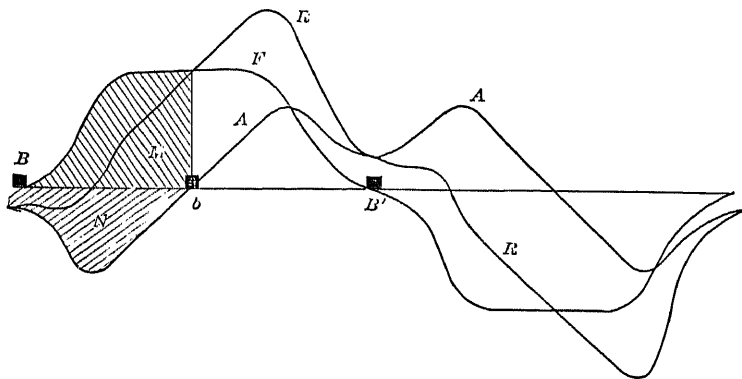



FIG. 127.—Flux Distribution in Air-gap of Third-brush Generator.

fore the machine will function as a generator. Then, as the speed increases, the current will increase and reach a definite maximum value. Beyond this point any further increase in speed will result in a decrease in the current, which will ultimately become zero.

A typical speed-current characteristic of a third-brush generator, when used in connection with a storage battery, is shown in Fig. 128. The maximum value of the current may be adjusted by shifting the position of the third brush. It should be noted that this type of characteristic holds only when the machine is connected to a storage battery. If it is connected to a resistance load, an increase in speed will always result in an increase in current.

FIG. 128.—Current-speed Characteristic of Third-brush Generator Supplying a Storage Battery.

The third-brush generator is extensively used on automobiles

for charging the battery which supplies the current for ignition, lighting and starting.

For a complete mathematical analysis of the performance of a third-brush generator, the reader is referred to Langsdorf, *Principles of Direct-current Machines.*

PROBLEMS

A motor-generator set consists of a 550-volt motor and a 125-volt, 100-kilowatt generator. The efficiency of the generator at rated load is 93 per cent, and the efficiency of the motor 89 per cent. The motor is connected to a 550-volt circuit. What is the value of the current supplied by the generator and what is the value of the current taken by the motor when the generator is delivering its rated load?

2. The resistance of the low-voltage armature circuit of a certain dynamotor is 0.01 ohm and the resistance of the high-voltage armature circuit is 18 ohms. The high-voltage armature winding has 40 times as many active conductors as the low-voltage armature winding. The core-loss, friction and windage of the armature is 35 watts. The resistance of the field circuit (which is connected in shunt with the low-voltage winding) is 4 ohms.

(*a*) The low-voltage side of this dynamotor is connected to an 8-volt storage battery. Assuming the terminal voltage of the storage battery to be exactly 8 volts, what will be current output of this battery when 1 ampere is taken from the high-voltage winding of the dynamotor? (Give exact solution taking into account the various losses.)

(*b*) What will be the voltage between the high-voltage terminals of the dynamotor?

(*c*) What is the efficiency of the dynamotor at this load?

3. It is desired to keep the voltage at the terminals of the motor of a motor-generator set, located at a distance of 1500 feet from the power-house, equal at all loads to the voltage at the power-house switchboard. The transmission line between the power-house and the motor generator set is a pair of 500,000 circular-mil, stranded copper wires. The generator of the motor-generator set is rated at 100 kilowatts, the overall efficiency of the set is 85 per cent, and the voltage at the power-house switchboard is kept constant at 250 volts.

(*a*) How large a booster will be required for this service, i.e., what should be its rated current and rated voltage?

(*b*) What kind of a voltage regulation curve should this booster have?

(*c*) Were no booster provided, what would be the voltage at the terminals of the motor of the motor-generator set, when the output of the set is 100 kilowatts?

4. Referring to Fig. 114, the resistance of the section of track between the bus and the point of connection (P) of the feeder to the track is 0.02 ohm, the resistance of the feeder is 0.01 ohm and the internal resistance of the booster is 0.0008 ohm. The only car on the track is at a distance from the power house equal to three-fourths of the distance from the power-house to

the point P. This car takes a current of 700 amperes. The voltage regulation curve of the booster is a straight line whose equation is $V = 0.01\,I$, where I is the current through it and V its terminal voltage.

(*a*) What is the maximum drop of potential in the track, and between what points does it occur?

(*b*) How much current returns to the negative bus through the booster, and how much through the track?

(*c*) What would be the maximum potential drop in the track were the booster and feeder disconnected?

5. The currents (amperes) taken by the various loads in Fig. 116 are as follows:

$I_1 = 20$	$I_3 = 40$
$I_2 = 50$	$I_4 = 30$

The resistances (ohms) of the various sections of the system are:

$AB = 0.05$	$FK = 0.09$
$BC = 0.12$	$JM = 0.05$
$CD = 0.03$	$MH = 0.04$
$GF = 0.11$	$HL = 0.08$

The internal resistance of each generator is 0.01 and its terminal voltage is 120 volts.

(*a*) Determine the voltage across each load and the electromotive force of each generator.

(*b*) How many amperes enter the junction point, J, between the two generators?

6. The armature of each unit of a shunt balancer has a resistance (including brushes and brush contacts) of 0.1 ohm. Each shunt-field winding has a resistance of 60 ohms. This balancer is connected across the outside wires of a 250-volt, three-wire system, with the shunt fields in series, as shown in Fig. 118. The total core-loss, friction and windage of the two units is 1 kilowatt. The current taken by the load on one side of the system is 1000 amperes and the current taken by the load on the other side of the system is 900 amperes.

Neglecting armature reaction, determine,

(*a*) The electromotive force of each unit of the balancer.

(*b*) The armature current in each unit.

(*c*) The terminal voltage of each unit, i.e., the voltage across each side of the system.

(*d*) The total current output of the main generator.

7. A 4-stage homopolar machine of the type shown in Fig. 124 has the following dimensions:

Diameter of armature	24 inches
Active length of each armature conductor (axial length of air-gap)	12 inches
Speed of armature	3000 r.p.m.
Flux density in air-gap	60,000 lines per sq. in.
Diameter of armature core under slip rings	16 inches

Calculate the electromotive force of this generator.

8. A copper disc, 18 inches in diameter, is rotated, at a speed of 1500 r.p.m., in the air-gap between the two poles of a horse-shoe shaped electromagnet. The average flux density in this air-gap is 50,000 lines per square inch.

(*a*) What is the potential difference between the center and the periphery of this disc?

(*b*) Were the disc made of iron, instead of copper, and the exciting current of the electromagnet kept unaltered, would this potential difference have the same value? Explain.

(*c*) Make a sketch showing how such an arrangement could be used as a direct-current generator.

9. If, for a given value of the current supplied through the main brushes, the flux distribution in the air-gap of a third-brush generator is as indicated by the curve R, in Fig. 127, what would be the voltage across the shunt field, expressed as a percentage of the voltage between the main brushes? Neglect internal drop due to armature resistance.

CHAPTER IX

TESTING OF DIRECT-CURRENT DYNAMOS

88. Introduction.—From an operating point of view the four fundamental characteristics of a direct-current generator are:

1. Efficiency.

2. Voltage regulation (or variation of terminal voltage with current output).

3. Commutation.

4. Temperature rise under load.

Similarly, the four fundamental characteristics of a direct-current motor are:

1. Efficiency.

2. Speed regulation (or variation of speed with torque).

3. Commutation.

4. Temperature rise under load.

In this chapter are briefly described the usual methods of determining these characteristics by test, and also some of the methods used for detecting and locating defects which may develop during the operation of a direct-current dynamo. The methods of test here described are based upon the rules given in the *Standards of the American Institute of Electrical Engineers*, a copy of which should be owned by every student of electrical engineering.

89. General Directions.—Before running a machine, either as a generator or motor, make sure that the bearings are properly supplied with oil. Also see that the commutator is clean and that the brushes make uniform contact. The brushes should bear on the commutator with a uniform pressure of about $1\frac{1}{2}$ pounds per square inch of contact area.

If the commutator is oily or dirty, it should be carefully cleaned with a soft rag. If rough, it should be sandpapered and then wiped off. Never use emery cloth on a commutator, as the parti-

cles of emery tend to embed themselves in the copper, and will cause rapid wearing down of the brushes.

Whenever a running test of a generator is made, the speed of the armature should be held constant throughout the test. Unless there is some special reason to the contrary, this speed should be the rated speed of the machine (see Article 31).

Whenever a running test is made of a motor, the terminal voltage should be kept constant throughout the test. Unless there is some special reason to the contrary, this voltage should be the rated voltage of the machine.

Always use voltmeters and ammeters of such range that the deflection of each instrument will be well up on its scale for all important readings.

The temperature of the surrounding air, also called the **ambient temperature,** should be noted at the beginning of each test, as should also the temperature of the various parts of the machine, in case these differ appreciably from the ambient temperature. If the machine is loaded, the ambient temperature and the temperature of the various parts should also be noted at frequent intervals throughout the test.

Ordinary mercury-glass Centigrade thermometers, which have been properly checked at 0 and 100°, are sufficiently accurate for measuring the ambient temperature and the surface temperatures of the parts of a machine. In order to obtain the average temperature of the surrounding air when there is an appreciable draft, the American Institute of Electrical Engineers recommend that the thermometer be placed in an **oil cup,** consisting of a massive metal cylinder with a hole drilled partly through it. This hole is filled with oil and the thermometer is placed therein with its bulb well immersed. This cylinder should not be less than 1 inch in diameter and 2 inches in height.

A thermometer used for measuring the hot temperatures of windings, cores, commutators, etc. should be placed against the part whose temperature is desired, and covered with a small felt pad, or with putty. A convenient size of felt pad is $1\frac{1}{2} \times 2$ inches, and $\frac{1}{8}$ inch thick. The pad should be attached to the surface of the winding or core with glue. If neither felt nor putty is available, a small wad of cotton waste may be used.

The readings of all meters and other measuring instruments should be corrected in accordance with their calibration curves. When a high degree of precision is required meters, and instruments should be calibrated just prior to the test; and also immediately after the test, in order to make sure that they have not changed in the meantime.

90. Setting of Brushes.—The brushes of a generator or motor should always be set in that position which will give minimum sparking throughout the range in load for which the machine is designed, and should be kept in this position throughout all tests made on the machine, unless there is some special reason to the contrary.

In commutating-pole machines, and in all motors designed to be run in either direction (e.g., railway motors), this condition is secured by setting the brushes in the mechanical neutral (see Article 32.

In non-interpole generators the brushes must be set ahead of the mechanical neutral, and in non-interpole motors intended for operation in one direction only, the brushes must be given a backward shift.

The mechanical neutral of either a non-interpole or of a commutating-pole machine may be determined by driving the machine as a separately excited generator, with the brushes raised from the commutator, and locating the position occupied by an armature coil when it has no electromotive force generated in it. This position may be found as follows:

Place in one of the brush-holders an insulating block of the same size as one of the brushes, with two holes through it to accommodate two small copper wires (about No. 12 A.W.G.), as shown in Fig. 129. These two wires are connected to a low-reading voltmeter, and their free ends are kept in contact with the commutator. While the armature is rotating, shift the rocker-ring back and forth until the voltmeter reads zero. The test brush will then be in the mechanical neutral.

In the case of a multipolar machine, the test brush should be moved successively from one brush-holder to the next, and the mean of the several positions of the rocker-ring thus found be taken as that corresponding to the mechanical neutral.

When commutating-poles are employed, the centers of these poles should always be symmetrically located with respect to the conductors which form the two sides of a coil which is short-circuited by a brush, as otherwise the commutating poles will produce a compounding or differential action (see Article 50). When the armature has a full-pitch winding this means that the centers of the commutating poles should be directly over the conductors which are short-circuited by the brushes, when the latter are in the mechanical neutral.

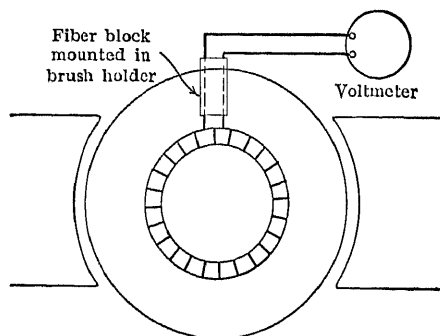



Fig. 129.

Whether or not the commutating poles produce a compounding action, may be readily determined by running the machine as a generator at no-load with normal shunt-field excitation, but with the commutating-pole winding connected to a separate source through a reversing switch, as shown in Fig. 130. A change in the reading of the voltmeter, when the current in the commutating pole winding is reversed, indicates that, for the given setting of the brushes, the commutating pole winding has a compounding action (either cumulative or differential). By shifting the brushes slightly one way or the other, until there is no change in the voltmeter reading on throwing the reversing switch, this compounding action may be eliminated.

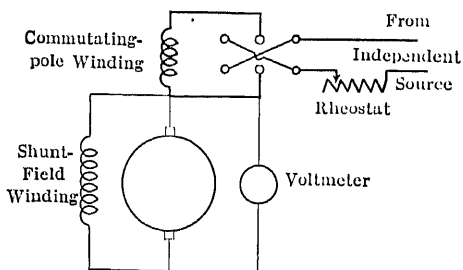



Fig. 130.

When it is not feasible to shift the brushes, as in reversing motors, it is customary to use a small series field to neutralize the compounding action (if any) of the commutating poles.

91. Efficiency and Losses.—The efficiency of a machine is the ratio of the useful power output P_o to the power input P_i, this ratio usually being expressed as a percentage, viz.:

$$\text{Percent efficiency} = 100 \frac{P_o}{P_i} \tag{1}$$

The power input and power output must, of course, be expressed in the same unit. The unit ordinarily employed in such calculations for electric machines is the kilowatt. One horse-power equals 0.746 kilowatt.

The difference between the power input and the useful power output is equal to the total power loss L in the machine, viz.:

$$L = P_i - P_o \tag{2}$$

It is evident that if the total power loss L for a given power output P_o are known, then the power input to the machine is $P_i = (P_o + L)$ and the efficiency may therefore be written

$$\text{Percent efficiency} = 100 \frac{P_o}{P_o + L} = \frac{100}{1 + \frac{L}{P_o}} \tag{3}$$

Similarly, if the total power loss L for a given power input P_i are known, the power output is $P_o = P_i - L$, and the efficiency of the machine may be written

$$\text{Percent efficiency} = 100 \frac{P_i - L}{P_i} = 100\left(1 - \frac{L}{P_i}\right) \tag{4}$$

The efficiency of a generator or motor may be determined either by direct measurement of both the power input and power output (**directly measured efficiency**), or the various losses may be determined experimentally and the efficiency calculated therefrom (**efficiency calculated from losses**). When the efficiency is calculated from the losses, and conventional values are assigned to one or more of these losses (see Article 92, and example), the efficiency is called the **conventional efficiency** of the machine.

The efficiency of a dynamo can usually be determined more accurately by measuring the various losses in the machine, than by the direct measurement of input and output. Since the efficiency of such a machine is usually in the neighborhood of 90 percent, it is evident that an error of, say, 1 percent in the

determination of the losses will introduce an error of only 0.1 percent (or approximately this much) in the calculated efficiency. On the other hand, an error of 1 percent in measuring either the input or output, will introduce approximately the same error in the efficiency calculated from input and output measurements.

Another advantage of the loss method is that an efficiency test by this method requires much less power (and therefore costs less) than an input-output test. This is particularly important in the case of large machines, which frequently have to be tested at a place at which there is not available sufficient power to make an input-output test.

It is therefore the usual practice, in testing a generator or motor, to determine the losses experimentally, and from these measured losses to calculate the efficiency.

Some of the more common methods of determining these losses will now be briefly described. The detailed calculation of efficiency from these measured losses will then be given.

92. Classification of Losses.—The total power loss in a direct-current dynamo is made up of the following losses:

(*a*) *The RI^2 Loss in the Various Windings*, i.e., in the armature winding, shunt-field winding, series-field winding, commutating-pole winding, and compensating winding. The RI^2 loss in the shunt-field rheostat and the RI^2 loss in any shunt used in connection with the series field, commutating-pole winding or compensating winding, are included in the total RI^2 loss of the machine.

(*b*) *Brush-contact Loss.*—This loss is due to the resistance of the contact between the brushes and the commutator. As noted in Article 40, the contact resistance between a brush and the commutator is not constant, but, except at light loads, is approximately inversely proportional to the current, resulting in an approximately constant *potential drop* at the brush contact.

The A.I.E.E. recommended that 1 volt drop per brush set shall be considered as the standard brush contact drop, *at all loads*, for carbon and graphite brushes with pig-tails attached. The corresponding total drop at the positive and negative brush sets is therefore 2 volts. This makes the total brush-contact loss in watts equal to twice the total armature current in amperes.

When no pig-tails are provided and the contact between the carbon brush and the brush holder is relied upon to carry the cur-

rent, the A.I.E.E. recommend that the brush contact drop be taken as $1\frac{1}{2}$ volts per brush set, or 3 volts for the positive and negative brushes together.

When metal-graphite brushes are used, and in the case of low-voltage machines, such as the motors and generators used on automobiles, the actual contact drop should be determined by test, and this test value used in the calculation of efficiency.

(*c*) *Core-loss.*—This name is applied to the total loss caused by hysteresis and by the eddy-currents in those parts of the machine which move with respect to the magnetic lines of force, or within which there is a fluctuating or alternating magnetic field; see Articles 16 and 18.

(*d*) *Friction and Windage.*—This term is used to designate the losses caused by the friction of the bearings, the friction of the brushes against the commutator and the churning of the surrounding air by the motion of the armature.

(*e*) *Stray Load-losses.*—As discussed in detail in Chapter V, the flux distribution in the magnetic circuit of a dynamo is different under load from what it is when there is no current in the armature. Consequently, the eddy-current and hysteresis losses under load are different from what they would be for the same flux per pole at no load. The term "stray load-losses" is used to designate this increase in the eddy-current and hysteresis losses caused by the change in flux distribution produced by the armature current. These load-losses in direct-current machines are usually of low magnitude, and are generally neglected, except in the case of railway motors (see Article 97).

The reader should not confuse the term "stray load-loss" with the term "**stray-power loss,**" which latter term is sometimes used to designate the combined loss due to friction, windage, hysteresis and eddy-currents, viz., items (*c*) and (*d*) above. The stray load-loss is a loss due to the *distortion* of the magnetic field by the *load* current. The stray-power loss is always present when the machine is running, whether or not the machine is carrying a load.

93. Measurement of Field and Armature Resistance.—The simplest way of measuring the resistance of an electric circuit is to establish a direct current through it, measure this current I by means of an ammeter, and measure the drop of potential V

across the terminals of the circuit by means of a voltmeter. The resistance of the circuit between the two terminals is then

$$R=\frac{V}{I} \tag{6}$$

This is the method commonly employed for measuring the resistances of the field and armature windings of a dynamo.

In the case of heavy windings of very low resistance (0.001 ohm or less), it is often more convenient, and likewise more accurate, to determine the resistance by means of a **Kelvin double bridge.** The scheme of connections known as a Kelvin double bridge is shown in Fig. 131.

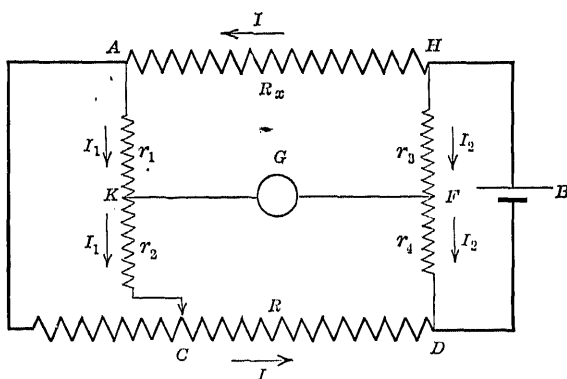



Fig. 131.—Kelvin Double Bridge.

Referring to this figure, r_1, r_2, r_3 and r_4 are adjustable known resistances, for which plug or dial resistance boxes having a range from 1 to 10,000 ohms may be conveniently employed. R is also an adjustable known resistance, usually a fraction of an ohm. R_x is the unknown resistance to be determined. G is a galvanometer, and B a battery.

The resistances r_1, r_2, r_3 and r_4 are adjusted so that

$$\frac{r_1}{r_2}=\frac{r_3}{r_4} \tag{7}$$

and the resistance R is adjusted until the galvanometer shows no deflection. Under these conditions there will be no current from K to F through the galvanometer; the current I_1 in r_1 will

be the same as in r_2; the current I_2 in r_3 will be the same as in r_4; therefore the current I in R_x will be the same as in R.

The application of Kirchhoff's Second Law to the two loops $HAKF$ and $KCDF$ gives

$$R_xI + r_1I_1 = r_3I_2$$
$$RI + r_2I_1 = r_4I_2$$

Multiply the first of these equations by r_2 and the second by r_1, and subtract. There results

$$(r_2R_x - r_1R)I = (r_2r_3 - r_1r_4)I_2$$

But from equation (7), $r_2r_3 - r_1r_4 = 0$. Whence

$$R_x = \frac{r_1}{r_2}R \quad (8)$$

Hence, when the ratio $\frac{r_1}{r_2}$ is known, and the ratio $\frac{r_3}{r_4}$ is made equal to $\frac{r_1}{r_2}$, the unknown resistance R_x is given directly by equation (8), upon substitution in this equation of that value of R which gives zero deflection of the galvanometer.

The resistance of each winding of a machine should always be determined when the machine is at room temperature, i.e., before the field is excited or the machine is loaded in any way. The resistance of a winding at room temperature is called its **cold resistance.**

The current to be used in making a resistance measurement should always be less than the rated current of the winding, preferably not more than ¼ of this current. This is to avoid heating of the winding by the test current, for the resistance of a copper winding increases approximately 1 percent for every 2.5° C. rise in temperature. For the same reason, a switch should be placed in the test circuit, and this switch should be closed only while readings of the meters are being taken.

Care should be taken in measuring the **resistance of a shunt-field winding** to connect the voltmeter leads to the terminals of the field winding itself, and not to include the potential drop through the shunt-field rheostat. The resistance of the field rheostat, for a given setting of its handle, may of course be deter-

mined at the same time by noting the drop of potential through it, and dividing this drop by the field current.

It is common practice, in ordinary laboratory testing, to include the brush contact resistance as part of the **armature resistance,** i.e., to measure the drop of potential between brush studs of opposite polarity, when a current is sent through the armature at rest, and to take as the armature resistance this potential drop divided by the test current.

When the rules for calculating efficiency given in the Standards of the A.I.E.E. are followed, it is necessary to determine the resistance of the armature winding *only*, exclusive of the brush contact resistance. If the resistance at each brush contact of like polarity has the same value, this may be readily done by measuring the drop of potential between two commutator bars which are respectively directly under the centers of a positive and a negative brush. This drop of potential divided by the total current supplied to the armature is then the true armature resistance.

In the case of a multi-polor machine, should the resistances at the several brush contacts differ by an appreciable amount, the test current will divide unequally among the several parallel paths formed by the armature winding. Under these conditions the potential drops between successive pairs of commutator segments (located under the centers of successive pairs of brushes) will not be the same. However, when these potential drops do not differ from one another by too great an amount, their average value may be taken as the potential drop which the given test current would produce were it equally divided among the several parallel paths.

In measuring resistance by the drop of potential method, care should be taken to disconnect the voltmeter before interrupting the test current, as the "inductive kick" on opening the circuit may damage the instrument.

94. Calculation of Hot Resistance and Temperature Rise.—The resistance to be used in calculating the loss of power in a winding should, strictly speaking, be the resistance corresponding to the temperature of the winding under the specified condition of operation. The resistance of a winding under load conditions is usually refered to as its "hot resistance." The hot resistance of a winding may, of course, be determined experimentally in the same manner as the cold resistance, provided the machine is

operated under load for a sufficient time to allow the winding to reach the steady temperature corresponding to the given conditions of operation. However, for the purpose of efficiency calculations, it is more convenient, and usually sufficiently accurate, to assume a hot temperature corresponding to average operating conditions, and to calculate the hot resistance from the measured cold resistance. The American Institute of Electrical Engineers recommends that 75° C. be taken arbitrarily as this hot temperature for all windings, irrespective of the load.

Let R be the measured cold resistance of a copper winding at a temperature of t° C. Then the hot resistance of this winding corresponding to a temperature of 75° C., is from equation (8*a*), Article 5,

$$R' = \frac{309.5R}{234.5+t} \tag{9}$$

When a winding is carrying a current its temperature is never uniform throughout, but its interior portions are always hotter than its surface. Its *average* temperature, however, may be determined by measuring its actual hot resistance R', provided the temperature coefficient of the conductor is known, see Article 5. For a copper winding, the average temperature t' corresponding to a hot resistance R' is

$$t' = (234.5+t)\frac{R'}{R} - 234.5 \tag{10}$$

where R is the cold resistance of the winding at room temperature t.

When a heat-run is made on a machine (see Article 110) the hot resistance of each of its windings should be determined experimentally. Either the drop of potential method or a bridge may be used. When the machine is shut down at the end of a heat-run, the resistance measurements should be made as quickly as possible, in order to avoid any cooling of the winding prior to the reading of the instruments.

95. Core-loss and Friction.—The core-loss in a direct-current machine (i.e., the combined hysteresis and eddy-current losses) is a function both of the speed at which the armature rotates and of the resultant flux per pole. See Articles 16 and 18, and note that the frequency of variation of the flux is directly proportional to the speed of the armature.

Since, for a given speed, the resultant flux per pole is directly proportional to the electromotive force generated in the armature, the variation in the core-loss with the flux per pole can be expressed directly in terms of the armature electromotive force and the speed. However, since the hysteresis loss is *directly* proportional to the speed and to the *1.6th power* of the flux, the core-loss is not a function of the armature electromotive force only, but is a function of both the armature electromotive force *and* the speed.

Were it not for the change in the distribution of the lines of force caused by the current in the armature, the core-loss, *for a given speed and armature electromotive force*, would be constant, irrespective of the load, and would have at all loads the same value as at no load. Except in the particular case of railway motors, the extra core-loss due to this flux distortion (i.e., the so-called "stray load-losses") is relatively small, and as already noted, the American Institute of Electrical Engineers recommends that it be neglected.

Therefore, except in the special case of railway motors, the core-loss in a given machine under any normal condition of load may, to a close approximation, be taken equal to the core-loss in this machine when it is run without load *at the speed and armature electromotive force corresponding to this load*. This no-load core-loss may be readily determined by test, as will be explained presently.

The friction and windage losses, which will hereafter be referred to simply as the *friction*, depends upon the speed only, assuming of course, constant conditions of lubrication. In the case of a belted machine, the bearing friction increases with the load, due to the pull of the belt, but this loss is not chargeable to the machine.

As already noted, the combined core-loss and friction is sometimes called the "stray-power" loss.

Three different methods are used for determining experimentally the core-loss and friction (or stray-power loss), namely,

(1) The power input to armature is measured when the machine is run as a motor without load. This test is sometimes called the "running light test."

(2) The machine is driven as a generator, without load, by means of a small auxiliary motor, and the power output of this auxiliary motor is determined experimentally. As the auxiliary

motor is usually belted to the machine under test, this method is usually referred to as the "belted method."

(3) The machine is brought up to the highest safe speed, the power is then shut off, and the armature is allowed to slow down, the field being kept excited. From the observed rate of retardation of the armature, the core-loss and friction may be calculated. This test is usually referred to as the "retardation test," or the "deceleration test." This method is satisfactory only for large machines, in which the kinetic energy of the armature is sufficient to maintain the rotation for several minutes.

96. Running-light Test.—The connections for a running-light test, when applied to a shunt machine, are shown in Fig. 132. A

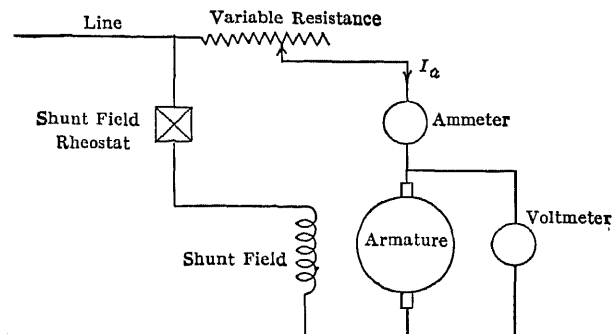



FIG. 132.—Connections for Running-light Test.

compound machine is tested in the same manner, the series field being disconnected. In the case of a series machine, the field winding is separately excited. By properly adjusting the field rheostat simultaneously with the variable resistance* in the armature circuit, the speed may be held constant while the armature terminal voltage is varied over a wide range.

In the case of a generator, the core-loss and friction should be determined at the rated speed of the machine, and for such values of the armature terminal voltage as will at least cover the variation in the armature electromotive force of the machine under all ordinary conditions of load. Readings are usually taken for the lowest voltage which can be conveniently obtained and for

* When available, a booster may be used to advantage in place of the variable resistance in the armature circuit.

several successively higher values, up to about 25 percent in excess of the rated voltage of the machine.

In the case of a motor, the core-loss and friction should be determined at rated speed and for such other speeds as will cover the variation in the speed of the machine under load. For each speed the loss should be determined for several values of the armature terminal voltages, as in the case of a generator.

Railway motors are considered in detail in the next Article.

The armature electromotive force corresponding to any value V_a of the armature terminal voltage and armature current I_a is $(V_a - R_a I_a)$, where R_a is the armature resistance, including the brush contact resistance measured as explained in Article 88. The total core-loss and friction $(L_c + L_f)$ is equal to the power input $(V_a I_a)$ to the armature, less the armature resistance loss, $(R_a I_a^2)$, that is,

$$L_c + L_f = (V_a - R_a I_a) I_a \tag{11}$$

The results of this calculation for the successive values of the terminal voltage are usually plotted as a curve, either with loss as ordinates and armature terminal volts as abscissas (Fig. 136), or with loss as ordinates and volts as abscissas. Since the current I_a in this test is relatively small, the resistance drop $R_a I_a$ is likewise small. This curve therefore gives, to a close approximation, the relation between the combined core-loss and friction, at the given speed, and the armature electromotive force.

As already noted, the combined core-loss and friction of a series motor may be determined in the manner just described, by separately exciting the series field. In the special case of a series motor the friction loss may also be determined independently of the core-loss, by operating the motor with the series field normally connected, but at a very much reduced impressed voltage. Under these conditions the current taken by the motor will be relatively small, and the flux per pole will therefore likewise be small. In practice this flux is so small, compared with the flux under ordinary load conditions, that the hysteresis and eddy-current loss due to it may be neglected.

Hence, to determine the friction of a series motor at any speed N, run the motor without load, with sufficient resistance in series with it to give the desired speed. Measure the voltage V_a across

its armature terminals and the current input I. Then, to a close approximation, the friction at this speed N is

$$L_f = (V - R_a I)I \tag{12}$$

where R_a is its armature resistance (including the brush contact resistance), measured as explained in Article 93.

The friction-loss should be determined in this manner for at least five different speeds, ranging from the highest to the lowest at which the motor is designed to run, and a curve plotted with speed as abscissas and friction-loss as ordinates. See Fig. 133.

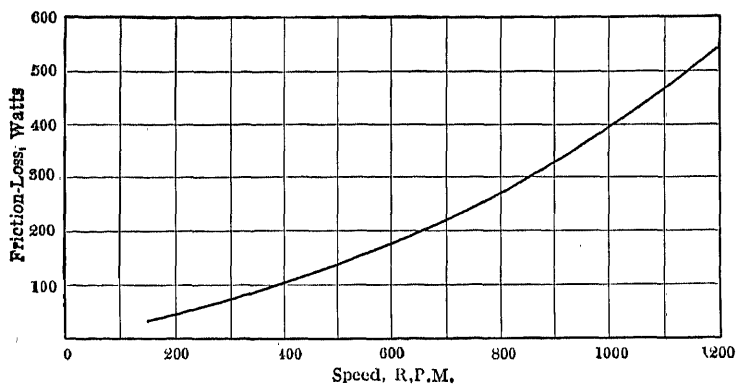



FIG. 133.—Friction Loss of a 500 H.P., 500-volt Railway Motor (without Gears).

97. Core-loss of Railway Motors.—On account of the relatively high armature reaction in railway motors, and consequent distortion of the lines of force in the field poles and armature, the core-loss of such a motor is not only a function of the speed and armature electromotive force, but depends also upon the armature current.

Let L_c' be the core-loss of such a motor at a given speed N and given value I of the field current, when the motor is separately excited and run without load, as explained in the preceding article. The core-loss under these conditions is called the **no-load core-loss** corresponding to the speed N and field current I.

Let L_f be the friction-loss at this same speed N. The combined friction and no-load core-loss at this speed and field current is then

$$L_c' + L_f = (V_a - R_a I_a) I_a \tag{13}$$

where V_a is the voltage which must be impressed upon the armature when the motor is operated as a separately excited motor with a field current I in order to obtain the speed N, and I_a is the corresponding armature current.

The friction-loss L_f may be determined by operating the machine as a series motor at reduced terminal voltage, as explained in the preceding article. The no-load core-loss at the speed N and field current I is then

$$L_c' = (V_a - R_a I_a) I_a - L_f \tag{14}$$

This core-loss L_c' is usually plotted as ordinates against the *current in the field winding* as abscissas; see Fig. 134

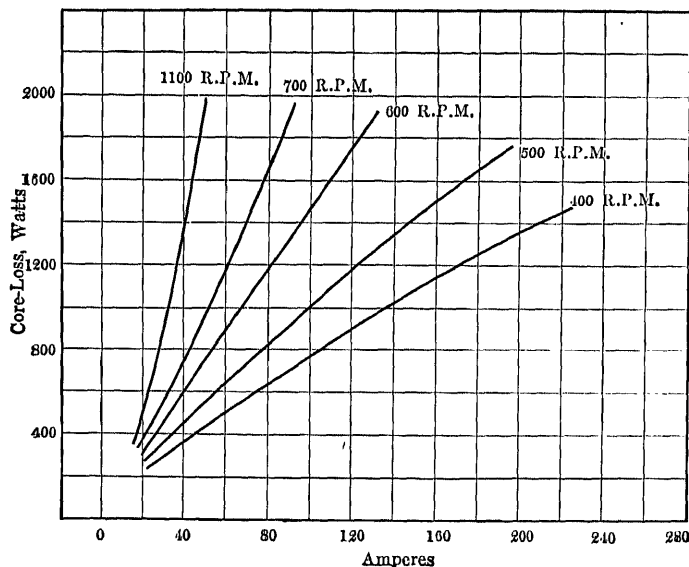



FIG. 134.—No-load Core-loss of a 50 H.P., 500-volt Series Motor.

The procedure just described is that recommended by the American Institute of Electrical Engineers for railway motors. For such motors the Institute recommends that the core-loss under load be taken greater than the corresponding no-load core-loss, as indicated in Table I. The figures in this table are averages derived from elaborate tests of a large number of motors of various sizes and makes.

TABLE I.—CORE-LOSS IN D-C RAILWAY MOTORS AT VARIOUS LOADS

Percent of Input at Nominal (1-hour) Rating	Loss as Percent of No-load Core-loss
200	165
150	145
100	130
75	125
50	123
25 and under	122

98. Belted Method for Determining Core-loss and Friction.—The connections for a belted core-loss and friction test are shown

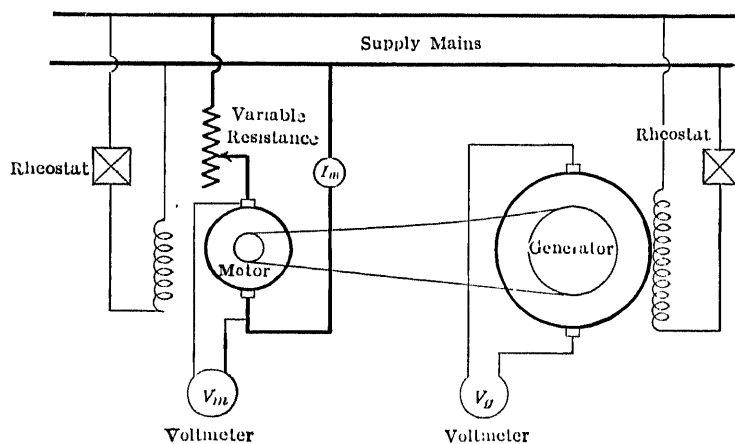



FIG. 135.

in Fig. 135. The auxiliary motor, which should have a rating about 10 percent of that of the machine to be tested, is shown to the left. Both the auxiliary motor and the machine under test are separately excited, the latter being driven by the motor as a generator.

The field of the auxiliary motor is adjusted to about its normal value, and is held constant at this value throughout the test. The speed of the machine under test is adjusted by means of a variable resistance in the armature circuit of the auxiliary motor,

and its armature terminal voltage is adjusted to any desired value, by means of the rheostat in its field circuit.

Let V_m be the voltage impressed across the armature of the auxiliary motor, I_m the current taken by this motor, and R_m the armature resistance of this motor. The mechanical power output of the auxiliary motor is then $V_mI_m - R_mI_m^2$ less the core-loss and friction of this motor.

The core-loss and friction of the auxiliary motor may, to a sufficiently close approximation for the purpose in hand, be taken equal to the power input to its armature when its belt is thrown

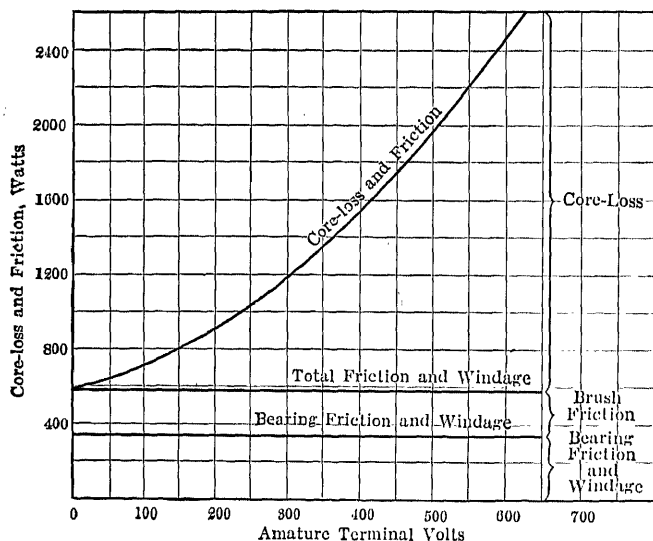



FIG. 136.—Core-loss and Friction of a 100 kw., 525/575 volt, 600 r.p.m. Generator at Rated Speed.

off and it is allowed to run light at the same speed and field excitation as before. Its speed can be kept constant by adjusting the variable resistance in its armature circuit.

Let V_m' and I_m' be the armature terminal voltage and armature current of the auxiliary motor when running light with belt off. Then, neglecting the loss in the belt, which is usually trifling, the mechanical power input to the armature of the machine under test, which in turn is equal to its core-loss and friction, is

$$L_c + L_f = (V_m - R_mI_m)I_m - (V_m' - R_mI_m')I_m' \qquad (15)$$

This same method of test is also applicable to the determination of the friction and windage only, for if the measurements just described are made with the field circuit of the machine under test *open*, there will be no core-loss. The calculation indicated by equation (15) will then give the friction and windage only.

This method is also applicable to the determination of the brush friction, alone by determining first the mechanical input to the armature of the machine under test with the brushes down on the commutator, and then with all the brushes raised from the commutator. The difference between the power inputs in the two tests is then the brush friction.

The core-loss and friction curves, determined by the belted method, for a 100 kw., 525/575 volt, compound generator are shown in Fig. 136.

The belted method here described is that recommended by the American Institute of Electrical Engineers for determining the core-loss and friction of all d-c. generators and motors except railway motors.

99. Retardation Test.—As stated in Article 95, the core-loss and friction can be obtained by bringing the armature up to a speed in excess of its rated speed, and then disconnecting the power supply to the armature, and allowing it to slow down under the action of the opposing forces due to the eddy-currents, hysteresis and friction. If the field is kept excited during the retardation of the armature, this test gives the combined core-loss, friction and windage. If the field circuit is open while the armature slows down, this test gives the friction and windage only.

The armature may be brought up to speed either by operating the machine as a motor without load, or by driving it by means of an auxiliary motor and belt. In the first case, the circuit connecting the armature to the source of supply is opened, by means of a switch or circuit breaker, when the proper speed has been reached. In the second case, the armature is disconnected from the source of supply by throwing off the belt.

When the armature is running at a speed of N revolutions per minute, the kinetic energy stored in it is $\frac{1}{2}AN^2$ watts, where A is a constant proportional to its moment of inertia. When there is no energy being supplied to the armature, the *rate* at which its kinetic energy decreases is equal to the power absorbed by all

the opposing forces, namely by friction, windage, hysteresis and eddy currents. Let N_1 be the speed of the armature at any instant of time t_1, and let N_2 be its speed at an instant t_2 a few seconds later. Then the average rate of change of the kinetic energy of the armature during the interval t_2-t_1, which is equal to the average value of the power absorbed by the core-loss and friction while the speed is falling from N_1 to N_2, is

$$L_c+L_f=\tfrac{1}{2}A\frac{N_1^2-N_2^2}{t_2-t_1} \tag{16}$$

As a close approximation, the value of the total loss given by this formula may be taken as the core-loss and friction at the average speed $\frac{N_1+N_2}{2}$, provided the two speeds N_1 and N_2 do not differ by more than, say, 5 percent.

The simplest way of determining the constant A in equation (16) is to make a running-light test, and to determine therefrom the value of the total loss at some definite speed and field excitation. This value substituted in equation (16), when N_1 and N_2 are so chosen as to make $\frac{N_1+N_2}{2}$ equal to this speed, and (t_2-t_1) is taken as the observed interval of time for the armature to slow down from N_1 to N_2, enables one to calculate A. Since the constant A depends only upon the moment of inertia of the armature, when it has once been found the retardation method may be used to determine the core-loss and friction under any conditions of speed and field excitation, and also to determine the brush friction.

The chief difficulty in performing a retardation test lies in the accurate determination of the speed of the armature and in the measurement of the short intervals of time between successive speed readings. One of the best types of speed indicator for this purpose is a small magneto, connected to a sensitive voltmeter, and belted to the armature shaft of the machine under test.

In the case of a generator which has no projecting shaft to which a pulley can be attached, the retardation method is the only one available for separating the core-loss, journal friction and windage, and the brush friction. When a pulley can be used, the belted core-loss and friction test described in the previous Article is preferable.

100. Calculation of Efficiency of a Generator from its Measured Losses.—A long-shunt compound generator with commutating poles will be considered. The modifications necessary for other types of generators will be evident. To illustrate the procedure, the calculations will be applied to the special case of a 100 kw., 6-pole, 600 r.p.m., over-compound generator, designed to give 525 volts at no-load and 575 volts at full load.

Measure the cold resistance of the various windings (in parallel with their shunts,* if any), and calculate therefrom, as explained in Article 94, the hot resistance of each at 75° C. For the 100 kw. machine here used as a numerical example, the cold resistances at 23° C. were found to have the values given in Table II. The corresponding calculated hot resistances are given in the last column of the table.

TABLE II.—RESISTANCES OF WINDINGS

	Ohms at 23° C.	Ohms at 75° C.
Armature (exclusive of brush contacts).............	0.0802	0.0965
Series field (in parallel with shunt, if any)..........	0.0181	0.0218
Commutating-pole (in parallel with shunt, if any)...	0.0036	0.0043
Shunt-field (exclusive of field rheostat).............	122	147

Measure the shunt-field current at rated no-load voltage and rated no-load speed. For the 100 kw. machine this was found to be 3.1 amperes. To a sufficiently close approximation, as far as the calculation of efficiency is concerned, the terminal voltage may be assumed to increase uniformly from its no-load value, 525 volts, to its full-load value, 575 volts, and the shunt-field current to increase accordingly. From full load to 150 per cent load the terminal voltage and shunt-field current are assumed

* When a winding, such as a series-field winding, has a shunt around it, equation (9), Article 94, is not strictly applicable, since the shunt usually has a temperature coefficient different from that of copper. When greater accuracy is required, the cold resistances of winding and shunt should be determined separately. and their hot resistances (at 75° C.) calculated by using the proper temperature coefficient of each (see Article 5). The resistance equivalent to these two hot resistances in parallel is then calculated in the usual way (see Article 8).

constant. The shunt-field current at various loads will then be as given in Table III. If greater accuracy is desired, the exact voltage regulation and shunt-field current may be obtained from a load test; see Article 108.

Calculate the armature current, as indicated in Table III, corresponding to 0, 25, 50, 75, 100, 125 and 150 percent of full-load line current. Calculate the total resistance drop in the armature circuit, for each value I_a of the armature current, namely, the drop $(R_a+R_s+R_c)I_a+2$, where R_a, R_s and R_c are respectively the resistances, at 75° C., of the armature, series-field, and commutating-pole windings, and 2 is the total brush-contact drop at the positive and negative brushes; see Article 92. The total drop $(R_a+R_s+R_c)+2$, added to the corresponding line voltage, gives the corresponding armature electromotive force.

From the core-loss and friction curve, Fig. 136, determined preferably as described in Article 98, find the core-loss and friction corresponding to the calculated value of the armature electromotive force at each load, and insert in Table III as indicated. This assumes (see Article 95) that the core-loss and friction is a function of the total flux per pole and speed only, and is not affected by the distorting effect (cross-magnetizing action) of the armature current. This method of determining the core-loss and friction is that recommended in the Standards of the American Institute of Electrical Engineers.*

The loss in the shunt-field winding, exclusive of the shunt-field rheostat, is equal to the hot resistance of the shunt-field winding times the square of the shunt-field current. The loss in the shunt-field rheostat, however, is always included as part of the total losses of the machine. The total loss in the shunt-field circuit, including the field winding and field rheostat, is most conveniently calculated by taking the product of the voltage impressed on this circuit and the shunt-field current. In the case

* In an engine-type generator, i.e., one which is directly coupled to an engine in such a way that it cannot be run independently of the engine, the Institute recommends that only the *brush friction* be included in the friction losses. This recommendation is based on the fact that the bearing friction and windage in such a machine is usually very small (only a fraction of 1 percent), and there is no simple method by which it can be measured experimentally.

of a shunt generator, or of a long-shunt compound generator, the voltage across the shunt-field circuit is the line voltage. In the case of a short-shunt compound generator the voltage across the shunt-field circuit is the armature terminal voltage.

The brush-contact loss is calculated on the assumption of 1 volt drop at each brush set (see Article 92). This loss in watts is the equal to twice the total armature current.

The RI^2 losses in the armature, series-field, and commutating-pole windings are calculated from the hot resistances of these windings; see Table II.

The total loss L is then found by adding the separate losses. The power output P_o is the product of the line voltage by the line current. The power input is then $P_i = P_o + L$, and the per-cent efficiency is $100 \frac{P_o}{P_i}$.

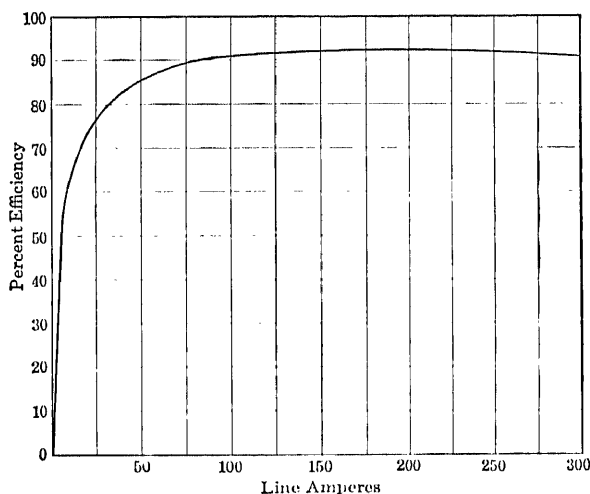



FIG. 137.—Efficiency of Generator.

The efficiency curve of the particular generator to which the numerical calculations in Table III apply is shown in Fig. 137. It is the usual practice, in drawing such curves, to take the line current (rather than kilowatts output) as abscissas.

It will be noted that the efficiency of the particular machine under consideration has a maximum value at about full-load.

TABLE III.—EFFICIENCY OF 100 KW., 525/575 VOLT, 6-POLE, 275 R.P.M., D-C. GENERATOR

Percent Full-load Current		0	25	50	75	100	125	150
Line Voltage	V	525	537.5	550	562.5	575	575	575
Line Amperes	I	0	43.5	87	130.5	174	217.5	261
Shunt Field Amperes	I_f	3.10	3.18	3.25	3.32	3.40	3.40	3.40
Armature Amperes	$I_a=I+I_f$	3.1	46.7	90.3	133.8	177.4	220.9	264.4
Total Res. Drop in Arm. Circuit	RI_a+2 (See Note)	2.4	7.7	13.0	18.4	23.8	29.1	34.4
Armature Electromotive Force	$V+RI_a+2$	527.5	545.2	563.0	580.9	598.8	604.1	609.4
Core Loss and Friction		2110	2190	2260	2380	2470	2490	2520
Shunt-Field Loss, including Shunt-Field Rheostat	VI_f	1630	1710	1780	1870	1950	1950	1950
Brush Contact Loss		6	93	181	268	355	442	529
RI^2 Armature	$R_aI_a^2$ (Note)	1	203	760	1660	2920	4550	6500
RI^2 Series Field, including Shunt	$R_sI_s^2$ (Note)	0	48	178	390	688	1170	1540
RI^2 Com.-pole Wind., incl. Shunt	$R_cI_c^2$ (Note)	0	10	28	78	138	234	308
Total Losses, Kilowatts	L	3.747	4.254	5.187	6.646	8.521	10.836	13.347
Kilowatt Output	P_o	0.0	23.4	47.8	73.4	100.0	25.0	150.0
Kilowatt Input	$P_i=P_o+L$	3.7	27.7	53.0	80.0	108.5	135.8	163.3
Percent Efficiency	$100\frac{P_o}{P_i}$	0.00	84.5	99.3	91.7	92.2	92.1	91.9

NOTE.—R_a, R_s, and R_c are respectively the hot resistances of armature, series field and commutating-pole windings as given in Table II, and $R=R_a+R_s+R_c$

The shape of the efficiency curve in Fig. 137 is typical of all ordinary shunt and compound generators, except that the maximum efficiency need not necessarily occur at full load (see Article 102), nor is the efficiency curve always as flat as that shown in the figure.

101. Calculation of Efficiency of a Motor from its Losses.—The efficiency of a motor is calculated from its losses by the same method as used for a generator, with the following modifications:

(*a*) As in the case of a generator, calculations are made for successive values of the line current, equal to say 0, 25, 50, 75, 100, 125 and 150 percent of the approximate current input at rated voltage and rated power output. The approximate current input at rated load and voltage V may be calculated from the formula

$$I=\frac{746\times(\text{Horsepower})}{0.9V} \qquad (17)$$

(The 0.9 in this formula is a rough approximation to the full-load efficiency.)

(*b*) In the case of a shunt motor, the field current I_f should be taken at the value required to give rated speed of the armature when the machine is operating at rated voltage and is delivering rated load. The only way of determining this "normal" field current experimentally is to make a load-test; see Article 103. As a fairly close approximation, when such a test is not feasible, the field current required to give rated speed at *no-load*, when rated voltage less the armature-drop R_aI_a is impressed, may be taken as the value of I_f in the calcluations of efficiency.

(*c*) The armature current I_a of a shunt or compound motor is equal to the line I current *less* the shunt-field current I_f, viz..

$$I_a=I-I_f \qquad (18)$$

In a series motor the armature current is equal to the line current.

(*d*) The armature electromotive force is equal to the line voltage *less* the total internal drop in the armature circuit, viz.,

$$\text{Armature E.M.F.}=V-(RI_a+2) \qquad (19)$$

where R is the total resistance $(R_a+R_s+R_c)$ in the armature circuit.

(*e*) In the case of a *shunt motor*, it is common practice in calculating the core-loss and friction to neglect the slight decrease in speed with increase of load, and to assume the speed to be constant at its rated full-load value. When a greater degree of precision is desired, a load test must be made (see Article 103), and the variation of speed with current input determined. The core-loss and friction is then determined at two or more speeds within this speed range, and the loss corresponding to the proper speed for each value of the line current is taken from the proper curve, interpolating if necessary.

In the case of a *series motor*, on account of the wide variation of speed with load, the proper speed for each value of the line current must always be used in calculating the core-loss and friction. The speed-ampere curve is obtained from a load test, see Article 107. The core-loss and friction loss may then be read directly from the core-loss curves and friction-loss curves determined as explained in Article 99, the "no-load" core-loss being corrected as there explained to allow for the increase in this loss under load.

(*f*) The kilowatt input P_i is calculated for each value of the line current I, viz.,

$$P_i = \frac{VI}{1000} \tag{20}$$

The kilowatt output is then

$$P_o = P_i - L \tag{21}$$

where L is the sum of the separate losses, in kilowatts, corresponding to the line current I. The corresponding efficiency and horsepower output are then

$$\text{Percent Efficiency} = 100\frac{P_o}{P_i} \tag{22}$$

$$\text{Horsepower Output} = \frac{P_o}{0.746} \tag{23}$$

The efficiency curve of a motor, together with its other characteristics, such as speed, torque, and power output, are usually all plotted on the same sheet as ordinates against current input as abscissas; see Fig. 138.

102. Maximum Efficiency of a Dynamo.—From what has been said in the previous discussion of the losses in a dynamo, it is

evident that in a shunt or compound machine the RI^2 loss in the shunt field and the core-loss and friction are approximately constant, independent of the load on the machine. On the other hand, the RI^2 losses in the armature circuit, including the series-field and commutating-pole windings, vary with the load.

Let K be the constant losses (or those which are assumed to be constant), and let R be the total resistance of the armature circuit, including the series-field and commutating-pole winding (if

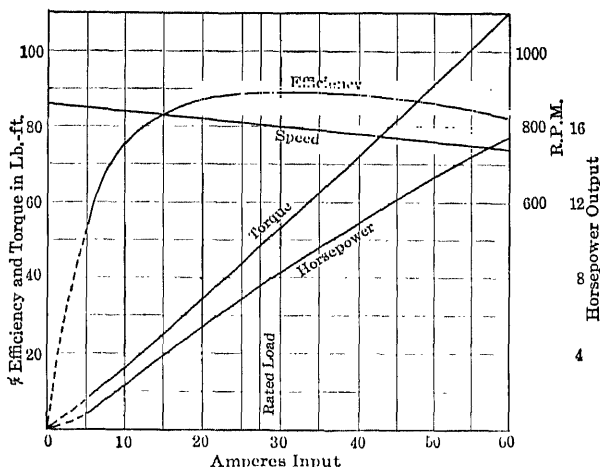



FIG. 138.—Efficiency and Other Characteristics of a Typical 7.5 H.P. Shunt Motor.

any) and the brush-contact resistance (here assumed constant). Neglecting that portion of the RI^2 loss in the armature circuit due to the shunt-field current, the total losses in the machine may then be written

$$L = K + RI^2$$

In the case of a generator the efficiency may then be written

$$\text{Efficiency} = \frac{VI}{VI + (K + RI^2)} = \frac{1}{1 + \frac{1}{V}\left(\frac{K}{I} + RI\right)}$$

where V is the line voltage.

Hence, to determine the value of the line current for which the efficiency of a generator is a maximum, it is merely necessary to

find the value of I which will make $\left(\frac{K}{I}+RI\right)$ a minimum. This is readily done by differentiating this expression with respect to I and setting the result equal to zero. This gives

$$-\frac{K}{I^2}+R=0$$

or,

$$RI^2=K \qquad (24)$$

That is, the condition for maximum efficiency is that the line current must have such a value as will make the variable RI^2 losses equal to the sum of the constant losses. It is evident, therefore, that the particular load at which the efficiency of a machine will be a maximum can be readily controlled by the designer, according to the relative amounts of copper and iron used. The greater the cross-section of the magnetic circuit, the lower will be the flux density, and therefore the less the hysteresis and eddy-current losses, (see Articles 16 and 18). On the other hand, the greater the cross-section of the armature copper the less will be the RI^2 loss.

The relation expressed by equation (24) was deduced specifically for a generator. It can readily be shown that the same relation holds for a motor.

103. Load Tests.—Load tests are made on generators primarily for the purpose of determining their voltage regulation and heating, and on motors primarily for determining their speed regulation and heating. In making a heat-run, the load should be kept on the machine until all parts of it reach a constant temperature, which will usually require 6 hours, or more. The larger the machine, the longer the interval required for it to reach a constant temperature.

The determination of the voltage regulation or speed regulation should be made immediately after the heat-run, in order that the resistances of the various windings will be at their working temperature. In a college laboratory, on account of the shortness of the usual laboratory period, voltage-regulation and speed-regulation tests are, as a rule, made with the machine at room temperature, and the results obtained are therefore not strictly representative of the performance of the machine in actual service.

The simplest way of making a load-test of a generator is to drive the machine by means of a motor, and to absorb its power output in a rheostat of proper resistance and current-carrying capacity. For small machines a wire or grid rheostat is usually employed. For larger machines a water rheostat is used. This method, however, is not usually practicable for very large machines, say of 100 kilowatts capacity or over, on account of the large amount of power required to drive the machine.

For such machines the "loading-back" method described in the next Article is usually employed. Irrespective of the manner in which the load is absorbed, the power output is readily determined by noting the terminal voltage of the machine and the current output.

Load-tests of small motors are usually made by connecting the machine to a source of supply of the proper voltage, and loading

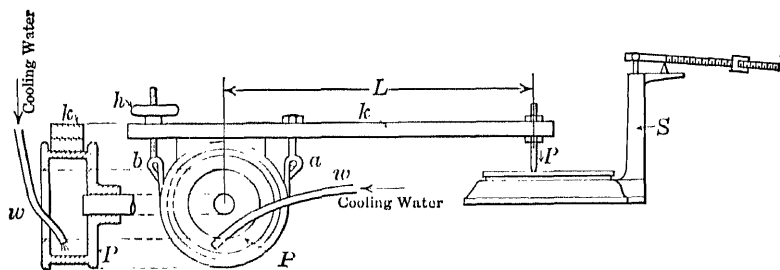



FIG. 139.—Water-cooled Prony Brake.

it by means of a Prony brake. A common form of Prony brake is shown in Fig. 139 (taken from Karapetoff's *Experimental Electrical Engineering*). A steel band *ab*, lined with wood or canvas, embraces the pulley of the motor and is fastened to the beam *k*, the free end of which rests on the scale *S*. The pressure of the brake is regulated by the hand-wheel *h*.

The product of the length L of the lever-arm of the brake, in feet, by the force F, in pounds, registered by the scale, gives the torque in pound-feet developed by the motor. If N is the speed of the motor in revolutions per minute, the horsepower output is then

$$\text{Horsepower} = \frac{2\pi FLN}{33,000} = \frac{FLN}{5252} \qquad (25)$$

Instead of a platform scale S, a spring balance may be employed.

In using a spring balance it is important that the balance always be kept perpendicular to the arm of the brake.

The power output of the motor, which is absorbed by the brake, is converted into heat by the friction between the band *ab* and the pulley. In order to prevent burning of the brake, it is usually necessary to cool it with water, as indicated in the figure.

Since the output of the motor can be determined from the reading of the balance and the speed of its armature, and the input determined from the impressed voltage and current input, as read by a voltmeter and an ammeter respectively, a brake test gives a direct method for the determination of efficiency. However, for the reasons pointed out in Article 91, this "directly measured efficiency" is usually less accurate than the efficiency calculated from the measured losses.

FIG. 140.—Brake for Testing Small Motors.

The brake test is used chiefly for determining the speed and torque characteristics of small motors. For large motors these characteristics are more conveniently determined by a "loading-back" test, as described in the next Article.

Another form of brake, suitable for small motors, is shown in Fig. 140. A belt *ab*, made of canvas, or preferably of automobile brake-lining, is attached to the hooks of two spring balances B_1 and B_2. The balances are suspended from threaded rods in a fixed support, and hand-wheels h_1 and h_2 are provided for adjusting the tension in the belt. Let F_1 and F_2 be the readings of the two balances, in pounds, and let D be the diameter of the pulley, in feet. Then the torque developed by the motor is $\frac{D}{2}(F_1-F_2)$ pound-feet. The horsepower output is then

$$\text{Horsepower}=\frac{(F_1-F_2)DN}{2626} \tag{26}$$

Instead of using a brake, it is sometimes more convenient to load a motor by belting or coupling it to a generator, and to absorb the output of the generator in a rheostat. This method of loading is used chiefly for making a heat-run or speed regulation test, in which case the output of the motor is calculated from its input and its efficiency, previously determined from its measured losses.

This last method is particularly well suited to the determination of the speed-current curve of a railway motor. Two identical motors are coupled together or geared to the same counter shaft. One motor is connected directly to the source of supply, and drives the other machine as a generator. The operation of the generator will usually be found more stable if its field is connected in series with the field motor, of the as shown in Fig. 141. When this con-

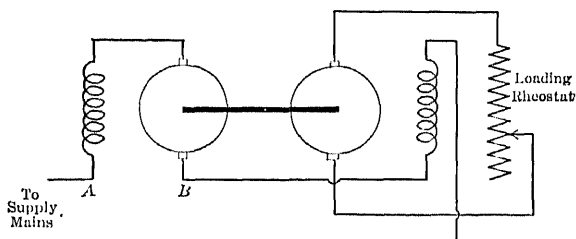



Fig. 141.—Connections for Load Test of Two Series Motors.

nection is employed, the voltage between A and B must be kept equal to the rated voltage of the motor for all values of the line current.

104. Loading-back Tests.—When a generator is loaded on a rheostat, or a motor on a brake, the entire power input to the machine is converted into heat, and is wasted. With large machines the amount of power required to make a full-load test in this way may exceed that available in the factory or testing room, to say nothing of the cost of the energy thus wasted. To overcome this difficulty, various methods of loading one machine on another have been devised, in which only the *losses* in the two machines are supplied from the external source of power.

The two machines are connected both mechanically and electrically, the connections being such that one machine acts as a motor and drives the other as a generator. The output of this generator is "fed back" to the motor, so that the line has to

supply only the difference between the input to the motor and the output of the generator, namely, the losses in the two machines. When the two machines are of the same, or approximately the same rating, and have a full-load efficiency of 90 percent or over, the power required for such a "loading-back" test is therefore less than one-fourth of that required for a brake test.

105. Kapp's Method of Loading-back.—The simplest arrangement for making a loading-back test is shown diagrammatically in Fig. 142. The two machines are connected in parallel and

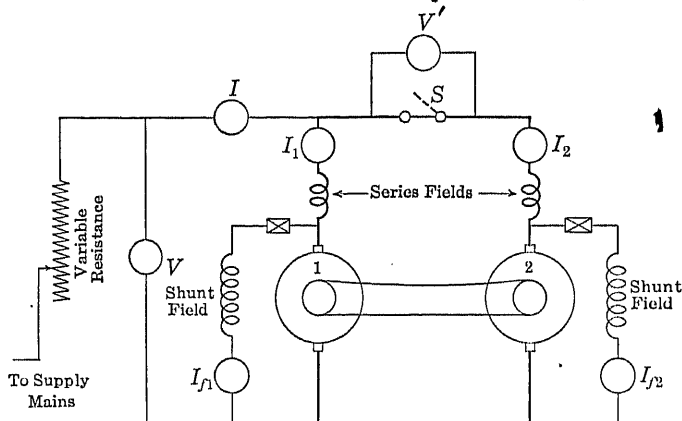



Fig. 142.—Connections for Kapp's Method of Loading-back.

belted (or otherwise mechanically coupled) together. This parallel circuit is connected to the supply mains through a variable resistance (or booster). The purpose of this variable resistance is to control the voltage which is applied to the two machines.

When the machine (No. 2) which is to operate as a motor is compound wound, its series field winding must be connected "cumulatively," i.e., so that the current in this winding tends to increase the flux per pole. If this is not done it may be found impossible to hold the load constant.

In starting up, the switch S is opened, and machine No. 1 is brought up to speed as a motor. Through the belt, No. 2 is driven as a generator. The field rheostat of No. 2 is then adjusted until its terminal voltage is equal to that impressed on No. 1, as indicated by zero reading of the voltmeter V'. The switch S is then closed.

If machine No. 1 is the one on which the test is to be made, and it is to be tested as a generator, the shunt field of No. 2 is weakened. This will reduce the electromotive force of No. 2 and will allow a circulatory current to flow in the loop formed by the two armatures and the buses. Machine No. 1 will then supply *electric* power to No. 2, and No. 2, through the belt, will supply *mechanical* power to No. 1, and the external source will supply only the losses in the two machines.

The field rheostat of the *generator* (No. 1) is adjusted so that when this machine is running at rated speed and delivering its rated armature current (preferably with all windings at their normal operating temperature), its terminal voltage is equal to the rated voltage of the machine. This rheostat is kept at this point throughout the test. The current output of the machine under test is adjusted to the desired value by manipulating the shunt-field rheostat of the motor (No. 2), and its speed is kept constant by manipulating the variable resistance in the supply circuit.

If machine No. 1 is to be tested as a *motor*, the voltage impressed on the circuit is kept constant, at the value of the rated voltage of the machine, by adjusting the variable resistance in the supply circuit. The field rheostat of this machine is adjusted so that when the machine is taking its rated full-load armature current (preferably with all windings at their normal operating temperature), its armature is running at rated speed. This rheostat is kept at this point throughout the test. The current input to the armature is controlled by manipulating the field rheostat of the other machine (No. 2).

When the two machines used in this test are identical in construction, an approximate value of the efficiency of each may be found by measuring, in addition to the terminal voltage V, the current I supplied by the line (or external source). The total losses in the two machines are then equal to VI, where V is the voltage across the *terminals of the machine*. Were the currents in the two machines equal, and were their back electromotive forces equal, the total power loss in *each* machine would then be $\frac{1}{2}VI$.

Actually, however, the back electromotive force of the machine which acts as a motor will always be less than the electromotive force of the one which acts as a generator. On the other hand,

the armature current of the motor will always be greater than the armature current of the generator, since the motor current is equal to the generator current *plus* the current supplied by the line. Consequently, the core-loss of the motor will be *less* than the core-loss of the generator, and the armature RI^2 loss of the motor will be *greater* than the armature RI^2 loss of the generator.

Since the back electromotive force of the motor is less than the electromotive force of the generator, the shunt field current of the motor is also less than that of the generator. The RI^2 loss in the shunt-field of the motor will therefore be *less* than the RI^2 loss in the shunt field of the generator.

Only as a very rough approximation, then, may the loss in each machine be taken equal to $\frac{1}{2}VI$. This method of determining efficiency is therefore not recommended, but should it be employed, the total loss for each machine $\frac{1}{2}VI$ should be taken as the loss corresponding to a load of $\frac{1}{2}(I_1+I_2)$ amperes; see Fig. 142.

106. Hopkinson's Method of Loading-back.—In the loading-back test described in the preceding Article, the losses in the two machines are supplied *electrically* (Kapp's method). It is also possible to supply the losses *mechanically* (Hopkinson's method) by means of an auxiliary motor, as shown diagrammatically in Fig. 143. The two machines are mechanically coupled as before, are brought up to speed by means of the auxiliary motor, and thrown into parallel by closing the switch S. By weakening the shunt field of one machine, or increasing the shunt field of the other, the desired value of the current may be caused to circulate in the two armatures. As in Kapp's method, one machine will then act as a generator and the other as a motor.

When the machine which is to operate as a motor is compound wound, its series field must be connected cumulatively, as in Kapp's method (see Article 105).

If it is desired to test machine No. 1 as a generator, its speed is held constant at its rated value by manipulating the rheostat (not shown in the figure) of the auxiliary motor. The field rheostat of the machine under test is set to give rated voltage at rated load. The load on this machine is then adjusted by manipulating the field rheostat of No. 2.

If it is desired to test machine No. 1 as a motor, its field rheostat is permanently set at the point which gives rated speed at rated

load. The voltage across the terminals of this machine is kept constant at its rated value. This can be done, for any value of the current input, by manipulating simultaneously the field rheostat of the auxiliary motor and the field rheostat of machine No. 2. The speed at which machine No. 1 operates will then be the speed at which it will normally operate at this same impressed voltage and current input.

When the losses are supplied mechanically as shown in Fig. 143, the current input to the one which acts as a motor is equal to the current output of the one which acts as a generator. When the two machines are identical, the armature RI^2 loss is therefore

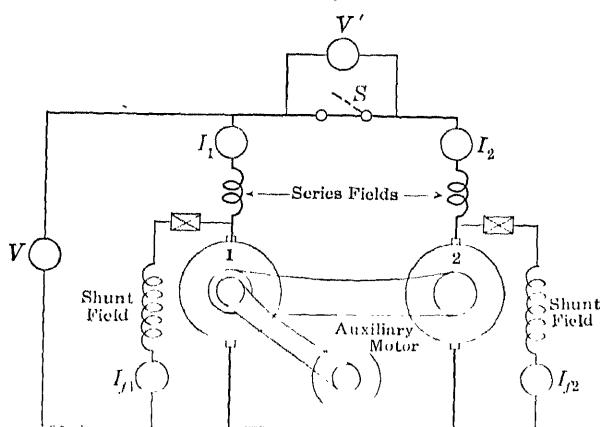



FIG. 143.—Connections for Hopkinson's Method of Loading-back.

approximately the same in each. However, since the back electromotive force of the motor is less than the electromotive force of the generator, the core-loss and shunt-field RI^2 loss in the former are less than they are in the latter.

Consequently, although the output of the auxiliary motor is equal to the total losses in *both* machines, it is only a rough approximation to take *one-half* of this output as the loss in *each* machine. Nevertheless, this method, though not recommended, is sometimes employed for determining the efficiency of a machine, when there is a second identical machine available.

Both Kapp's and Hopkinson's methods of loading are entirely satisfactory for regulation tests and heat runs, and are commonly

used for this purpose on the testing floors of the manufacturing companies.

107. Loading-back Test of Series Motors.—When the two machines used in a loading back test have shunt-field windings, their armature electromotive forces can be varied independently, and consequently a current of any desired value can be caused to circulate in their armatures. The electromotive force which produces this circulatory current is the *difference* of the armature electromotive forces of the two machines.

When the two machines have *series windings only*, their field excitations cannot be varied independently. Therefore, some external source of electromotive force must be provided in order to set up the required circulatory current. A convenient source of electromotive force for this purpose is a series booster, the armature of which is inserted in one of the buses which connect the two machines, as shown in Fig. 144. The current through each machine is then adjusted to any desired value by manipulating the booster field rheostat. The booster current must be established in the proper direction, or otherwise the motors will not "lock."

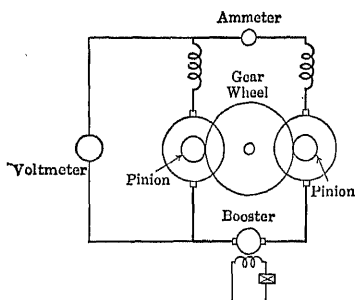



Fig. 144.—Connections for Loading-back Test of Railway Motors.

This arrangement is often used in testing a pair of railway motors. As the armature shaft of railway motors is usually provided with a pinion, the two machines are mechanically coupled by gearing them to the same counter-shaft, which shaft is driven by a small auxiliary motor (not shown in the figure). This motor supplies the core-loss and friction of the two machines under test, and the booster supplies the RI^2 losses.

The core-loss and friction may also be supplied electrically, by connecting the two motors (in parallel) to an auxiliary source of supply at the proper voltage.

Various other modifications of the loading-back test are sometimes employed; see Karapetoff's *Experimental Electrical Engineering*.

108. Experimental Determination of Voltage Regulation of Generators.—Connect the machine for a load test, employing that one of the methods described in the preceding Articles best adapted for the particular machine in hand. Drive the armature at rated speed, which must be held constant throughout the test, (or adjusted to the speed-load characteristic of the engine or driving motor, if specified). Adjust the shunt-field rheostat so that when the machine is carrying its rated load it develops rated terminal voltage,* and keep it set at this point throughout the test. In the case of the compound generator the series-field shunt (see Article 64) should also be set to give the proper degree of compounding.

If desired for some special purpose, the speed and the shunt-field rheostat may of course be given any other setting.

Measure the current output and terminal voltage of the generator for at least seven loads, preferably 0, 25, 50, 75, 100, 125 and 150 percent of rated load. In addition, record the speed and the shunt-field current at each load. Plot terminal voltage as ordinates against current output (line current) as abscissas. (See Fig. 81 for example.)

109. Experimental Determination of Speed Regulation of Motors.—Connect the machine for a load test, employing that one of the methods described in the previous Articles best adapted for the machine in hand. Keep rated voltage on the motor terminals throughout the test.* In the case of a shunt motor, set the field rheostat at that point which will give rated speed of the machine when it is delivering its rated load, and do not alter this setting after the proper point has been determined.

If desired for some special purpose, the impressed voltage and field rheostat may of course be given any other setting. For example, in tests at the factory, the speed curve is usuall taken at full field current (field rheostat cut out entirely).

Measure the current input to the motor and its speed for at least seven loads, preferably 0, 25, 50, 75, 100, 125 and 150 percent of the current input corresponding to rated load. In addition, record the impressed voltage and shunt-field current for each

* The machine should preferably be at the temperature corresponding to continuous operation at rated load. However, the difference in the "hot" and "cold" regulation of a machine is usually small.

load. Plot speed as ordinates against current input to the motor as abscissas.

When the load is applied by a brake, measure and record the torque for each value of the current input. Calculate the horsepower output from the torque and speed, and plot both torque and horsepower output as ordinates on the same sheet as the speed-current curve. (See Fig. 138 for example.)

110. Object of Heat-run.—Permissible Temperature Rises.—The object of a heating test, or heat-run, is to determine whether the temperatures of the various parts of the machine, when it is operating continuously at its rated load, are below the maximum temperature at which the insulation of the machine may be safely operated.

Since the hottest part of each winding and core is usually its interior, it is impossible, by any simple test, to determine the maximum, or hot-spot, temperature of each part. To determine the temperature of the surface of the various parts, ordinary glass-mercury thermometers are employed, as explained in Article 89. In the Standards of the American Institute of Electrical Engineers this method of temperature measurement is referred to as **Method 1.**

The average temperature rise of a copper winding may be conveniently determined by measuring its cold resistance and its hot resistance, as explained in Article 94. In the A.I.E.E. Standards this method of temperature measurement is referred to as **Method 2.**

In modern large machines it is becoming the practice to build into the windings small thermocouples, located as near as possible to the spots which are likely to reach the highest temperature. The leads from these couples are brought out to suitable terminals, to which a millivoltmeter may be connected. The temperature rise of the winding may then be determined from the reading of the millivoltmeter. In the A.I.E.E. Standards this method is referred to as **Method 3.**

In the A.I.E.E. Standards are given explicit rules in regard to what shall be considered as the safe operating temperature for various kinds of insulation; see Article 31. Of course, the actual temperature to which any given part of a machine will rise, when a given load is kept on it continuously, will depend upon the

temperature of the surrounding air. The temperature of the surrounding air is usually referred to as the **ambient temperature.** Since many machines may, at least during the summer months, be required to operate in an ambient temperature as high as 40° C. (104° F.), the Institute has adopted 40° C. as the "standard ambient temperature of reference" for all air-cooled machines.

On the basis of (1) this standard ambient temperature, (2) the permissible hot-spot temperatures (see Article 31), and (3) a reasonable allowance for the difference between the hot-spot temperature and the observable temperature, the Institute recommends that all d-c. generators and motors, with the exception of railway motors, be so rated that, when rated load is kept continuously on the machine, the *difference* between the temperature of its various parts and the *actual* ambient temperature shall not exceed the figures given in Table IV.*

A machine may be tested at any ambient temperature, preferably not less than 10° C. When the ambient temperature is less than 40° C., the temperature of the hottest spot will, as a rule, be proportionally less. However, since the machine may at some time be operated in an ambient temperature of 40° C., the temperature *rise*, irrespective of the ambient temperature during the test, must not exceed the values given in the table.

A **railway motor** is usually rated in terms of the mechanical load which it can supply continuously for *one hour* with a specified temperature rise. The output is usually stated in horsepower, although the A.I.E.E. recommends that it be stated in kilowatts. The A.I.E.E. definition of the one-hour, or "nominal," rating of a railway motor is as follows:

"The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90° C. at the commutator, and 75° C. at any other normally accessible part after one hour's continuous run at its rated voltage (and frequency in the case of an alternating-current motor) on a stand with the motor covers arranged to secure maximum ventilation without external blower,

* Machines designed for intermittent service in which the load is applied for short intervals only are sometimes given a special "short-time" rating; see the Standards of the A.I.E.E.

TABLE IV.—LIMITING OBSERVABLE TEMPERATURE RISES

Method by which Temperature is Measured	Parts of the Machine	Temperature Rise	
		For Class A* Insulation	For Class B † Insulation
1	All	50° C.	70° C.
2‡	All	55° C.	75° C.
3	For windings with two coil-sides per slot, with detectors between top and bottom coil-sides and between coil-sides and core.	60° C.	80° C.
	For windings with one coil-side per slot with detectors between coil-side and core and between coil-side and wedge.	55° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts).	75° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts).

* Class A insulation includes cotton, silk, paper and similar materials when so treated or impregnated as to increase the thermal limit, or when permanently immersed in oil; also enameled wire. When these materials are not treated, impregnated, or immersed in oil, they are not included in Class A.

† Class B insulation includes mica, asbestos and other materials capable of resisting high temperatures, in which any Class A material or binder is used for structural purposes only, and may be destroyed without impairing the insulation or mechanical qualities of the insulation. (The word "impair" is used in the sense of causing any change which could disqualify the insulation for continuous service.)

‡ In the application of Method 2, *thermometer measurements shall also be made whenever practicable without disassembling the machine,* in order to increase the probability of obtaining the highest observable temperature. The measurement indicating the higher temperature shall be taken as the "observable" temperature.

The rise in temperature, as measured by resistance, shall not exceed 100° C. The statement of the nominal rating shall include the corresponding voltage and armature-speed."

111. Procedure in Making Heat-run.—To make a heat-run on a generator or motor, connect the machine for a load test, employing that one of the methods described in Articles 103 to 107 which is best adapted for the machine in hand. Thermometers should

be attached, preferably by felt pads (see Article 89) to all the stationary parts whose temperature is to be noted. Additional thermometers and pads should be available for determining the temperature of the rotating parts when the machine is shut down.

The machine should be shielded from currents of air from open windows, doors, and adjacent pulleys or belts. To measure the ambient temperature, several thermometers, with their bulbs immersed in oil cups (see Article 89), should be used. These thermometers are placed around and half way up the machine at a distance of from 3 to 6 feet from it. The value to be adopted for the ambient temperature during a test, is the mean of the readings of these thermometers, taken at equal intervals of time during the last quarter of the duration of the test.

Before applying load, measure the cold resistance and temperature of all windings. Special precautions must be taken in measuring the armature resistance; see next Article.

If the test is begun with the machine cold, and rated load is applied, it will usually take from five to ten hours, or more, for its parts to come to a steady temperature. When the time for the test is very limited, the machine may be overloaded for the first hour or two, and the load then reduced to its rated value.

The test with rated load should be continued until at least six successive readings of the thermometers on the stationary parts, taken at ten-minute intervals, show no change of temperature. The machine is then shut down, and thermometers applied as quickly as possible to the rotatable parts. Readings of all thermometers should then be taken every two minutes until they indicate definitely that the machine has begun to cool off.

Should the first two readings of any one of the thermometers indicate a falling temperature, several successive readings of this theremometer should be taken and a cooling curve (temperature against time) plotted. By extending this curve back to the instant of shut-down, a close approximation to the actual temperature at this instant can be obtained.

At the same time that the temperature readings are being taken after shut down, measure the hot resistance of all the windings.

Calculate the temperature rise of each part of the machine from (1) the ambient temperature and the temperatures observed dur-

ing the last hour of the heat-run (this applies to the stationary parts only), (2) from the ambient temperature and the thermometer readings immediately after shut-down, and (3) from the hot and cold resistance measurements (this of course applies to the windings only). As the temperature rise of any given part of the machine is taken the *maximum value thus found.*

It is particularly important that during the last part of a heat-run, both the current and terminal voltage of the machine be held constant at their rated values. In the case of a generator, the speed also must be held constant at its rated value. In the case of a motor, the speed will take the value inherent to the design of the machine. Its value should be noted from time to time during the test.

112. Determination of Rise of Temperature of Armature from its Cold and Hot Resistances.—The hot and cold resistance of an armature can usually be determined with sufficient accuracy by the method described in Article 93, care being taken to have the armature in exactly the same position, relative to the brushes, in both tests. *The brush contact resistance should not be included.* This method, of course, assumes that the distribution of the test current in the several parallel paths through the armature will be the same in both tests, which will be true only in case the relative values of the brush-contact resistances at the several brush sets do not change. When there is any question as to this condition being fulfilled, the following procedure may be employed:

Mark * two commutator segments which are a pole-pitch apart, and provide two auxiliary brushes having a thickness less than the width of a commutator segment. To make the resistance measurement, insulate all the main brushes from the commutator, by inserting paper under them, place the auxiliary brushes in contact with the two marked segments, and send the test current through the armature by way of the auxiliary brushes. Measure the drop of potential between the two segments, by placing the voltmeter leads on these segments (not on the brushes). This potential difference divided by the test current gives the value of the armature resistance under the conditions of the test.

* A convenient way of marking a commutator segment is to put one or more punch marks in the *end* of the segment; not in its working surface.

In the case of a multipolar machine this resistance is not the normal armature resistance, i.e., is not the resistance of the several paths through the armature in parallel. This is readily seen by considering a 4-pole simplex lap winding. With respect to two brushes placed a pole pitch (180 electrical degrees) apart, such a winding forms two parallel paths, one path containing one-fourth of the armature-conductors and the other path the remaining three-fourths of these conductors. The resistance of the armature between these two brushes, when all the other brushes are insulated from the commutator, is therefore greater than the normal armature resistance.

For the purpose of determining the temperature rise, however, it is immaterial what combination of paths is used, provided the resistance of this *same* combination is determined at the two temperatures.

113. Experimental Determination of the Saturation Curve.— The usual method of determining by test the saturation curve of a dynamo has already been described; see Article 35.

The machine is operated at rated speed as a separately excited generator, and readings of the field current and terminal voltage of the armature are taken for several values of the field current, from zero up to the value which will give a terminal voltage at least 25 per cent in excess of the rated voltage of the machine. The field current is then decreased, in steps, from this maximum value to zero, and the armature voltage corresponding to these successively smaller field currents noted. The armature voltage plotted against field current gives the saturation curve. A typical ascending and descending saturation curve is shown in Fig. 38.

This test is usually made at the same time as the belted core-loss test (see Article 98), since the electrical connections and general set-up are the same for both tests.

When it is inconvenient, or impossible, to attach a pulley to the shaft of the machine, its saturation curve may be obtained by driving it without load as a separately excited motor. The voltage impressed on the armature is increased in steps from the lowest value which can be used (about 50 percent of rated voltage) up to at least 125 percent rated voltage, and then back again to its initial value. For each value of the impressed voltage the speed is adjusted to its rated value, by manipulating the rheostat in

the field circuit. Readings of field-current, armature terminal voltage, armature current and speed are taken at each step. Neglecting the armature resistance drop and armature reaction due to the small current taken by the armature, the curve showing the relation between the armature terminal voltage and field current is the saturation curve of the machine.

This test is usually made at the same time as the running-light test for core-loss and friction (see Article 96), since the electrical connections and the general set-up are the same for both test.

114. Location of Open-circuits, Short-circuits, and Grounds.—When a defect develops in any of the electric circuits of a machine, it is usually due to an open-circuit, short-circuit, or "ground." By a "ground" is meant a break in the insulation which results in a conductive connection between a conductor and the frame of the machine.

An open-circuit in a single-circuit winding is readily detected by impressing a potential difference across its terminals. If no current flows, the circuit is open. A short-circuit, on the other hand, is indicated by an abnormally low resistance.

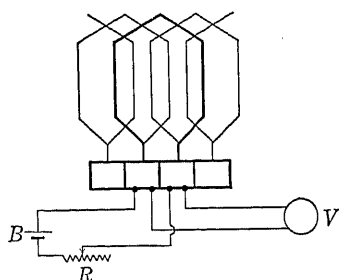



FIG. 145.—Test for Open-circuit or Short-circuit in Armature.

An open-circuit or short-circuit in the armature is readily located by a "bar-to-bar test." In this test the resistance between adjacent commutator bars is measured by the drop of potential method, as indicated in Fig. 145. A storage battery may be conveniently used as the source of current, and a millivoltmeter or galvanometer may be to determine the drop of potential. The leads are moved from bar to bar, the current being kept constant by adjusting the resistance R, when necessary. The potential drop between each pair of bars is noted. Should the potential drop between any pair of bars show a value *higher* than the rest, the armature coil which lies between these bars is *open-circuited.* Should the potential drop between any pair of bars show a value *lower* than the rest, the armature coil which lies between these two bars is *short-circuited.*

A ground between a winding and the frame of the machine is indicated by a flow of current, when a difference of potential is applied between this winding and the frame. In the case of a grounded armature, the particular coil which is grounded may be readily located by a bar-to-bar test with the millivoltmeter V (or galvanometer) connected between bar and frame of the machine, as shown in Fig. 146. A current of the proper value to give a readable deflection of the millivoltmeter is passed through the armature winding as indicated in the figure. This current is kept constant throughout the test, by adjusting the variable resistance R, when necessary. The millivoltmeter leads and battery leads are passed from bar to bar, until the pair of bars, say A and B, is located between which a relatively large deflection of the voltmeter is obtained, and then when the leads are moved one segment further, say to B and C, zero (or practically zero) deflection is obtained. The armature coil which lies between the bars A and B is then the one which is short-circuited.

FIG. 146.—Test for Ground in Armature.

PROBLEMS

1. The shunt motor whose saturation curve at 1000 r.p.m. as shown in Fig. 96 is initially at the temperature (20° C.) of the surrounding air. The resistance of its field circuit at this temperature is 25 ohms.

(*a*) What will be the no-load speed of this motor immediately after it is connected to 125-volt supply mains (i.e., before its field circuit has had time to heat up)?

(*b*) The setting of the field rheostat is left unaltered, and full load is kept on the motor for several hours. When this load is thrown off, it is found that the no-load speed is 5 per cent higher than it was initially. Explain.

(*c*) Assuming the field rheostat to have the same temperature coefficient as that of the field winding (copper), what is the average temperature rise of the field winding?

2. The full-load current of the 125-volt motor considered in the numerical example in Article 74 is 100 amperes. The friction loss in this motor when operation at rated load and rated voltage is 3 per cent of the power input.

Assuming the armature demagnetizing factor to be constant, what voltage

must be impressed on the terminals of this motor to make it run, without mechanical load, at a speed equal to its normal full-load speed?

3. The following additional data pertain to the 50-H.P., 500-volt railway motor whose friction-loss curve is given in Fig. 133, and whose no-load core-loss curves are given in Fig. 134:

Armature resistance (exclusive of brush contacts) at 21° C. . . . . 0.110 ohm
Field resistance at 21° C. . . . . 0.075 ohm

Current-speed Characteristic

Amperes.	30	40	60	90	150	200
R.P.M. at 500 volts	1100	870	720	600	500	450

(*a*) Assuming, as a preliminary figure, an efficiency of 90 per cent, what is the rated current input of this motor?

(*b*) Calculate and plot the conventional efficiency of this motor at rated voltage, following the A. I. E. E. rules in all particulars. Make calculations for current inputs of 25, 50, 75, 100, 150 and 200 per cent of the current found in (*a*).

(*c*) Plot on the same curve-sheet the speed-current, torque-current and horse-power-current characteristics of this motor.

(*d*) What is the exact value of the rated current input of this motor.

4. (*a*) Calculate and plot the conventional efficiency of the railway motor in Problem 3 for the same values of the current input as in Problem 3 but for an impressed voltage of 600 instead of 500 volts.

(*b*) Plot a curve showing the increase in the total losses against current input as abscissas, when this motor is operated at 600 volts instead of 500 volts.

(*c*) For the same current input in each case, will the torque developed by the motor at 600 volts be greater or less than at 500 volts?

(*d*) If the opposing torque which this motor has to overcome remains the same, how will the speed at 600 volts differ from the speed at 500 volts?

(*e*) Compare the power outputs of the motor at 500 and 600 volts, the opposing torque being the same in both cases.

(*f*) Compare the temperature rise of the motor at 500 and 600 volts respectively, the opposing torque being the same in both cases.

5. A retardation test was made on the compound generator whose core-loss and friction curves are shown in Fig. 136. The brushes were held off the commutator during this test and the shunt-field circuit was kept open. It was found that six seconds were required for the armature to slow down from 610 to 590 r.p.m.

(*a*) What is the value, for this machine, of the constant A in equation (16), Article 99?

(*b*) When the brushes were put down and the shunt-field circuit left open, how long would it take this armature to slow down from 610 to 590 r.p.m.?

(c) When the brushes are put down and the shunt field excited to give 525 volts at rated speed, how long will it take for the armature to slow down from 610 to 590 r.p.m.?

6. From test of a 5-kw., 125-volt, 1200 r.p.m. shunt generator, the following data were obtained:

No-load field current at rated speed and voltage.............. 1.3 amperes

Armature resistance (exclusive of brushes and brush contacts).. 0.119 ohm

Core-loss and Friction at Rated Speed.

Arm. term. volts...	30	60	90	120	150	180
Loss, in watts.....	50	66	85	108	139	175

(*a*) Calculate and plot the conventional efficiency of this generator for 25, 50, 75, 100, 150 and 200 per cent load.

(*b*) Assuming the combined core-loss and friction to be constant at its no-load value, and the brush-contact resistance to be constant at the value corresponding to rated load, what would be the maximum efficiency of this generator, and at what load would it occur? Compare with the curve plotted in (*a*).

7. (*a*) Calculate and plot the efficiency of the dynamo described in Problem (6) when this machine is operated as a motor on a 125-volt circuit. The resistance of the shunt-field circuit is kept unaltered. Make calculations for the same values of the line current as in Problem 6. (For the purpose of this problem, the speed of the motor is to be assumed constant at all loads.)

(*b*) Under the conditions stated in (*a*) what will be the speed?

(8) Two shunt motors, each of which is identical with the motor described in the numerical example in Article 75, are connected for a loading-back test by the Kapp method. The combined core-loss and friction of each of these motors, at 1000 r.p.m., is as follows:

Armature E.M.F	50	75	100	125	150
Loss, in watts..	.....	...	......	600	

It is desired to load machine No. 1 as a motor, at 125 volts, 1000 r.p.m., and current input of 100 amperes. Determine.

(*a*) The current output of machine No. 2.

(*b*) The total losses in each machine, and the sum of these losses.

(*c*) The current and power supplied from the external source.

(*d*) The efficiency of each machine, by calculation from its separate losses.

(*e*) The efficiency of each machine on the assumption that the total losses in each are equal to one-half the power supplied from the external source.

9. Make all the calculations called for in Problem 8, when the two motors are connected for a loading back test by the Hopkinson method.

10. Two railway motors, identical in all respects with the motor considered in Problem 3, are connected, as shown in Fig. 144, for a loading-back test.

(*a*) Calculate the terminal voltage and power output of the booster required to circulate its rated current through each motor.

(*b*) Will the load on the booster depend upon the speed at which the two motors are driven? Explain.

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